The spectrum of the torus profile to a geometric variational problem with long range interaction

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The profile problem for the Ohta-Kawasaki diblock copolymer theory is a geometric variational problem. The energy functional is defined on sets in $\mathbb{R}^3$ of prescribed volume and the energy of an admissible set is its perimeter plus a long range interaction term related to the Newtonian potential of the set. This problem admits a solution, called a torus profile, that is a set enclosed by an approximate torus of the major radius 1 and the minor radius q. The torus profile is both axially symmetric about the z axis and reflexively symmetric about the xy-plane. There is a way to set up the profile problem in a function space as a partial differential-integro equation. The linearized operator $\mathcal{L}$ of the problem at the torus profile is decomposed into a family of linear ordinary differential-integro operators $\mathcal{L}^m$ where the index $m = 0, 1, 2, ...$ is called a mode. The spectrum of $\mathcal{L}$ is the union of the spectra of the $\mathcal{L}^m$'s. It is proved that for each m, when q is sufficiently small, $\mathcal{L}^m$ is positive definite. (0 is an eigenvalue for both $\mathcal{L}^0$ and $\mathcal{L}^1$, due to the translation and rotation invariance.) As q tends to 0, more and more $\mathcal{L}^m$'s become positive definite. However no matter how small q is, there is always a mode m for which $\mathcal{L}^m$ has a negative eigenvalue. This mode grows to infinity like $q^{-3/4}$ as $q \to 0$.

This is joint work with Juncheng Wei.