Reaction-Diffusion Equations with Spatially Distributed Hysteresis in Higher Spatial Dimensions

Mark Curran, Free University of Berlin
Email: curran@zedat.fu-berlin.de

In this talk, I will treat the equation

$$u_t = \Delta u + f(u, \mathcal{H}(u)), \tag{1}$$

where u represents a diffusing substance and $\mathcal{H}(u)$ is a hysteresis operator defined at every spatial point. Such equations model processes where the non-diffusing substance $\mathcal{H}(u)$ can be in one of two states, and the switching mechanism between states is determined by a hysteresis law. These equations model a variety of biological and chemical processes that exhibit spatio-temporal patterns [F. Hoppensteadt, W. Jäger and C. Pöppe, 1980-83; A. Marciniak-Czochra 2006].

Numerical simulations of such models are in agreement with experiment, however questions of the existence and uniqueness of solutions, as well as a rigorous explanation of the mechanisms of pattern formation remain largely open. Well-posedness has only recently been addressed on one-dimensional domains [P. Gurevich, R.Shamin and S. Tikhomirov 2012-14].

The set of points where $\mathcal{H}(u)$ is in one state or the other naturally segregates the domain into two subdomains. Moreover, a switching mechanism implies that these subdomains are separated by free boundaries.

I will consider (1) on a higher dimensional domain and present conditions on the initial data that guarantee the existence and uniqueness of solutions. I will also give a description of how the hysteresis gives rise to a novel type of free boundary evolution.