Mathematical analysis of the collective motion of camphor disks

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Billiard and jamming like motions of camphor disks placed over water have recently been observed in an annular water channel. We investigate the mechanisms of these motions by constructing a mathematical model for the camphor system. In particular, we study the motion of two camphor disks by means of numerical simulation and mathematical analysis. As a result of our investigations, we have uncovered various morphologies of the camphor disk motions. Moreover, we were able to obtain the existence and stability of rotational and cluster motions numerically, by means of computer aided analysis. We have found that the billiard-like phenomenon is caused by the coexistence of uniform rotational and symmetrical oscillatory motions, and that the jamming phenomenon arises from a Hopf bifurcation of the uniform rotating motion.