Viscosity solutions to singular parabolic problems

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A prototype problem we study is

$$u_t = (\operatorname{sgn} u_x)_x, \quad u(x,0) = u_0(x), \qquad x \in (0,L), \quad t > 0,$$

augmented with boundary conditions. This equation may be equivalently written as

$$u_t = \left(\frac{dW}{dp(u_x)}\right)_x, \quad u(x,0) = u_0(x), \tag{1}$$

where $W(p) = |p|$. Here, an easy generalization is to consider a piecewise linear and convex function W. We define the notion of viscosity solutions to equations like (1). We show that weak solution to (1), where $u_0 \in BV(0,L)$, are in fact viscosity solutions. We recall that a comparison principle holds for viscosity solutions. We show that this tool is very useful to deduce properties of solution what we will illustrate by examples.

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