Patterns generated by a nondiffusive activator and a diffusive inhibitor

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Turing observed that interaction of two chemicals with different diffusivities can destabilize a steady state which is stable againt spatially uniform disturbances and considered this could be viewed as the onset of spontaneous formation of patterns in developmental processes. Twenty years after Turing's paper, Gierer and Meinhardt found that interaction between a slowly diffusing activator and a rapidly diffusing inhibitor can generate spatial patterns. Here, an activator promote both of its own production rate and that of the inhibitor, whereas an inhibitor suppresses the production of the activator.

In this talk we consider what happens if the activator does not diffuse. This may be viewed as a limit of "a slowly diffusing activator and a rapidly diffusing inhibitor". We take the FitzHugh-Nagumo system as an example and construct a family of continuous stationary solutions by applying the bifurcation theory and a family of stationary solutions with jump discontinuity. It turns out that all bifurcating solutions are very unstable in the sense that the linearized operators have infinitely many positive (hence unstable) eigenvalues. On the other hand, all stationary solutions with jump discontinuity are stable in an appropriate sense.

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