

## AN INVITATION TO $C^*$ -ALGEBRAS

The course will be an introduction to the main concepts in  $C^*$  algebra theory. Time permitting, we will also look at some more contemporary aspects of research in the classification programme for  $C^*$ -algebras. The following topics will be covered:

### SYLLABUS

*Introductory material:* A bit of historical background and contemporary context. Introduction of Banach algebras, spectrum of an operator, non-emptiness of spectrum, Gelfand–Mazur Theorem. Spectral radius, characters, Gelfand representation for Banach algebras. Definition of a  $C^*$ -algebra, some examples. The Gelfand theorem for  $C^*$ -algebras, the functional calculus. Positive elements, ideals, approximate units, hereditary subalgebras.

*Representations:* Positive linear functionals, representations and the Gelfand–Naimark–Segal (GNS) construction. Irreducible representations and pure states, Kadison’s transitivity theorem. Primitive ideals, hull-kernel topology.

*Further constructions and examples:* Inductive limits, tensor products (in particular, constructions for the max and min tensor products). Nuclearity and the completely positive approximation property (CPAP). Uniformly hyperfinite (UHF) algebras, transformation groups, universal  $C^*$ -algebras, Cuntz algebras, group  $C^*$ -algebras, etc.

*Brief introduction to  $K$ -theory and classification:*  $K_0$  and Elliott’s classification of approximately finite (AF) algebras.  $K_1$ , overview of classification programme. The nuclear dimension and the Jiang–Su algebra.

### REFERENCES

- [1] Davidson, Kenneth R.,  *$C^*$ -algebras by Example*, Fields Institute Monographs, Amer. Math. Soc., Providence, R.I., 1996.
- [2] Lin, Huaxin, *An introduction to the classification of amenable  $C^*$ -algebras*, World Scientific Publishing Co. Inc., 2001.
- [3] G. J. Murphy,  *$C^*$ -algebras and operator theory*, Academic Press, London, 1990.