AN INVITATION TO C*-ALGEBRAS

The course will be an introduction to the main concepts in C* algebra theory. Time permitting, we will also look at some more contemporary aspects of research in the classification programme for C*-algebras. The following topics will be covered:

Syllabus

Introductory material: A bit of historical background and contemporary context. Introduction of Banach algebras, spectrum of an operator, non-emptiness of spectrum, Gelfand–Mazur Theorem. Spectral radius, characters, Gelfand representation for Banach algebras. Definition of a C*-algebra, some examples. The Gelfand theorem for C*-algebras, the functional calculus. Positive elements, ideals, approximate units, hereditary subalgebras.

Representations: Positive linear functionals, representations and the Gelfand–Naimark–Segal (GNS) construction. Irreducible representations and pure states, Kadison's transitivity theorem. Primitive ideals, hull-kernel topology.

Further constructions and examples: Inductive limits, tensor products (in particular, constructions for the max and min tensor products). Nuclearity and the completely postiive approximation property (CPAP). Uniformly hyperfinite (UHF) algebras, transformation groups, universal C*-algebras, Cuntz algebras, group C*-algebras, etc.

Brief introduction to K-theory and classification: K_0 and Elliott's classification of approximately finite (AF) algebras. K_1, overview of classification programme. The nuclear dimension and the Jiang–Su algebra.

References

[1] Davidson, Kenneth R., *C*-algebras by Example*, Fields Institute Monographs, Amer. Math. Soc., Providence, R.I., 1996.

[2] Lin, Huaxin, *An introduction to the classification of amenable C*-algebras*, World Scientific Publishing Co. Inc., 2001.

[3] G. J. Murphy, *C*-algebras and operator theory*, Academic Press, London, 1990.

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