Jakub Gizbert-Studnicki

in collaboration with

Jan Ambjørn, Daniel Coumbe, Andrzej Görlich and Jerzy Jurkiewicz

Phase structure of Causal Dynamical Triangulations model in 4D

Singularities of general relativity and their quantum fate

29th June 2016

UNIWERSYTET JAGIELLOŃSKI

NATIONAL SCIENCE CENTRE

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- *Gravity as a QFT is perturbatively non-renormalizable in d > 2 dimensions*
- *But it could be renormalizable in a non-perturbative regime*
  - *assymptotic safety idea (S. Weinberg)*
  - *renormalization group flow can lead to a non-Gaussian UV fixed point (M. Reuter et al.)*
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  - *UV fixed point should be associated with a 2nd order phase transition*
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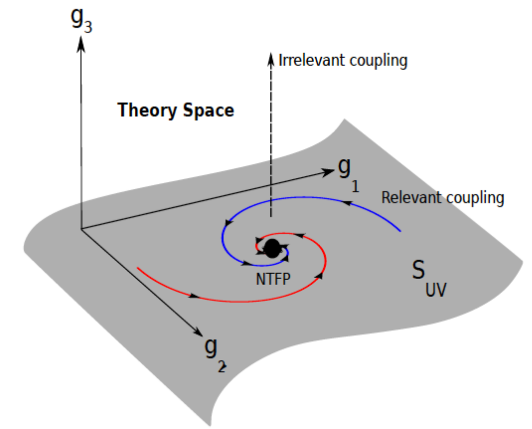

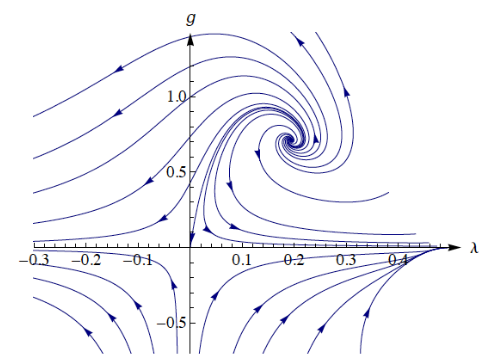

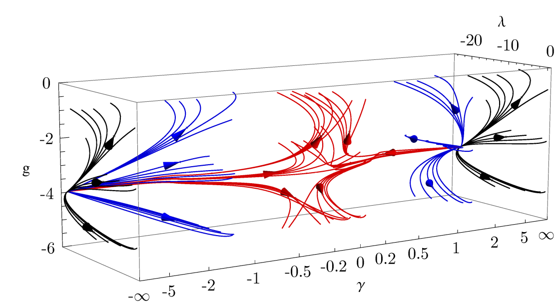

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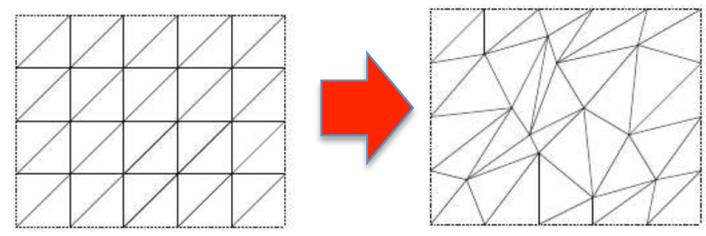

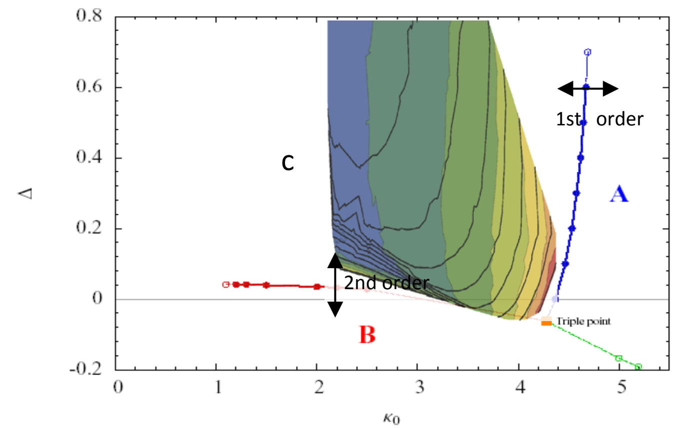

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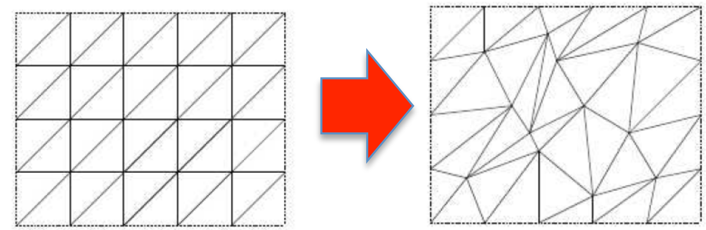

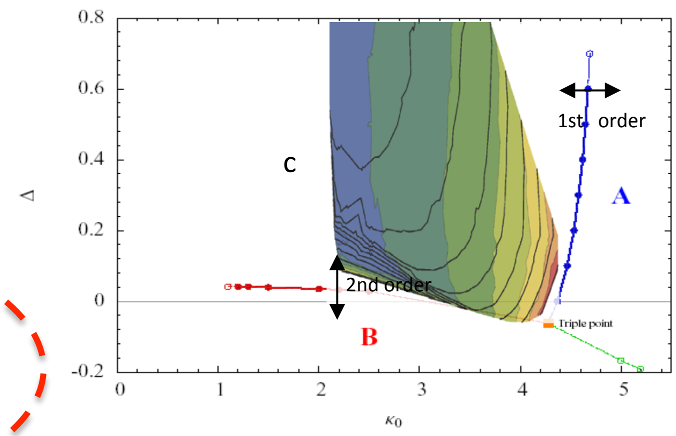

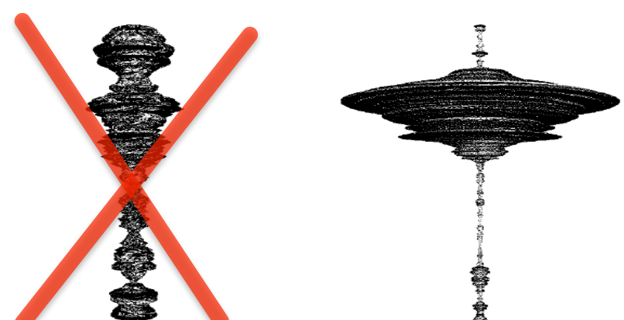

Outline

- ✧*CDT*
- ✧*Phase structure*
- ✧*Phase transitions*
- ✧*Transfer matrix method*
- ✧*Phase structure revisited*
- ✧*Phase transitions revisited*
- ✧*Signature change*
- ✧*Conclusions*

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✧ *Causal Dynamical Triangulations (CDT) is a Quantum Gravity model based on the path integral*

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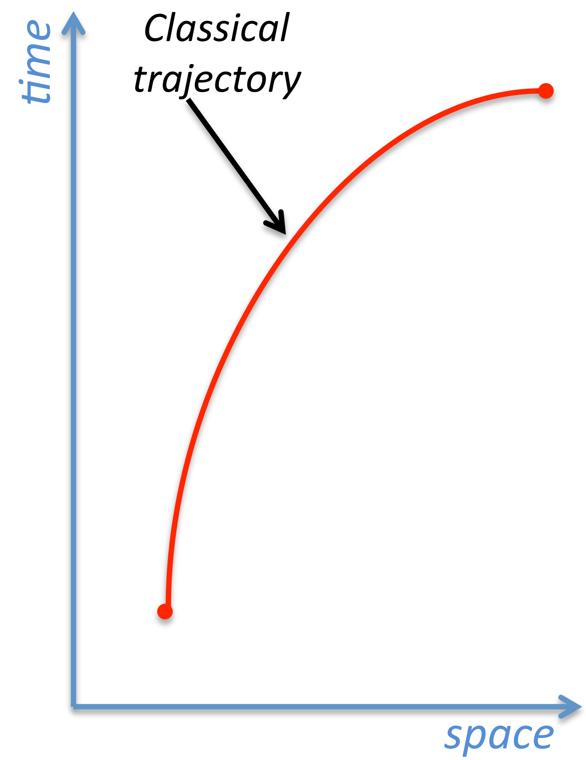

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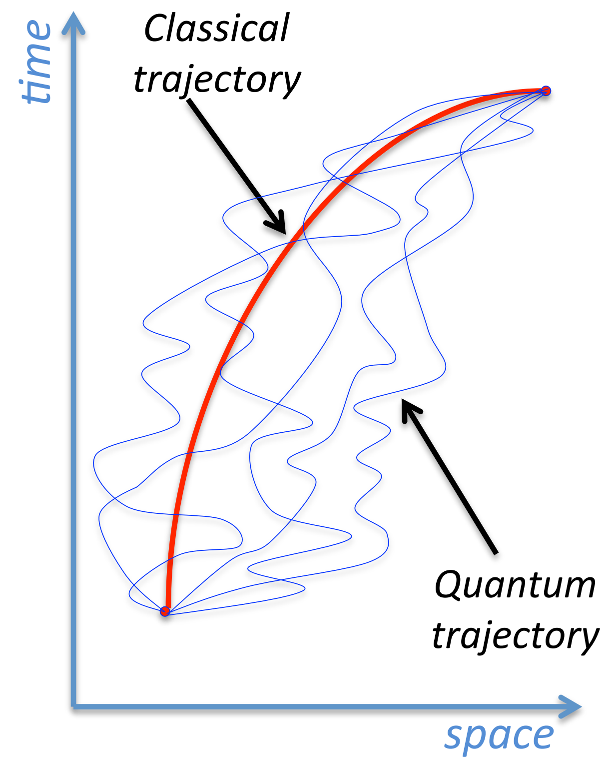

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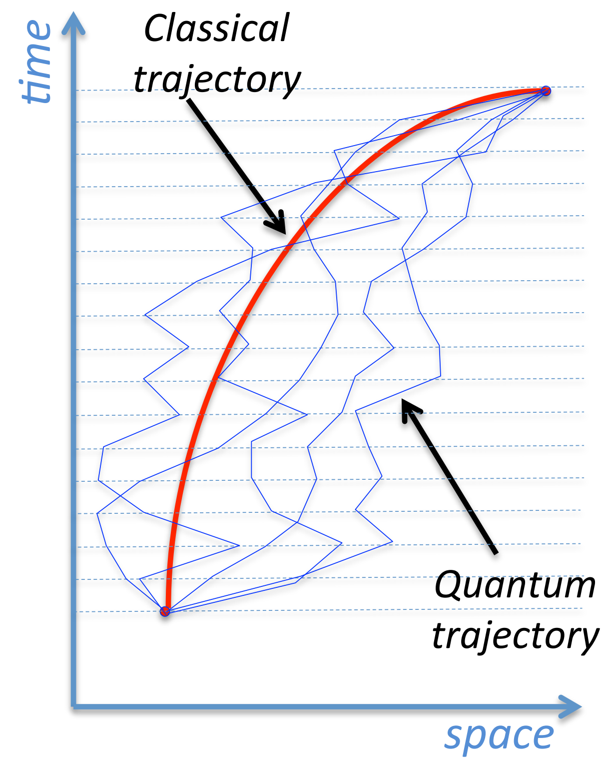

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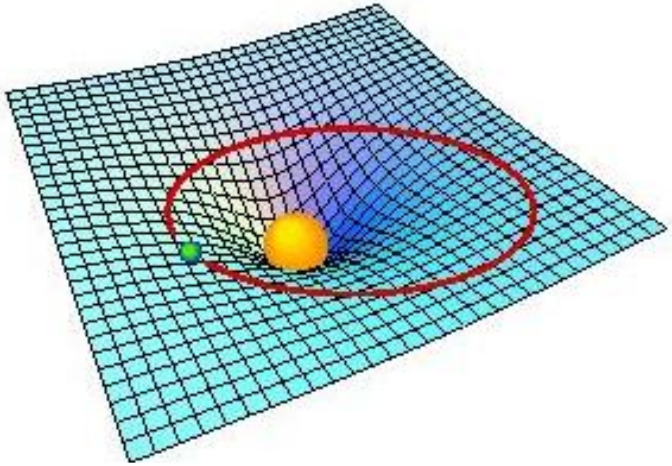

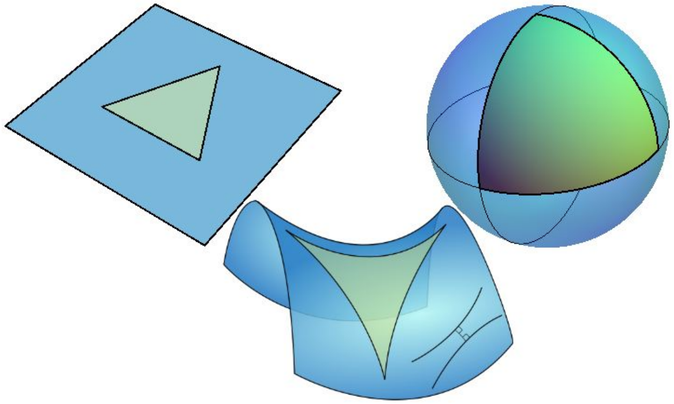

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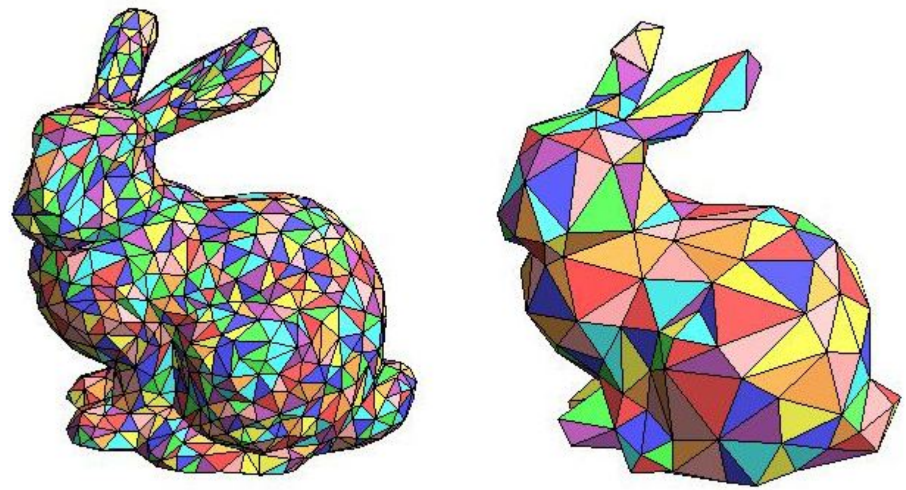

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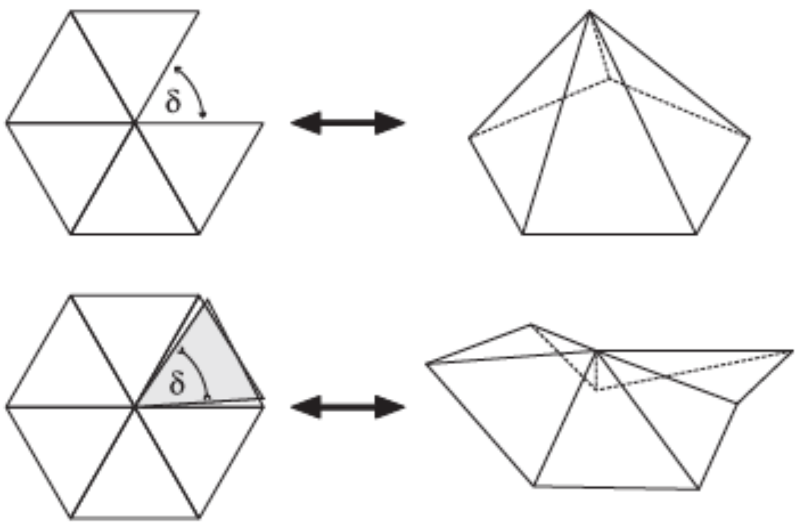

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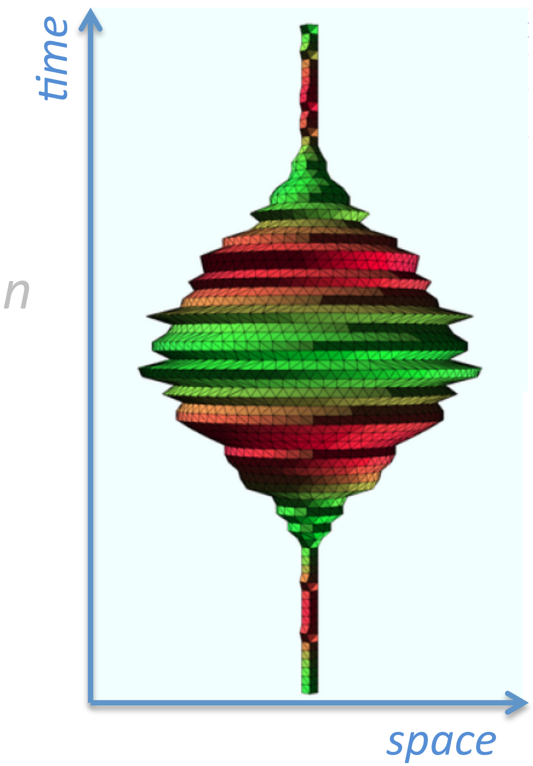

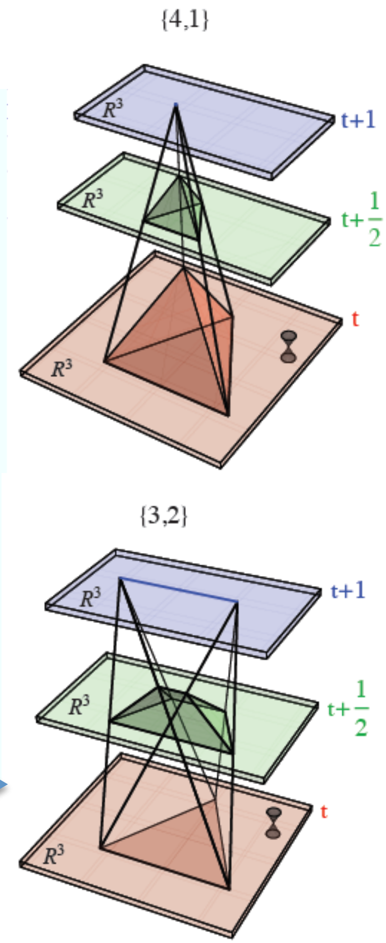

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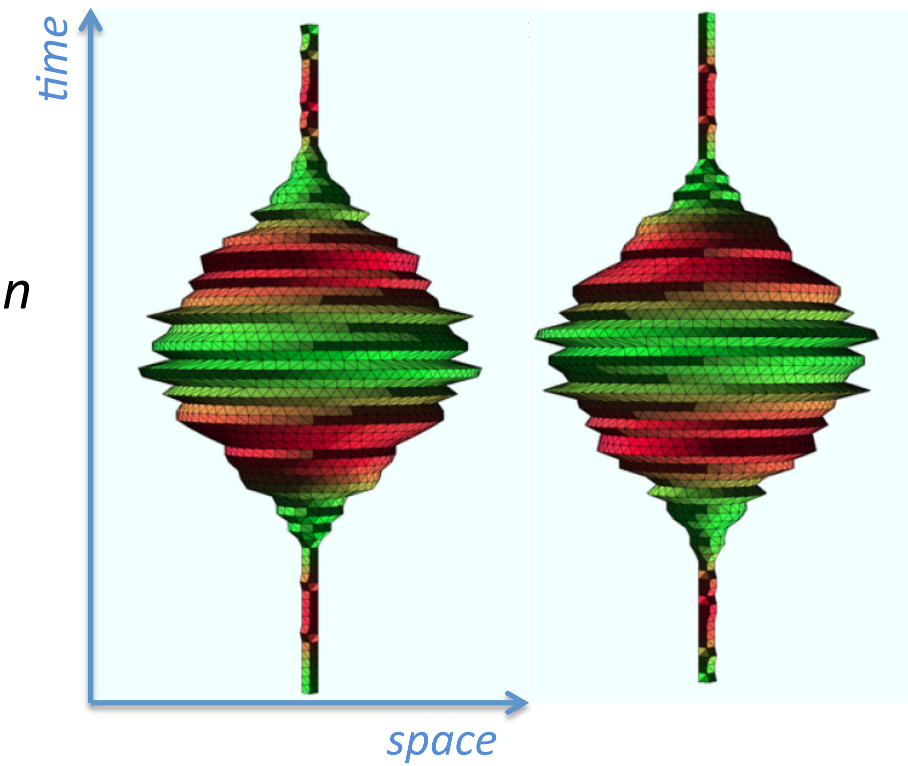

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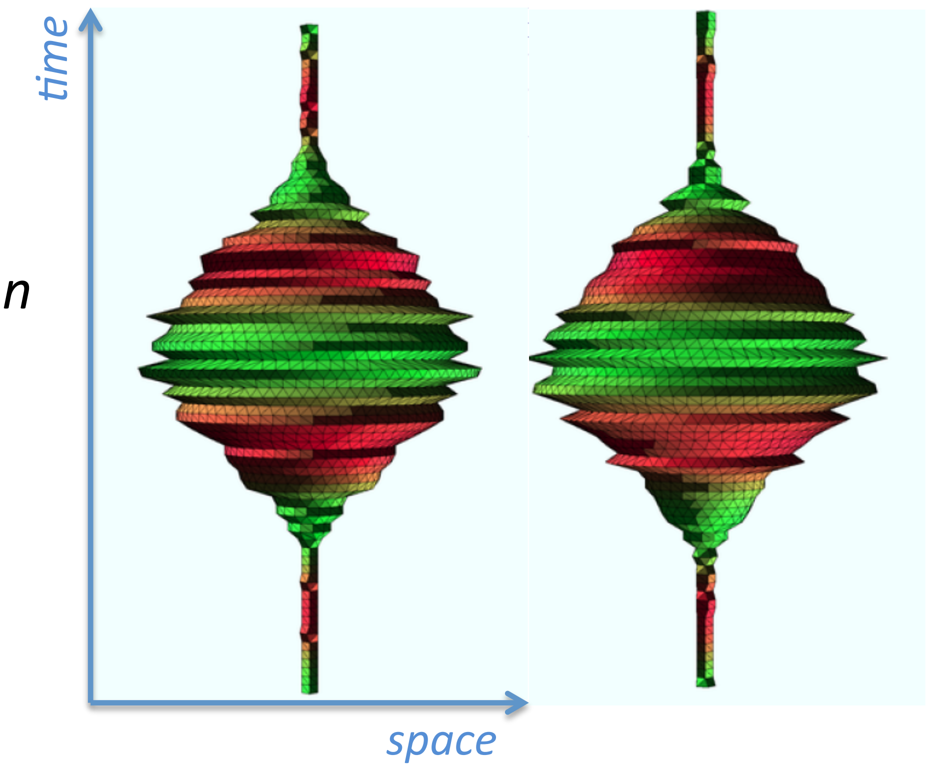

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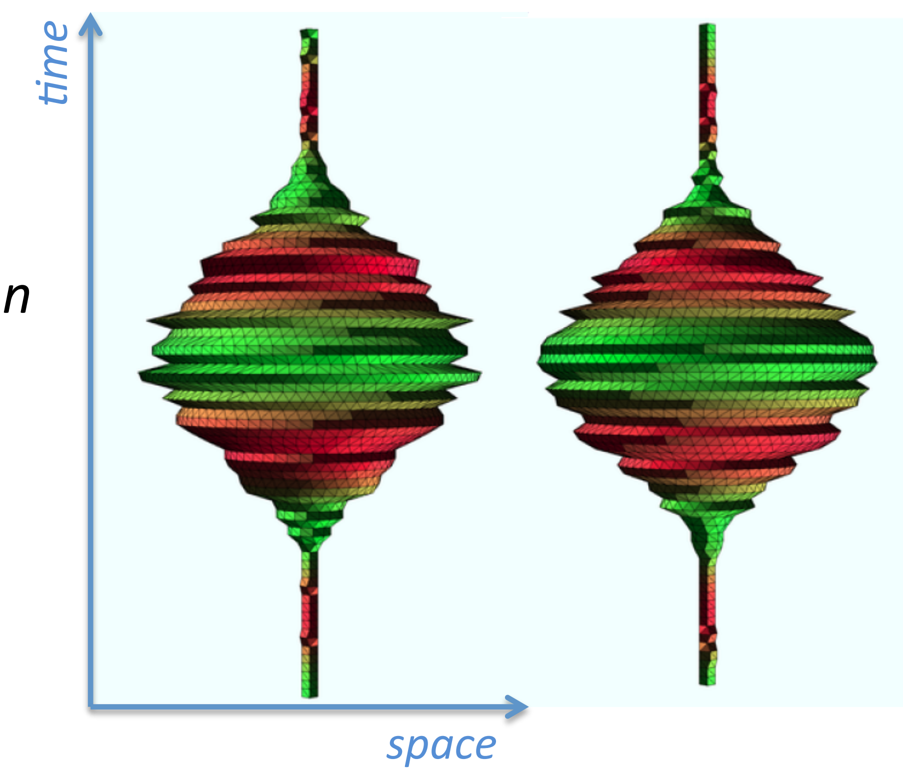

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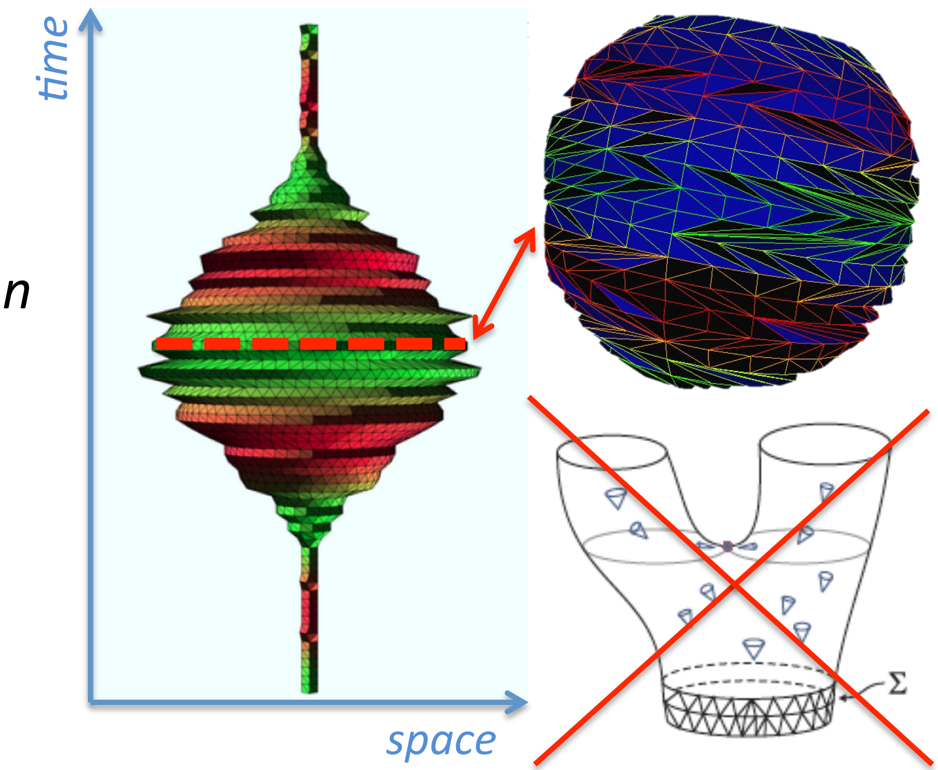

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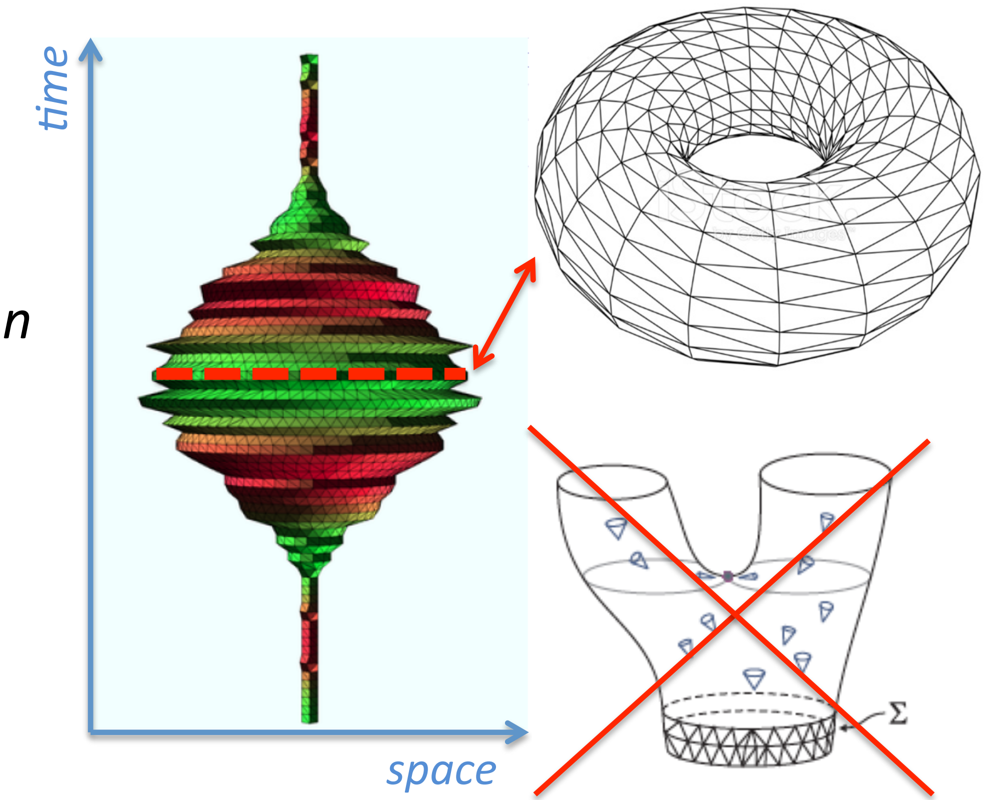

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- *Initially three phases (A, B, C) of various geometry were discovered*
- *Phase C (de Sitter phase) has good semi-classical properties (IR limit)*
  - *Hausdorff dim.: 4 , spectral dim.: 2 $\Rightarrow$ 4*
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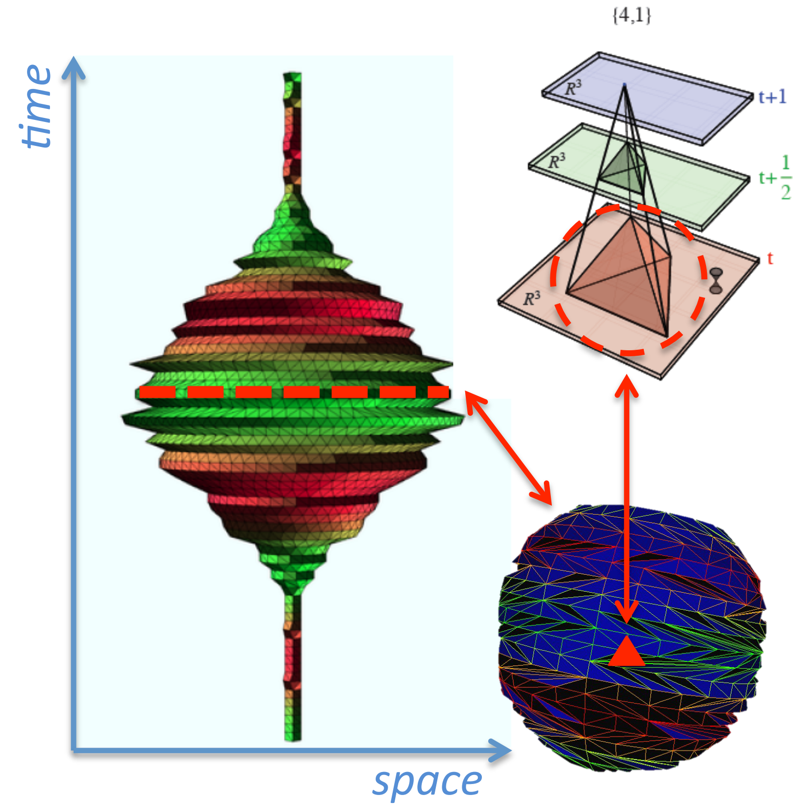

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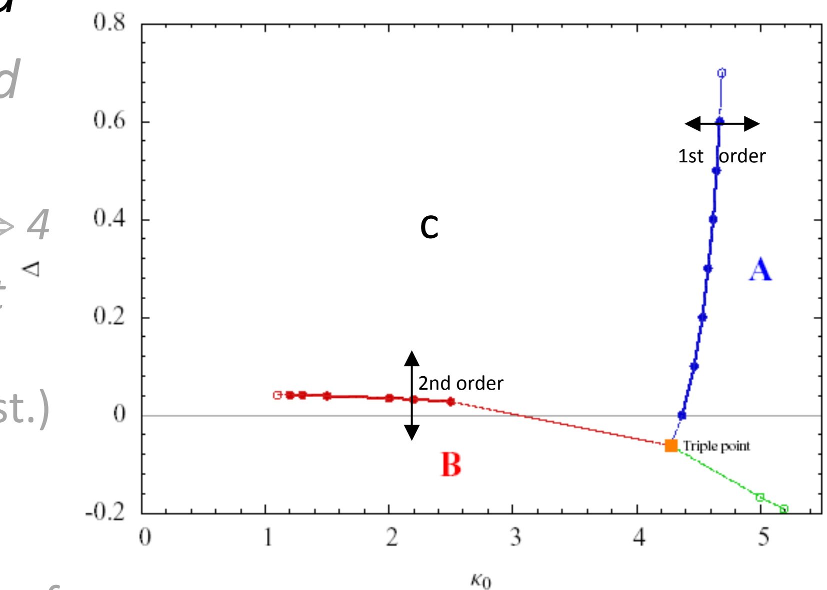

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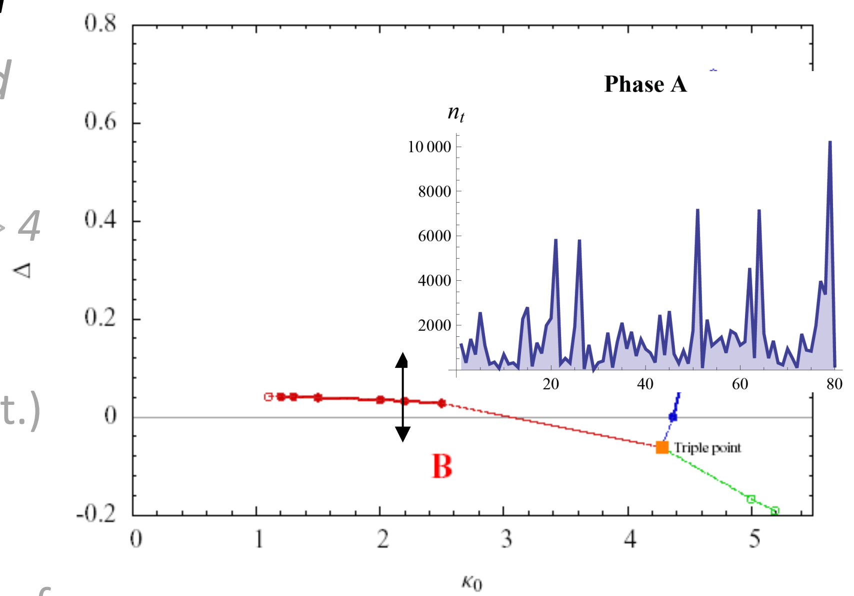

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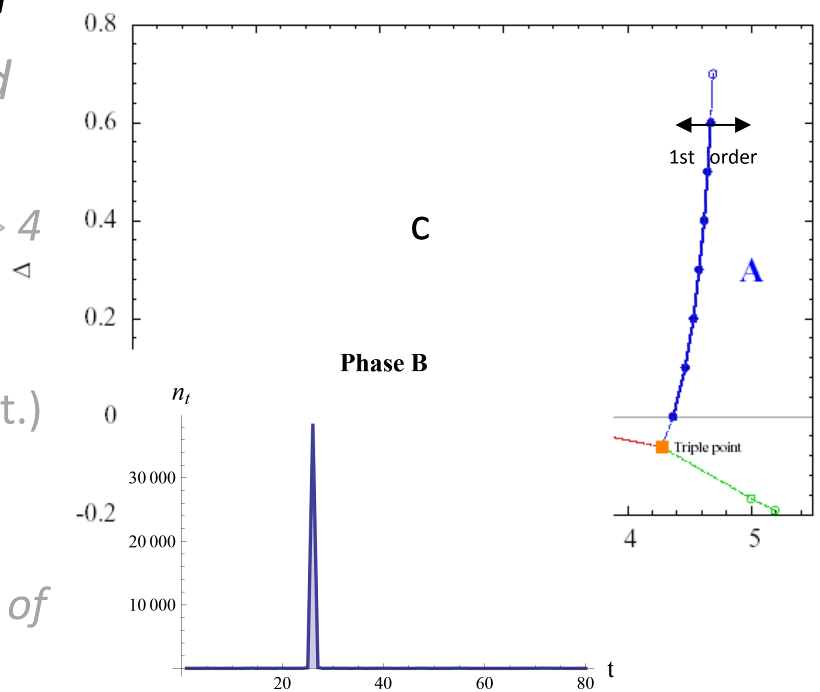

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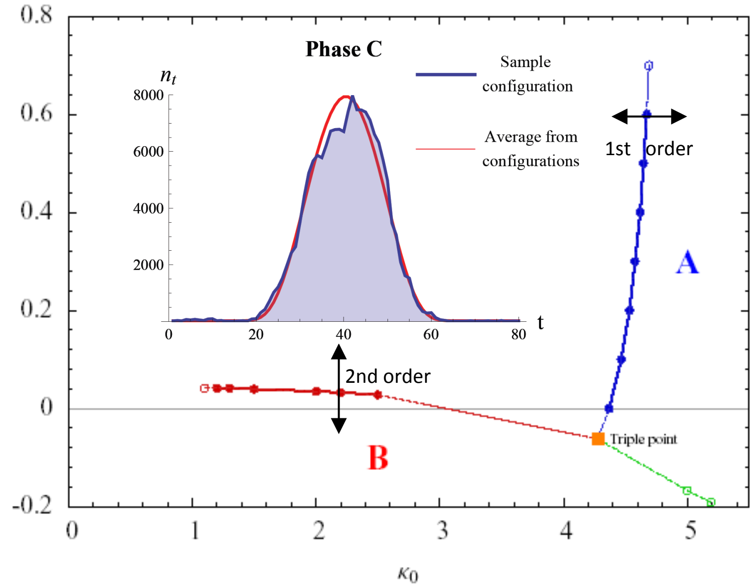

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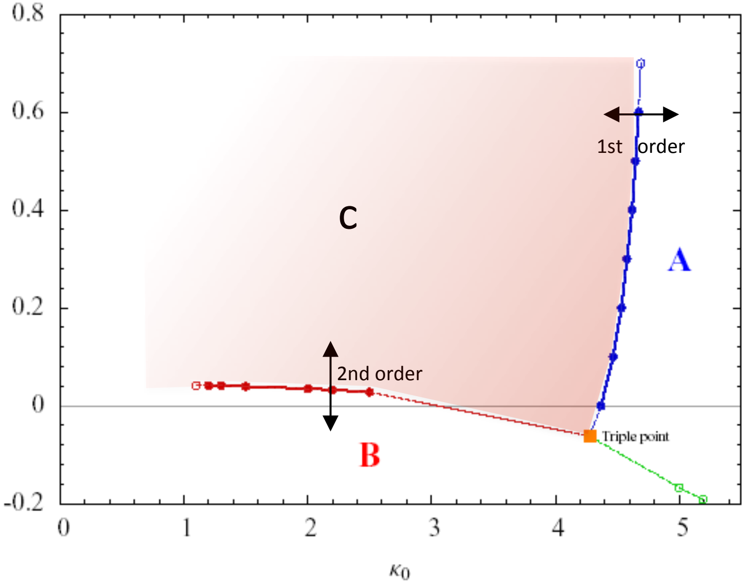

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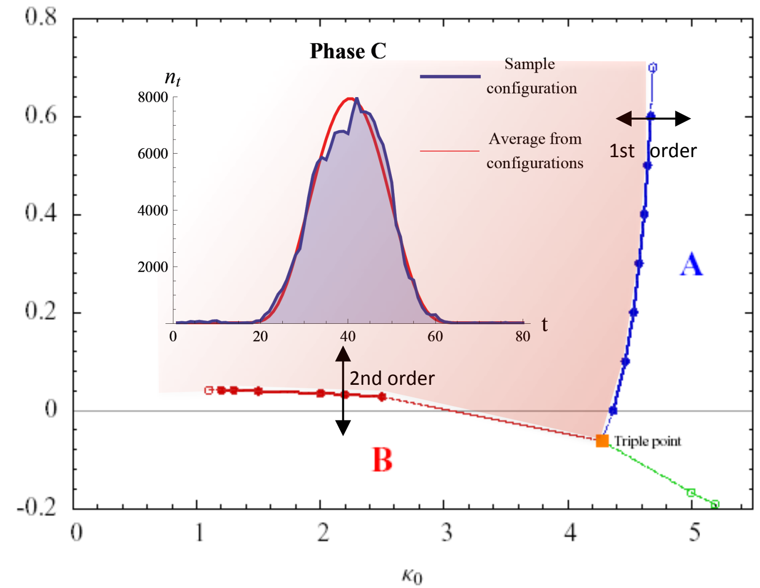

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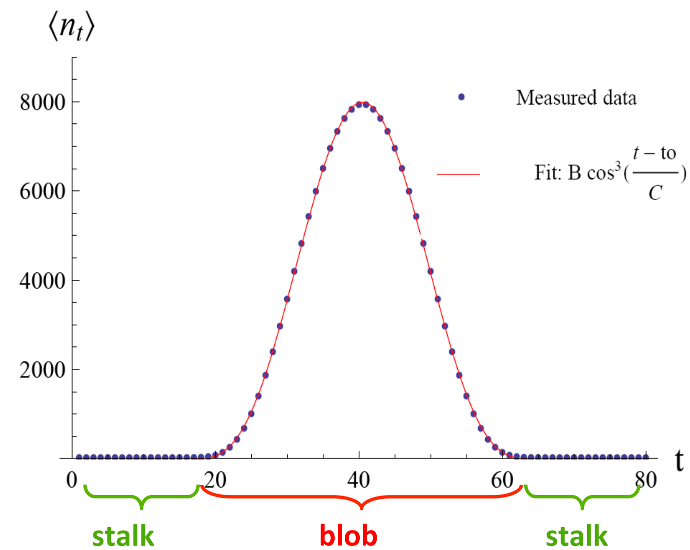

$$\bar{n}_t \equiv \langle n_t \rangle = \frac{3}{4}\tilde{V}_4 \frac{1}{\tilde{A}\tilde{V}_4^{1/4}} \cos^3\left(\frac{t-t_0}{\tilde{A}\tilde{V}_4^{1/4}}\right)$$

$$V_3(t) = \frac{3}{4}V_4 \frac{1}{A V_4^{1/4}} \cos^3\left(\frac{t-t_0}{A V_4^{1/4}}\right)$$

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- *The observable: 3-volume of spatial layers (foliation leaves of the global proper time): $V_3(t) \propto n_t \equiv N_{(4,1)}(t)$*
- *Initially three phases (A, B, C) of various geometry were discovered*
- *Phase C (de Sitter phase) has good semi-classical properties (IR limit)*
  - *Hausdorff dim.: 4 , spectral dim.: 2 ⇒ 4*
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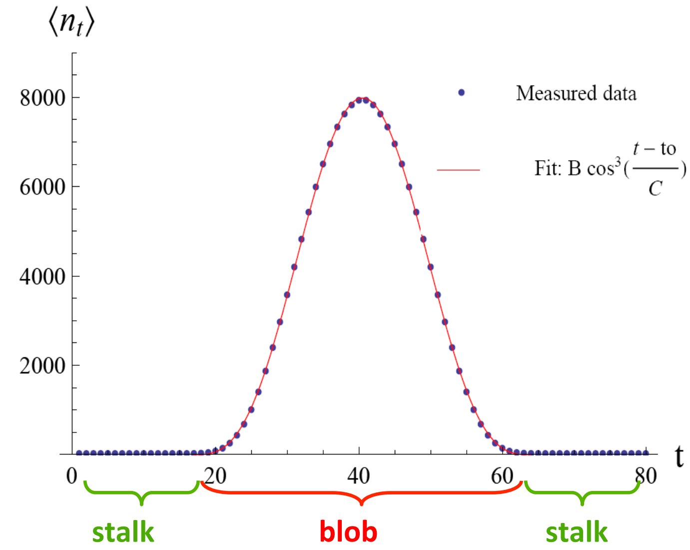

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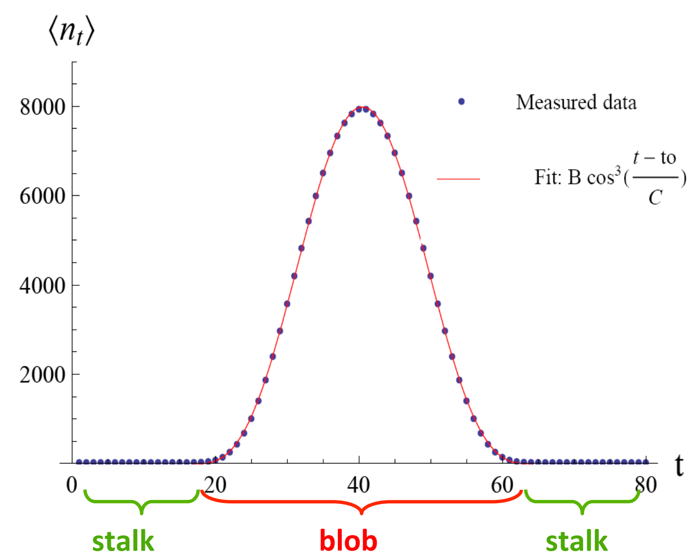

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Phase structure

- Phase C (de Sitter phase) continued
  - The effective action for the n_t observable ...
  - ... can be analyzed by looking at quantum fluctuations around the semiclassical solution
  - The (inverse of) covariance matrix $P = C^{-1}$ provides information about second derivatives of the effective action
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$$Z = \sum_T \exp(-S_R[T]) = \sum_{\{n_t\}} \sum_{T[\{n_t\}]} \exp(-S_R[T])$$

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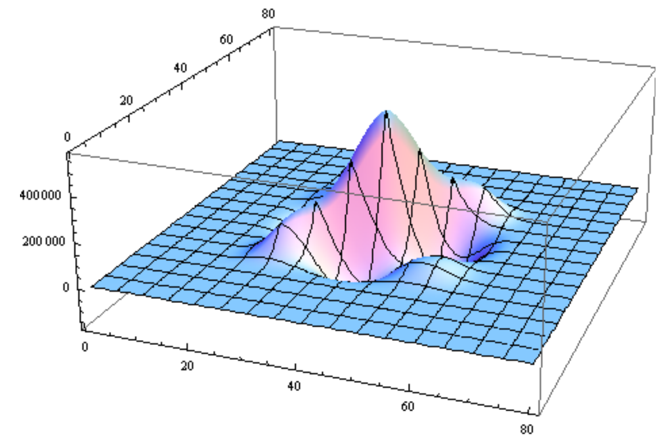

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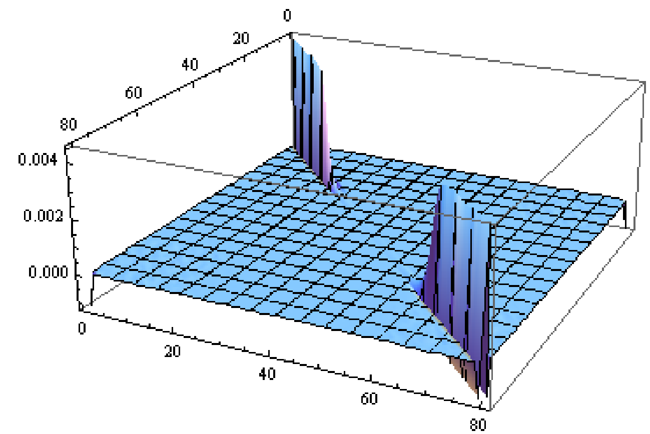

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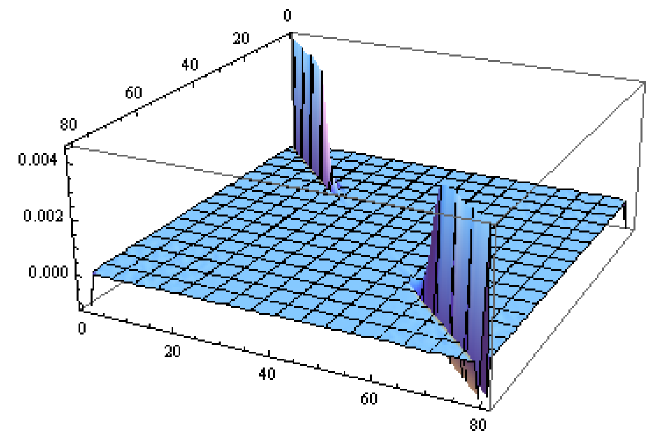

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Phase transitions

- *To analyze phase transitions one needs to define a suitable order parameter OP (e.g. N_4 , N_0 , ...)*
- *Here we look at OP conjugate to the varied coupling constant*
- *(Pseudo)critical point is signaled by max. of susceptibility*
- *Two-states jumping of OP (double peak structure of measured histograms) may signal a 1st order transitions*
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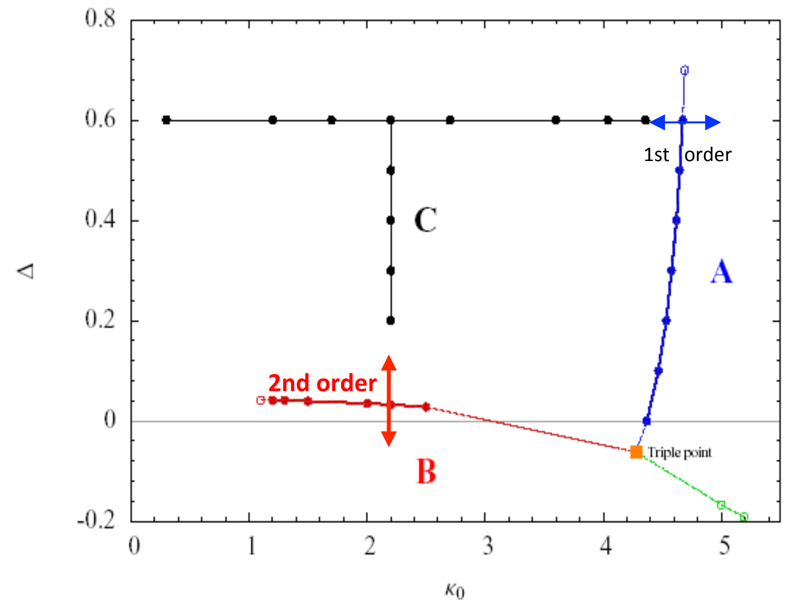

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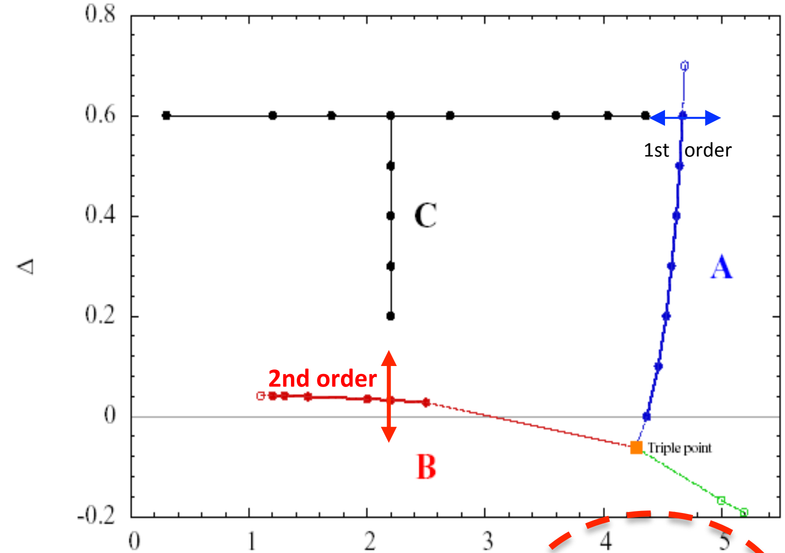

$$S_R = -k_0 N_0 + K_4 N_4 + \Delta\left(N_4^{(4,1)} - 6N_0\right)$$

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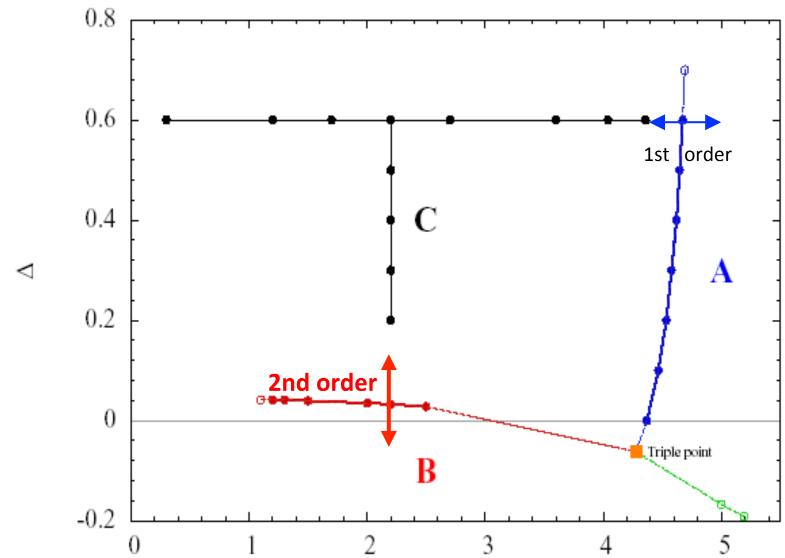

$$X_{OP} = \langle OP^2 \rangle - \langle OP \rangle^2$$

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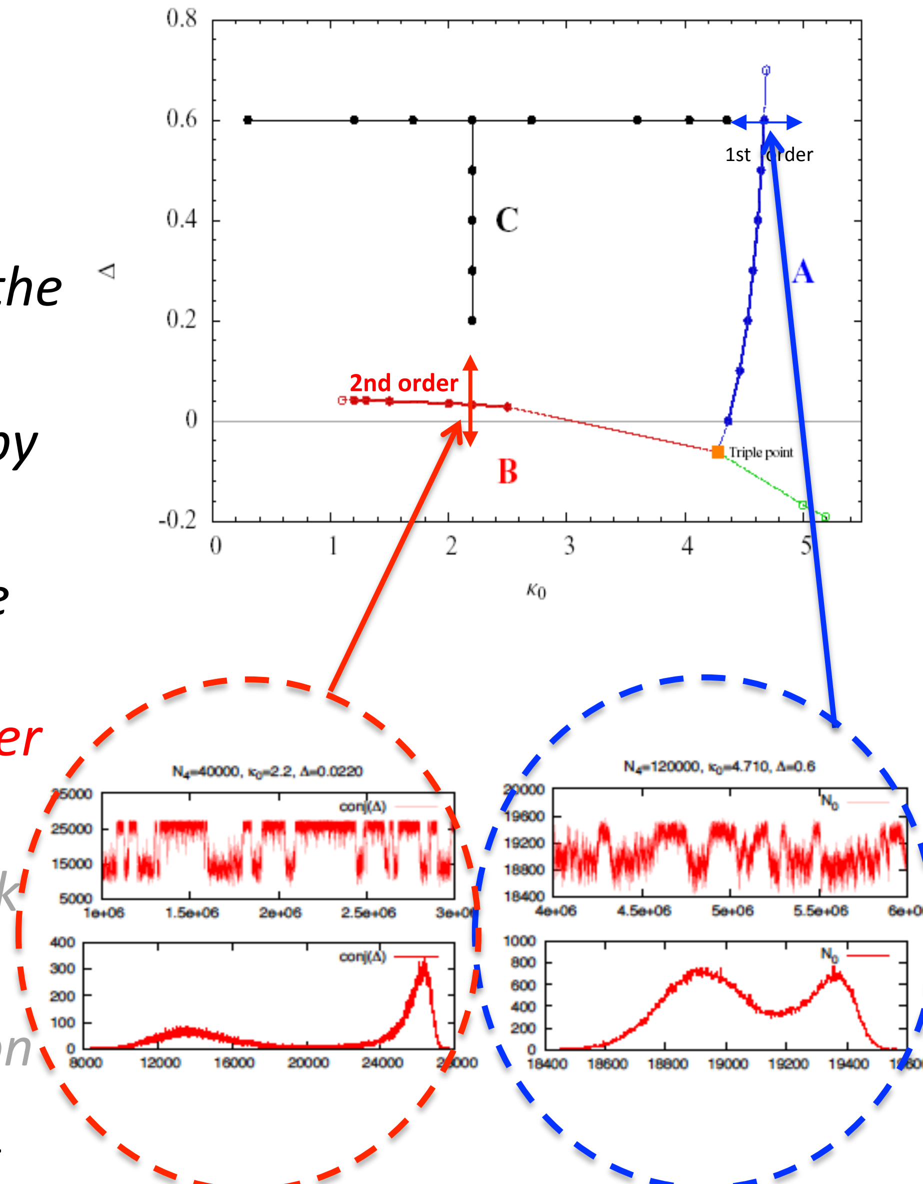

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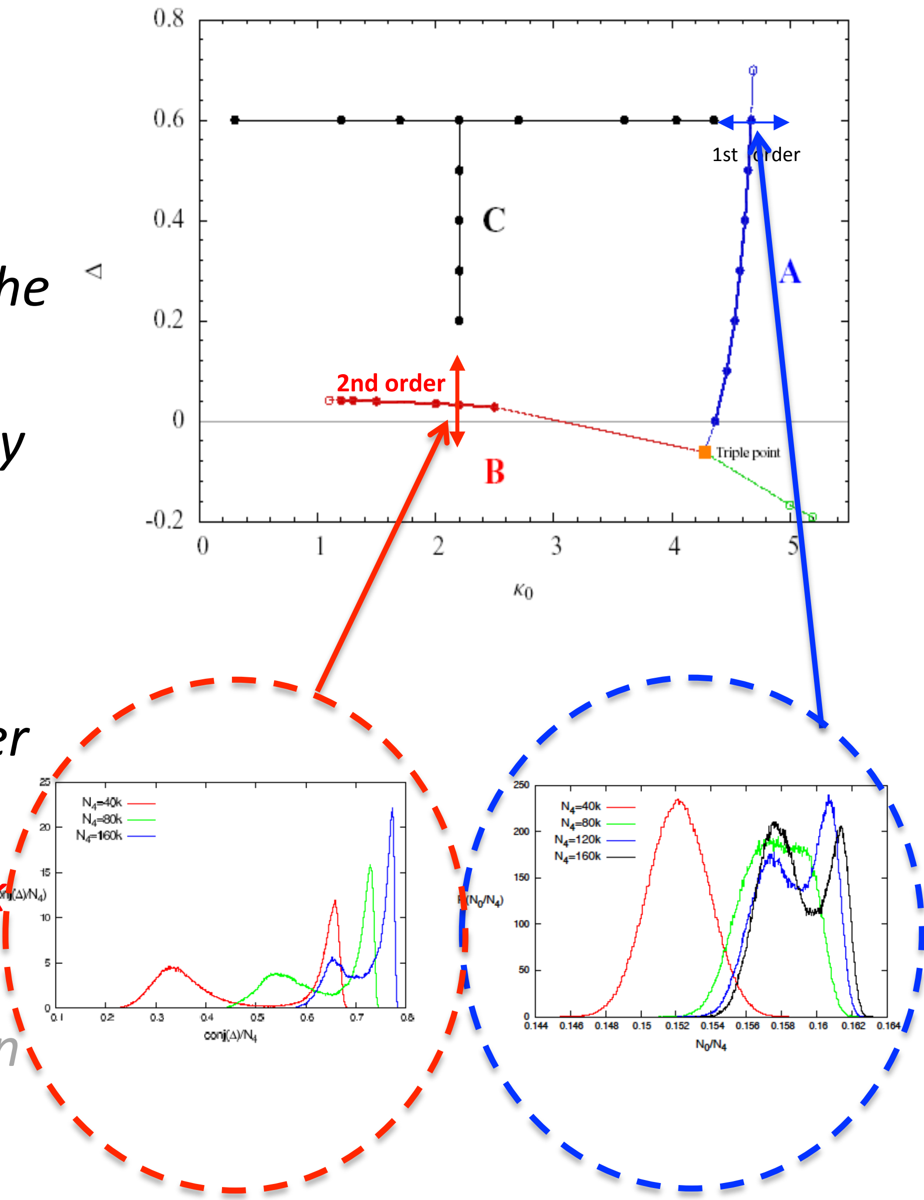

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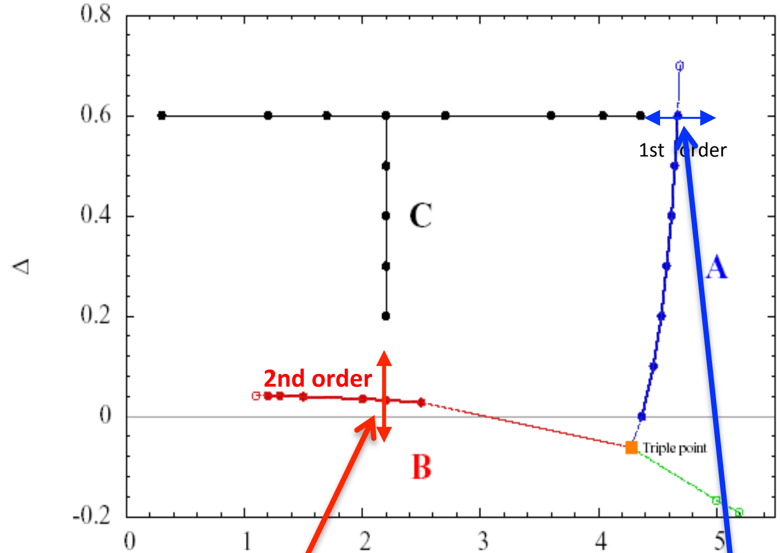
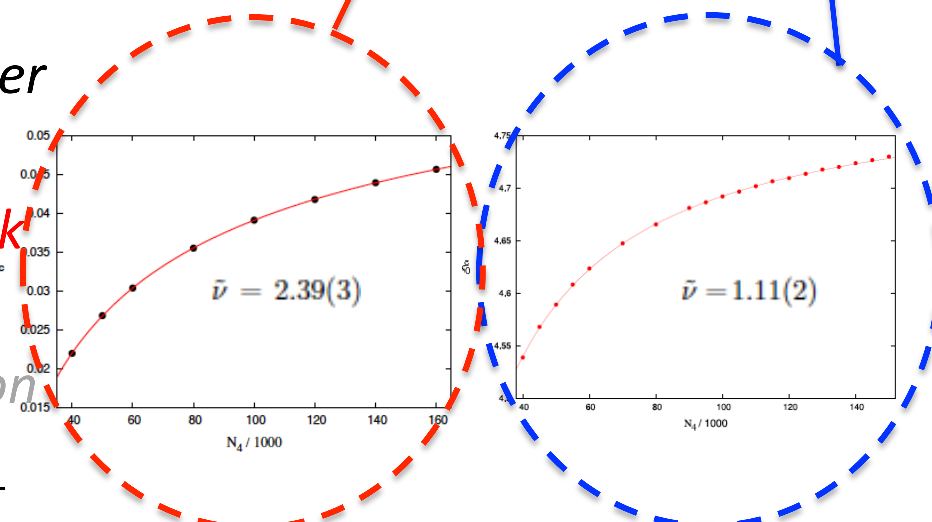


$$\Delta^c\left(N_4\right) = \Delta^c\left(\overset{\kappa_0}{\infty}\right) - \alpha N_4^{-1/\nu}$$



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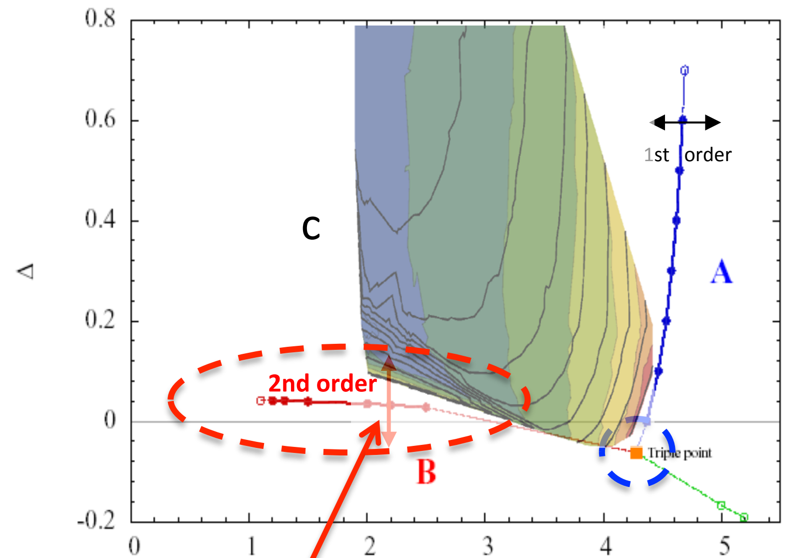

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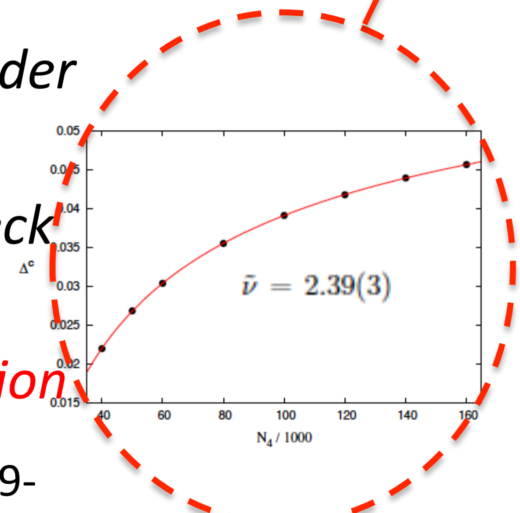
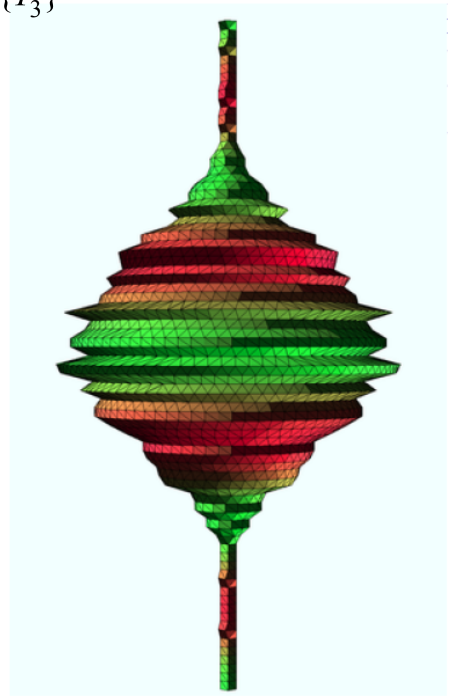


Transfer matrix method

- The transfer matrix method enables one to measure the effective action directly
  - CDT has by definition a transfer matrix parametrized by 3-dimensional spatial triangulations T_3
  - Local form of the effective action in Phase C suggests that a description by effective transfer matrix parametrized by spatial volume n_t is also viable
  - Measurement of the transfer matrix = direct measurement of the effective Lagrangian

$$Z = \sum_{\{T_3\}} \left\langle T_3 \,|\, M^T \,|\, T_3 \right\rangle = tr M^T$$



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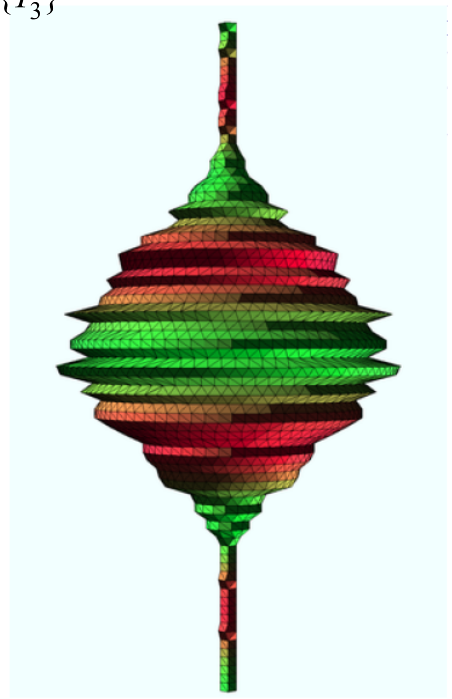

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    - Gaussian kinetic term & MS potential
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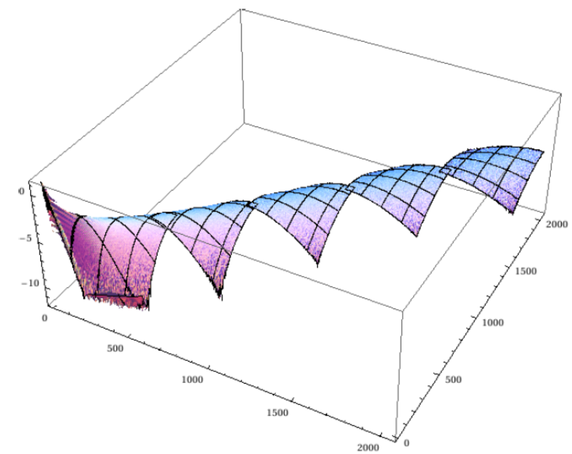

- But when we are are close to the BC phase transition
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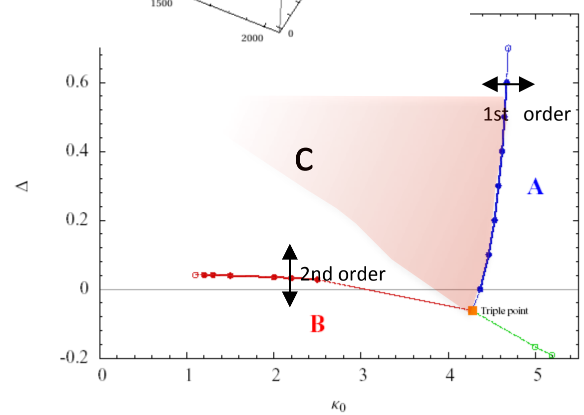

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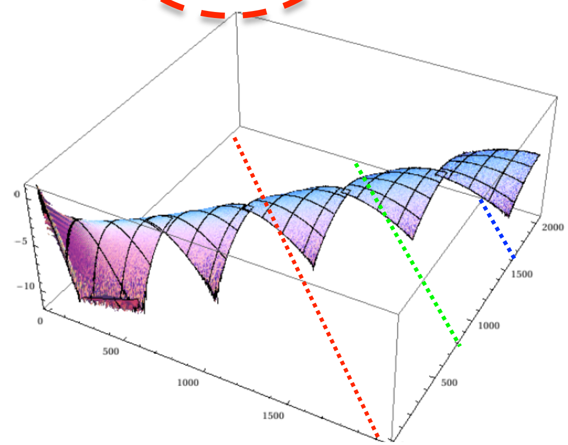

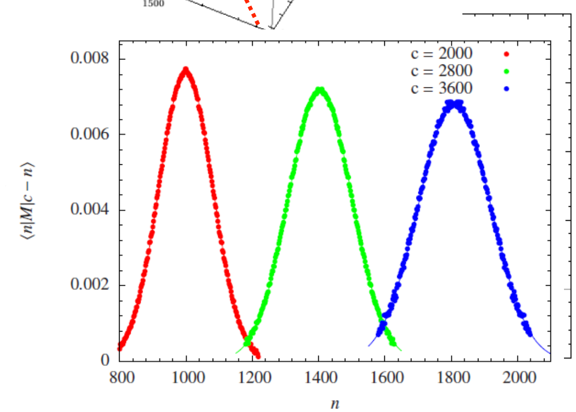

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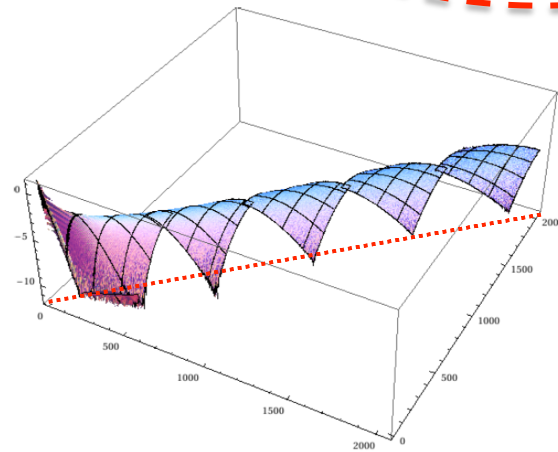

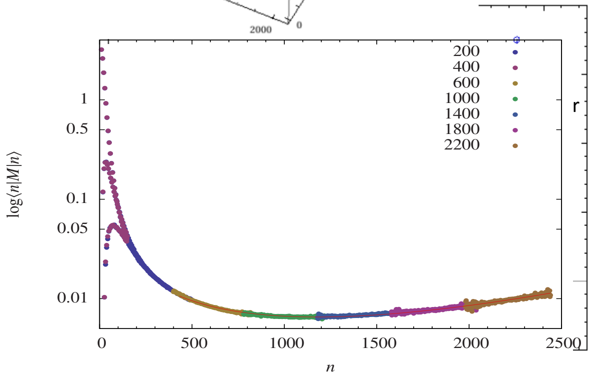

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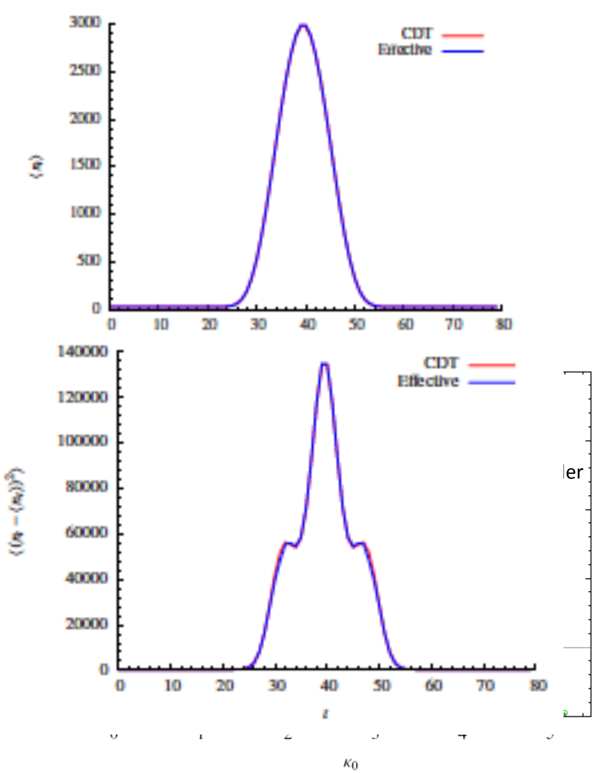

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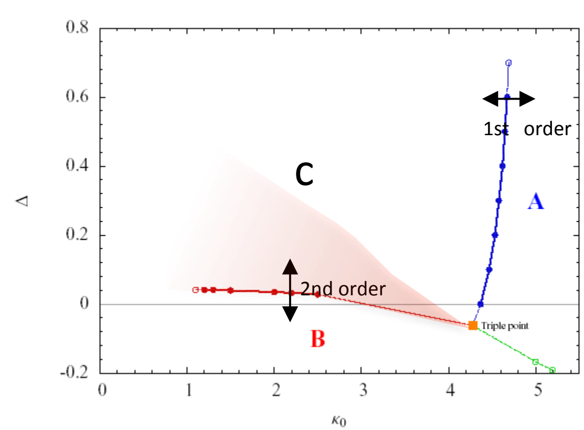

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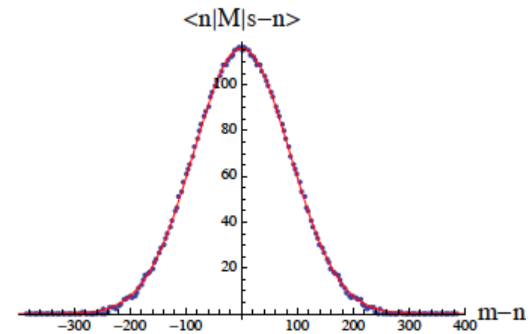

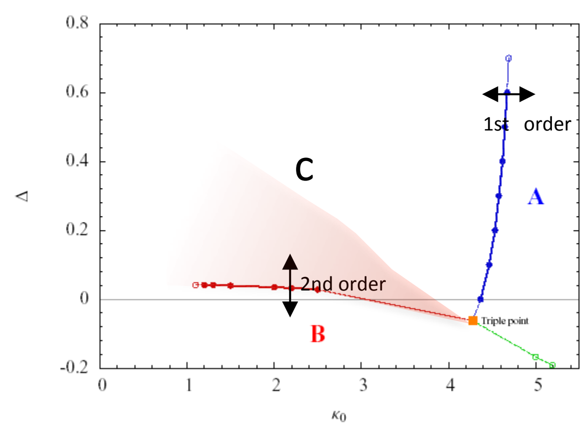

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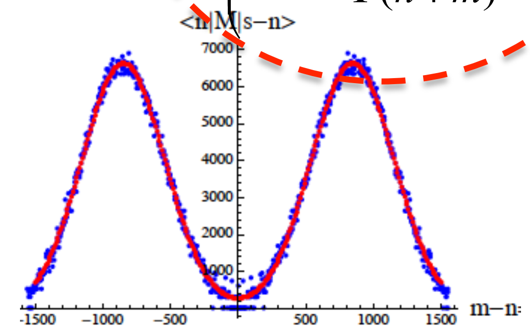

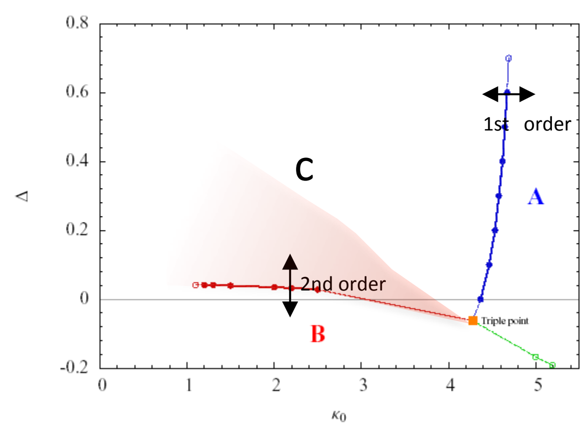

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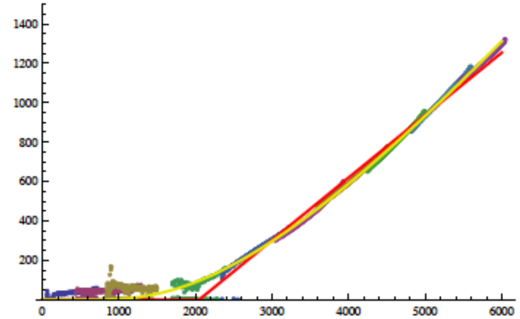
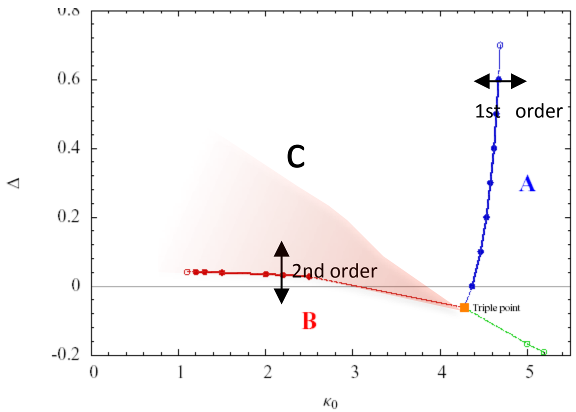
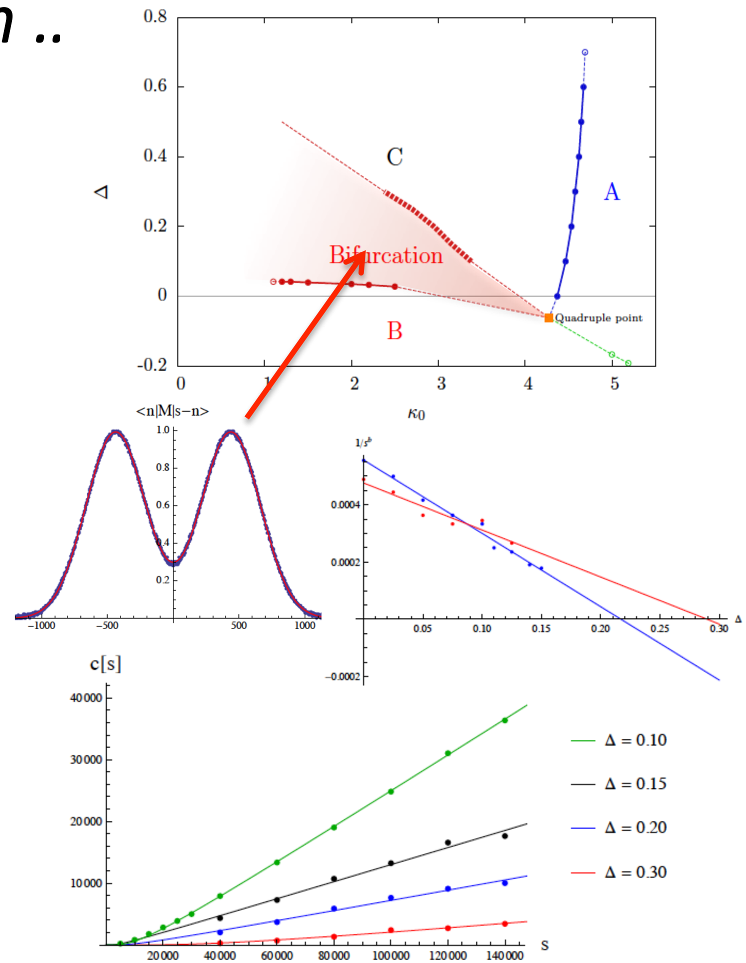




Phase structure revisited

✧ *The new phase separating phases B & C is related to a bifurcation of the effective action ..*



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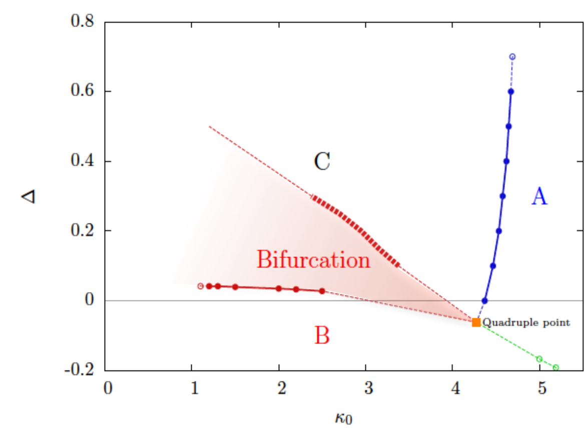
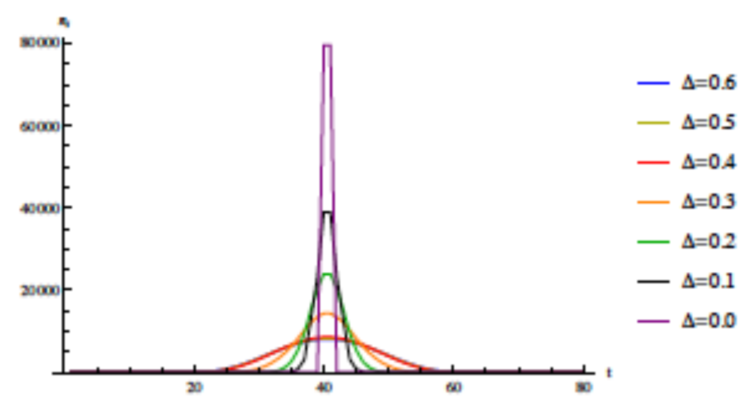


- ✧ *Average volume profile in the new phase resembles the profile observed in Phase C …*
- ✧ *… but the profile is shrinking in time direction …*
- ✧ *… which is well explained by the bifurcation of the transfer matrix kinetic term*



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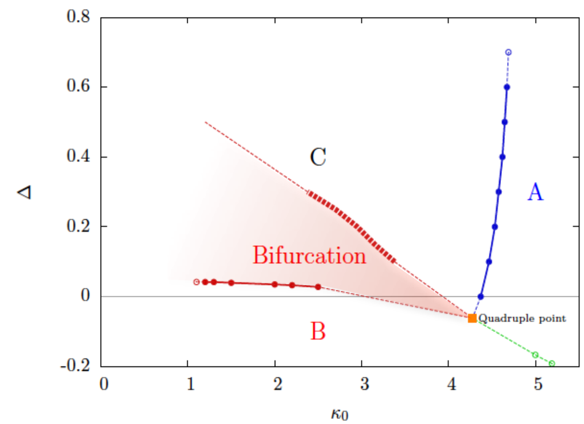

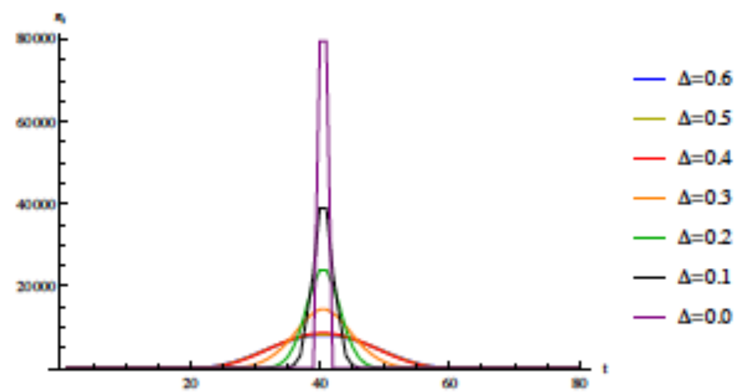

Phase structure revisited

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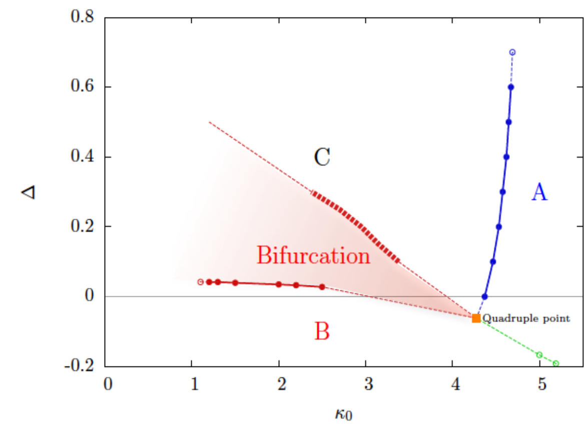

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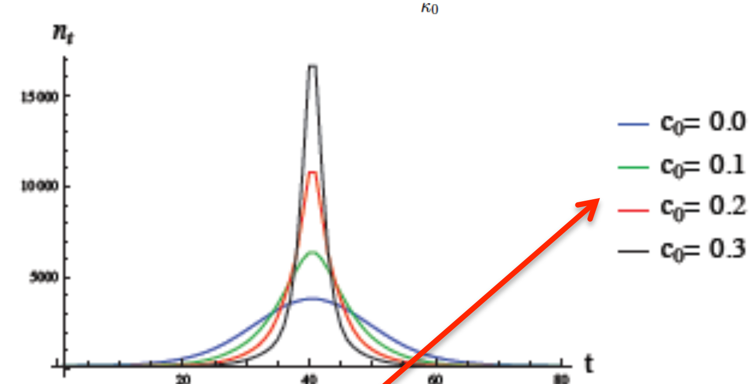
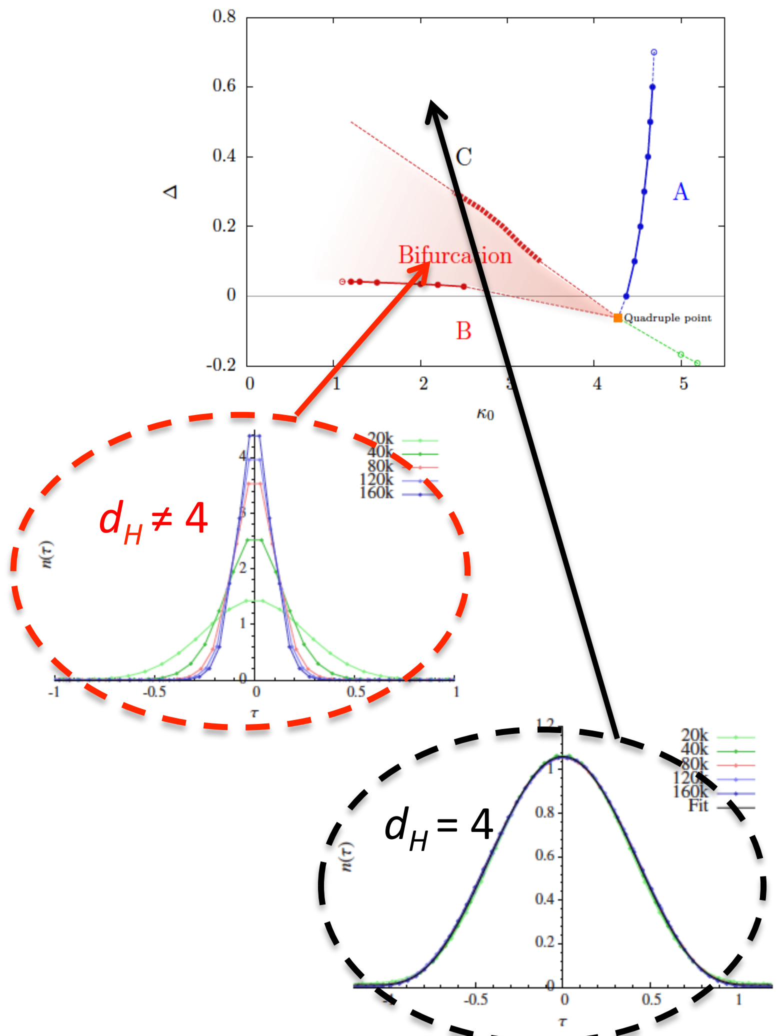


$$\langle n \,|\, M_B \,|\, m \rangle = N[n+m]\left[\exp\left(-\frac{\left(m-n-[c_0(n+m-s_b)]_+\right)^2}{\Gamma(n+m)}\right)+\exp\left(-\frac{\left(m-n+[c_0(n+m-s_b)]_+\right)^2}{\Gamma(n+m)}\right)\right]$$

Phase structure revisited

✧ … resulting from geometry considerably different than inside Phase C

- ✧ Infinite Hausdorff dimension?
- ✧ Spectral dimension > 4 and growing (to infinity ?) with growing volume
- ✧ This suggests high connectivity between the building blocks
- ✧ 4-volume is concentrated in short geodesic distance
- ✧ Such volume clusters appear every second time slice and are linked by „singular" vertices



Phase structure revisited

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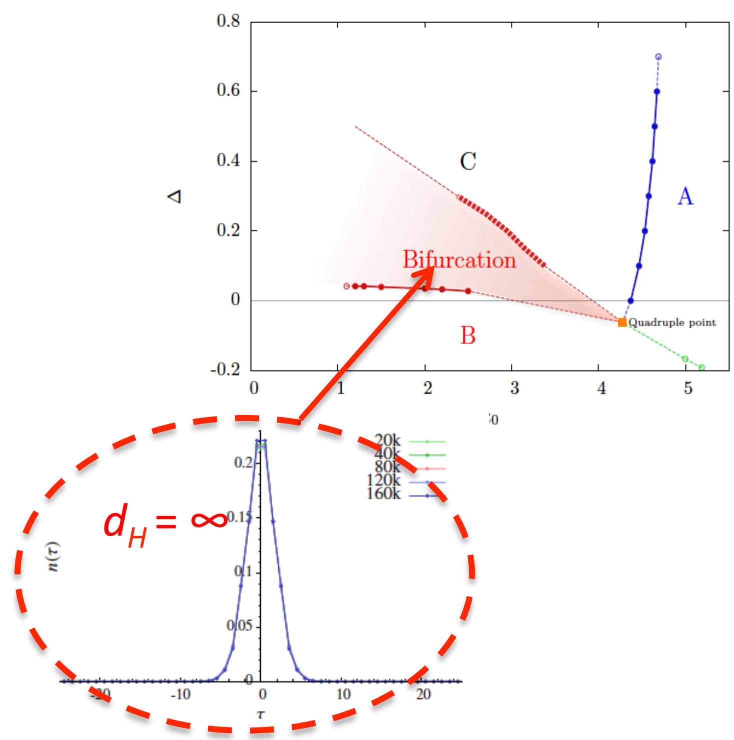

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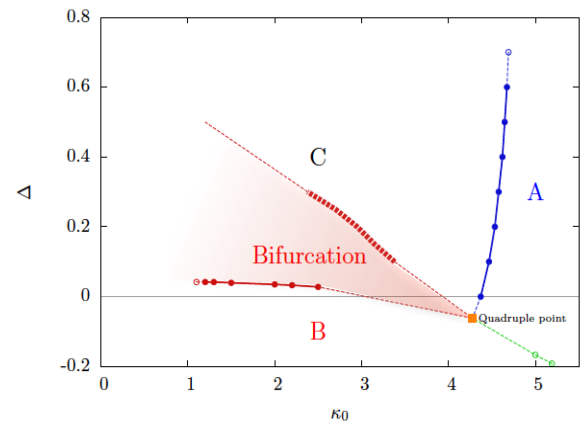

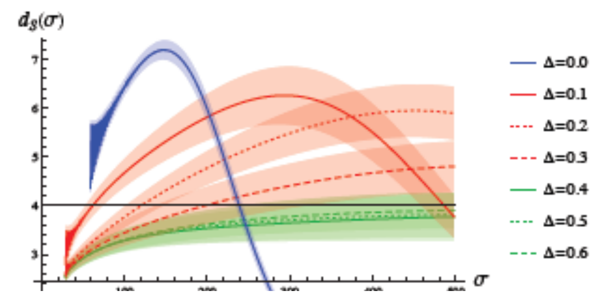

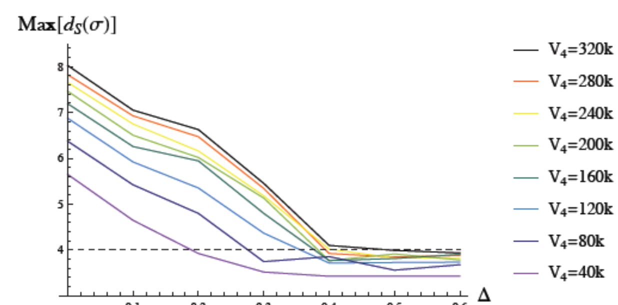

Phase structure revisited

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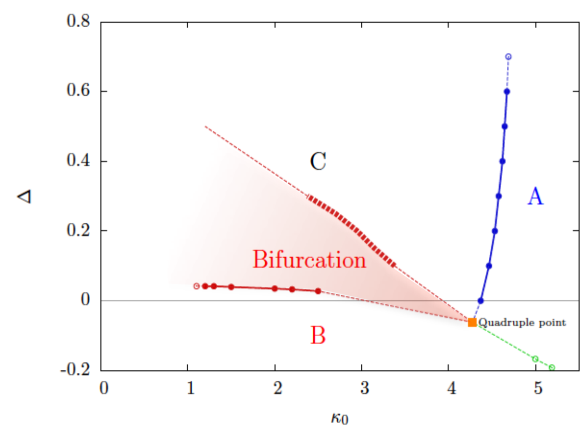

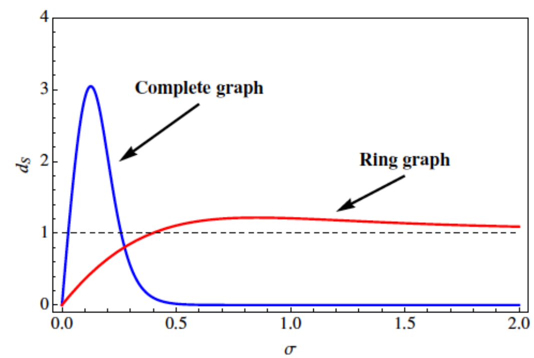

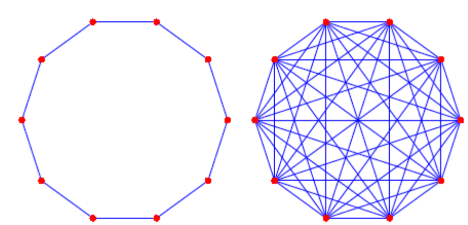

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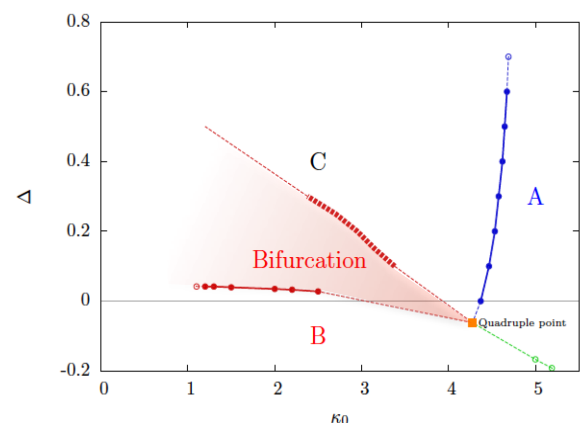
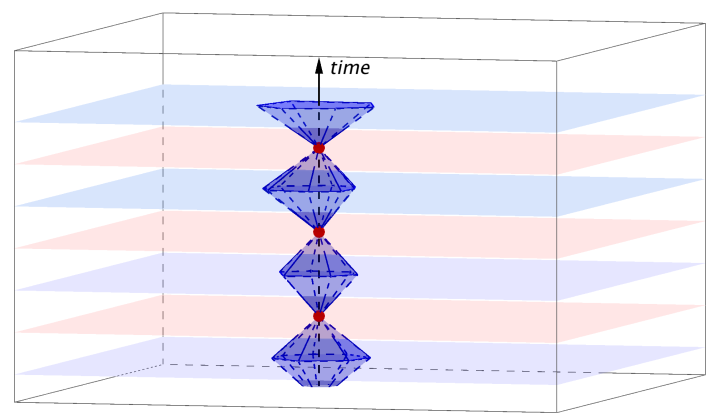
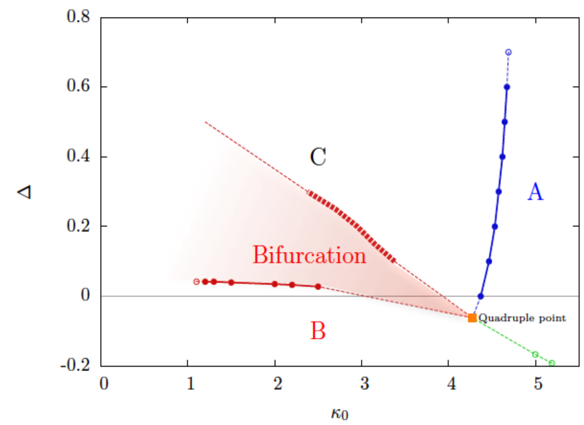
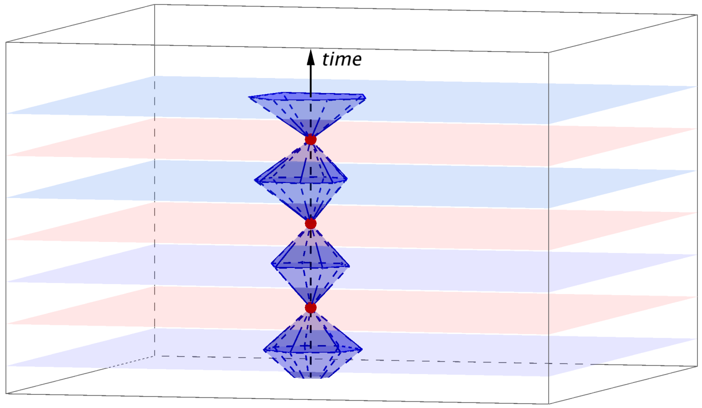




Phase structure revisited

- ... resulting from geometry considerably different than inside Phase C
  - Spatial (3-dimentional) geometries differ between odd and even t
    - average spatial curvature scalar
    - average extent of the universe
  - This is caused by geometric structures surrounding "singular" vertices
    - each such a vertex is shared by a compact cluster of spatial volume (tetrahedra)
    - whose boundary has topology of a 2-sphere (Euler characteristic = 2)
    - and this structure "evolves" in time
  - Do we observe a quantum BH ???





Phase structure revisited

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  - Spatial (3-dimentional) geometries differ between odd and even t
    - average spatial curvature scalar $\bar{R}_{(3)} = \dfrac{\int d^3x \sqrt{g_{(3)}}\, R_{(3)}}{\int d^3x \sqrt{g_{(3)}}}$
    - average extent of the universe
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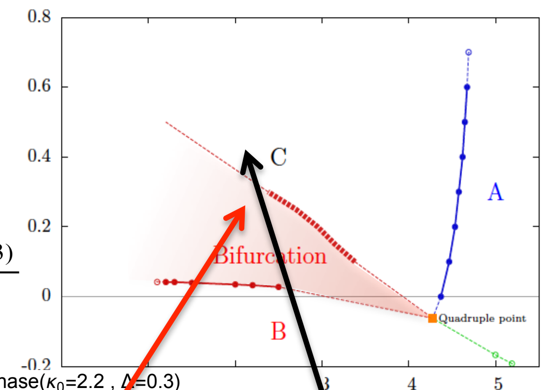

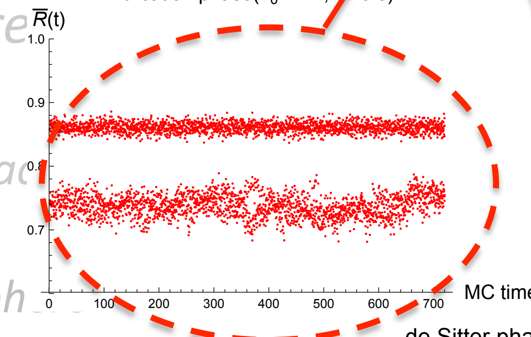

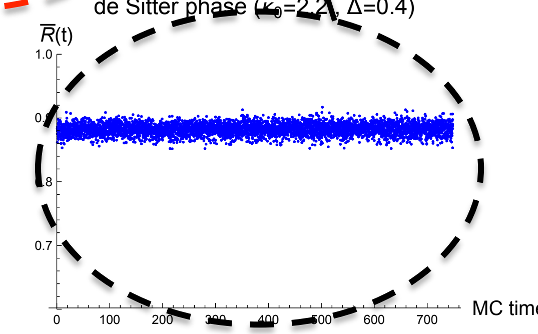

Phase structure revisited

- ... resulting from geometry considerably different than inside Phase C
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    - average extent of the universe $\langle r \rangle = \dfrac{\sum r V_3(r)}{V_3}$
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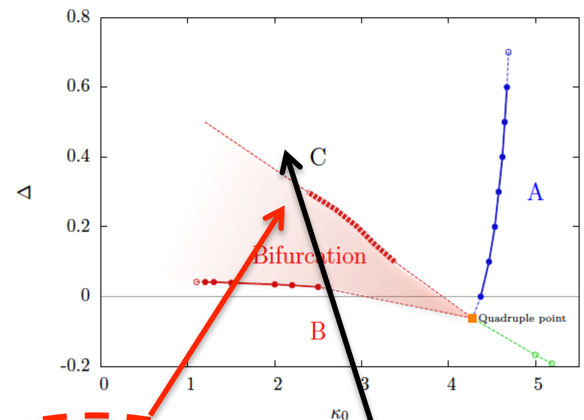

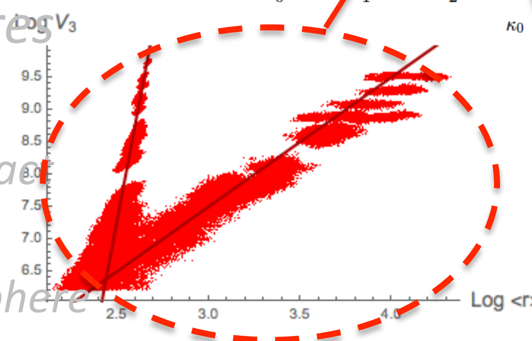

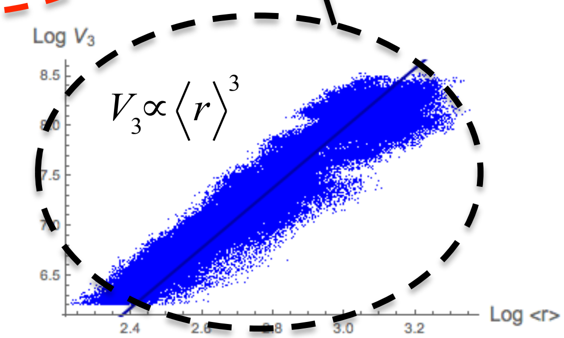

Phase structure revisited

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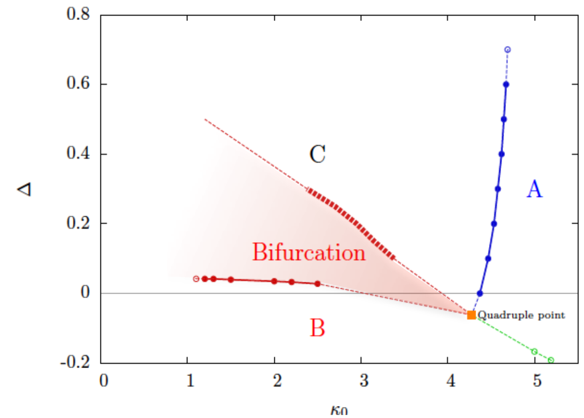

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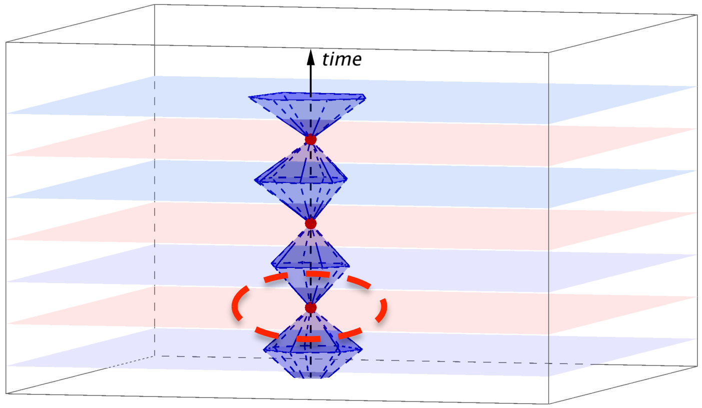

Phase structure revisited

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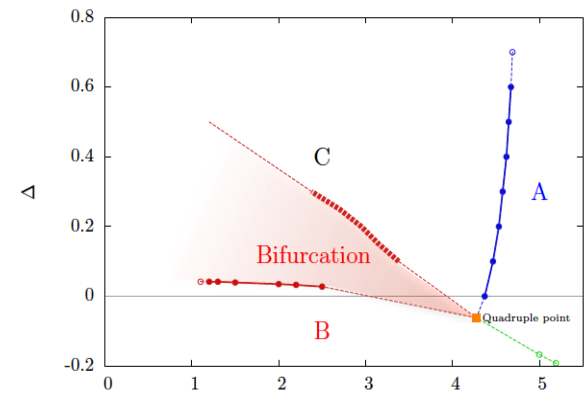

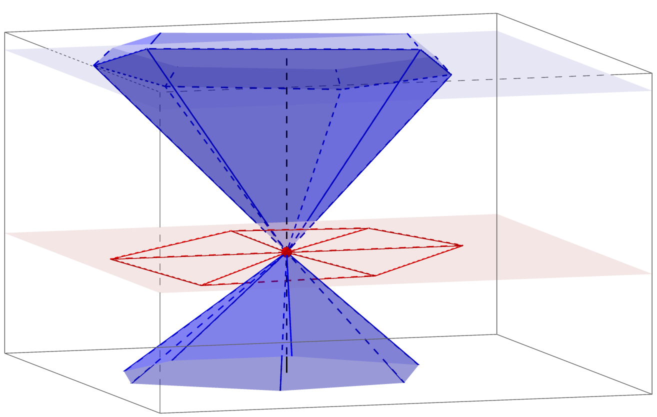

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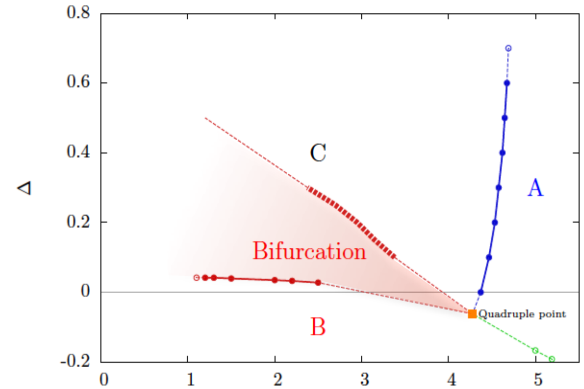

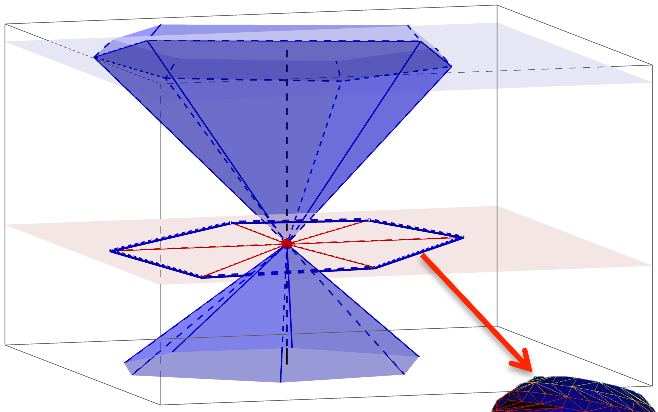

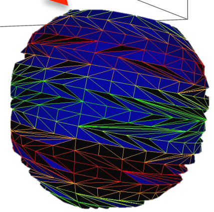

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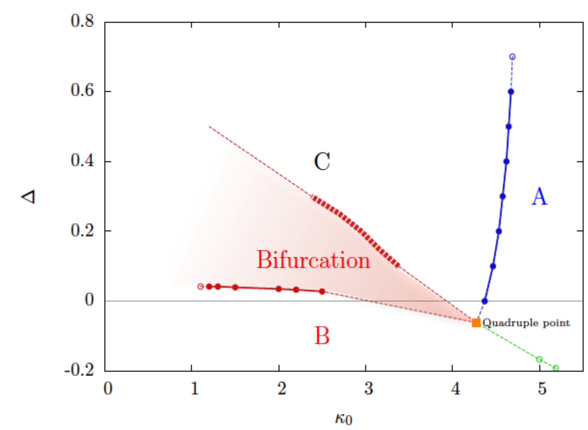

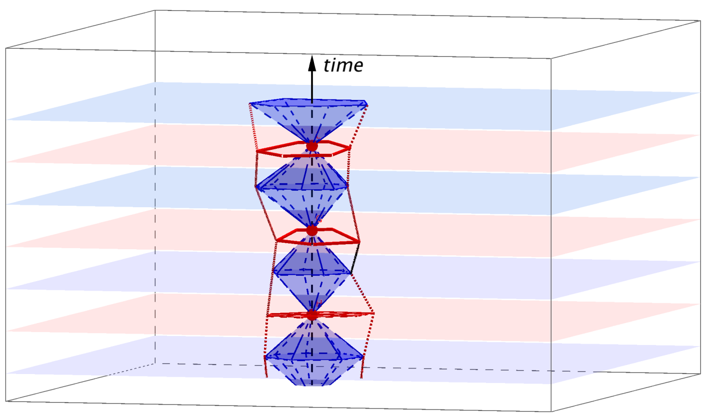

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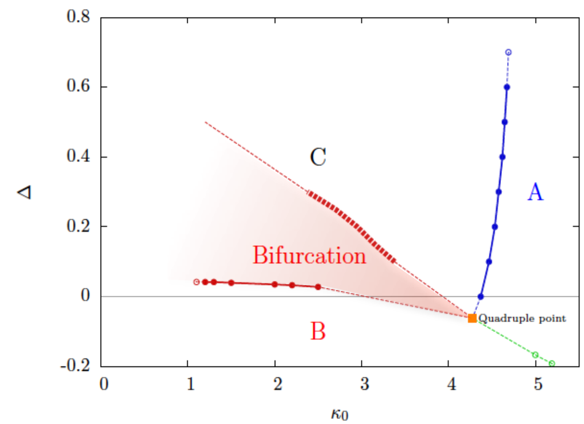
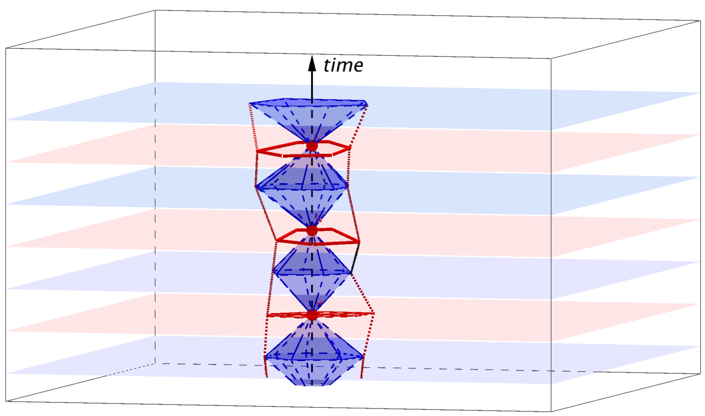
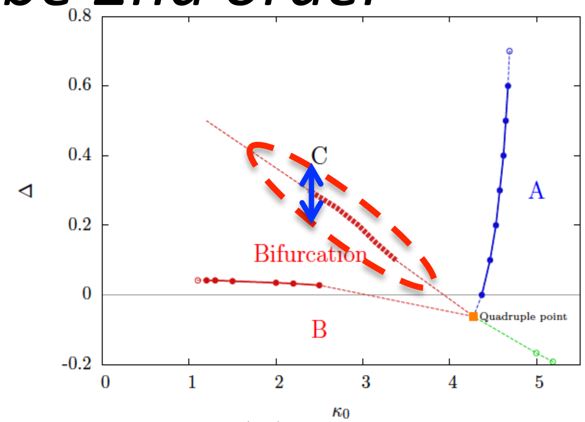


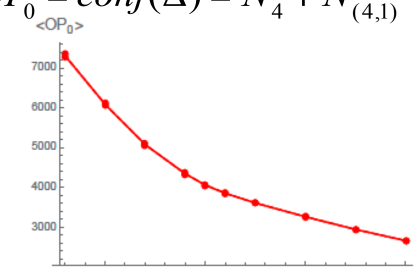


Phase transitions revisited

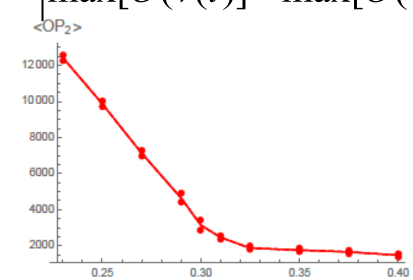
✧ *The new phase transition seems to be 2nd order*



$$OP_0 = conj(\Delta) = N_4 + N_{(4,1)} - 6N_0$$



$$OP_2 = \left|\max[O(v(t)] - \max[O(v(t+1)]\right|$$



- ✧ *To analyze phase transitions one needs to define a suitable order parameter OP*
- ✧ *(Pseudo)critical point is characterized by max in susceptibility X_{OP}*
- ✧ *At the transition point the OP jumps between two metastable states*
- ✧ *But the two states converge with increasing lattice volume*
- ✧ *Position of the (pseudo)critical point is moving with the total volume*
- ✧ *Critical exponent ($\nu = 3.0 \pm 0.3$) suggests 2nd order transition*

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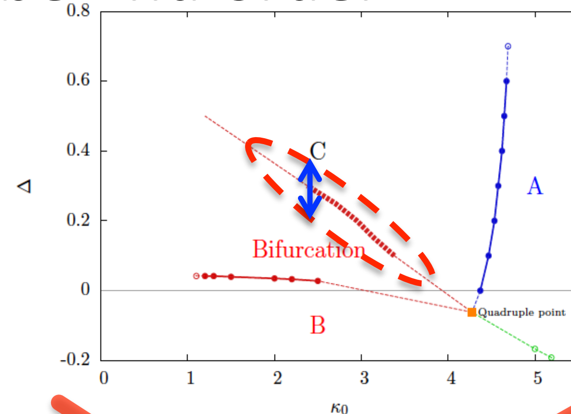

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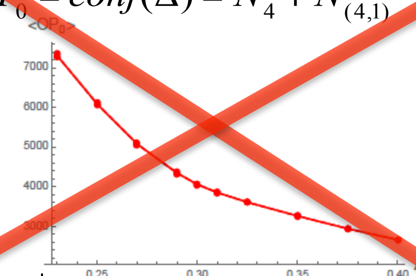

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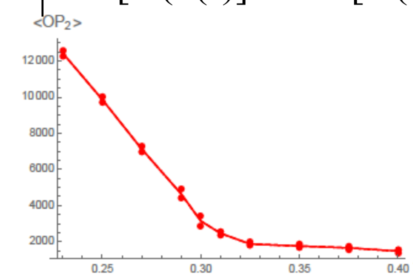

Phase transitions revisited

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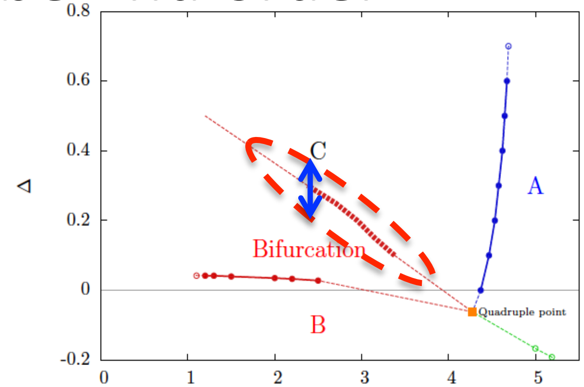

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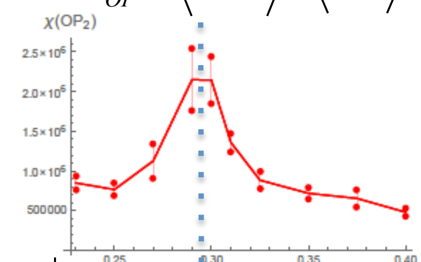
$$X_{OP} = \langle OP^2 \rangle - \langle OP \rangle^2$$



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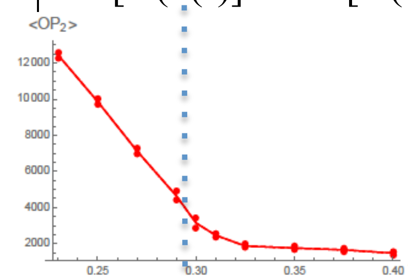

Phase transitions revisited

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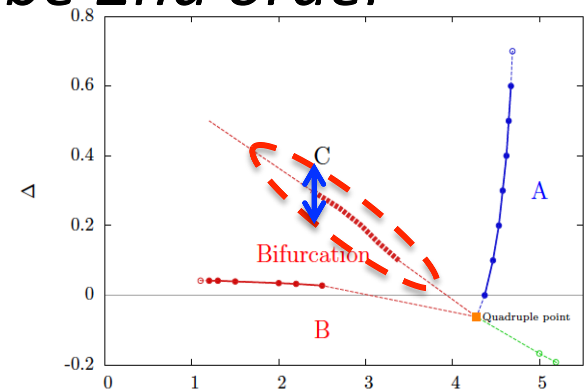

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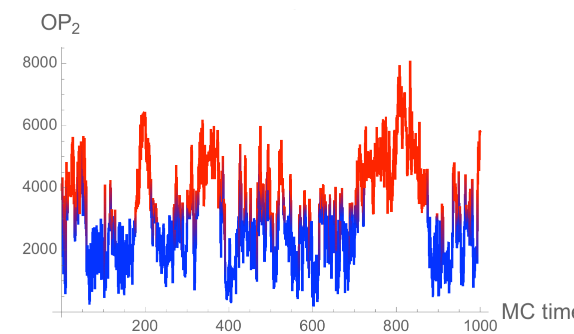
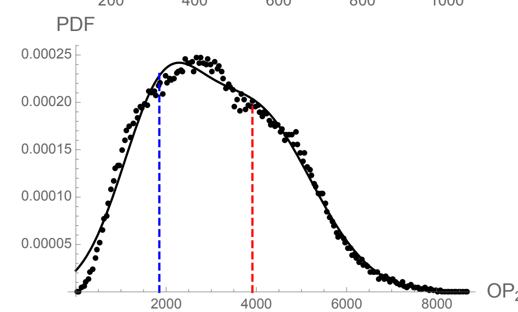
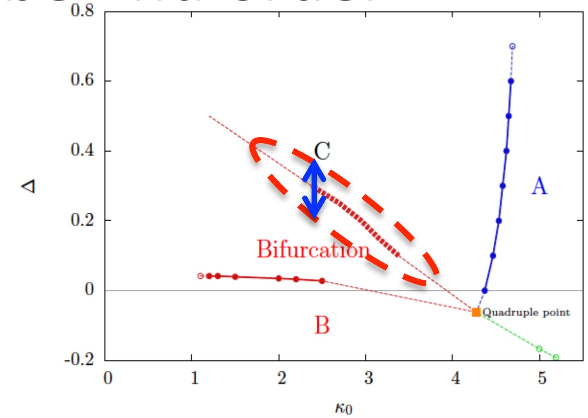




Phase transitions revisited

✧ *The new phase transition seems to be 2nd order*



- ✧ *To analyze phase transitions one needs to define a suitable order parameter OP*
- ✧ *(Pseudo)critical point is characterized by max in susceptibility X_{OP}*
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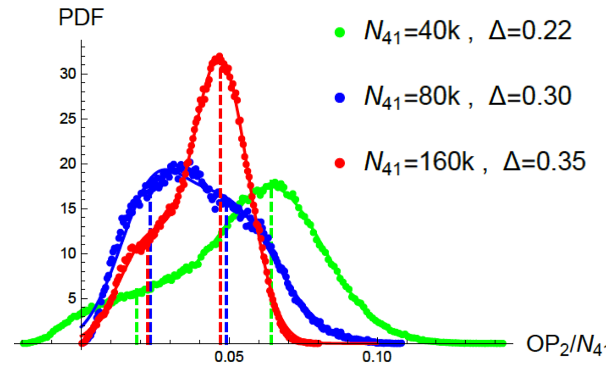

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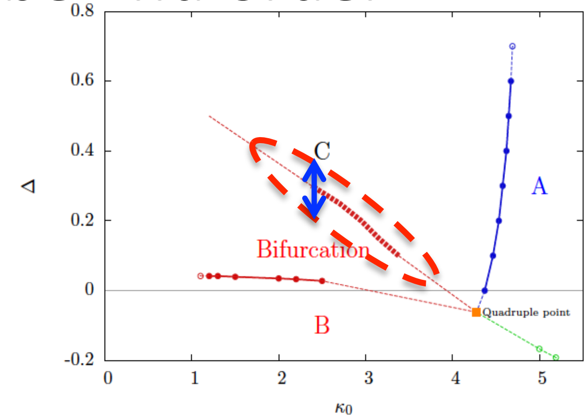

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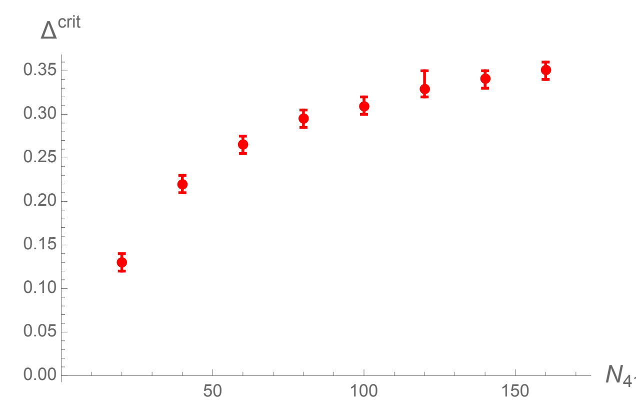

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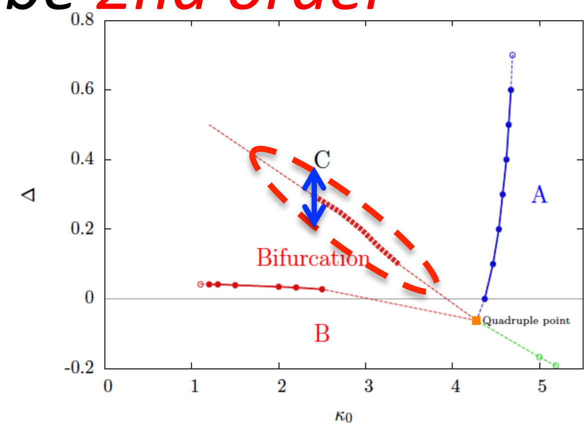

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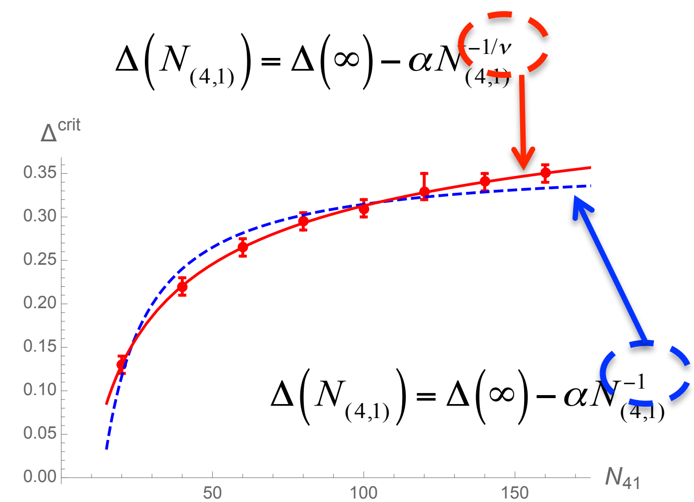
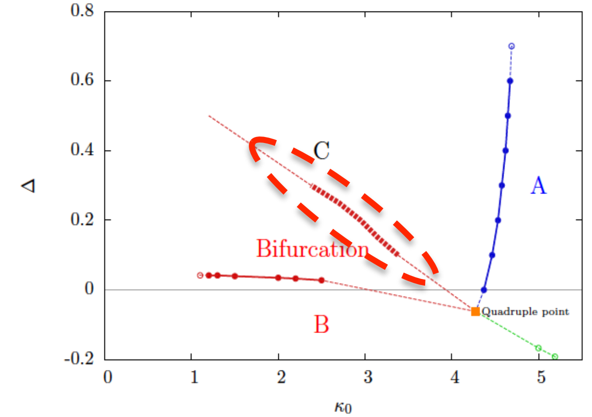


Signature change

✧ *Bifurcation of the effective action near phase transition can be interpretted as an effective signature change*



Signature change

✧ Bifurcation of the effective action near phase transition can be interpretted as an effective signature change

- ✧ The transfer matrix bifurcates at the new phase transition
- ✧ The phase transition is related to: $c_0 \rightarrow 0$ and $s_b \rightarrow \infty$ limit
- ✧ For large latice volumes ($n+m \rightarrow \infty$) one can expand in powers of $2c_0(n-m)/\Gamma << 1$
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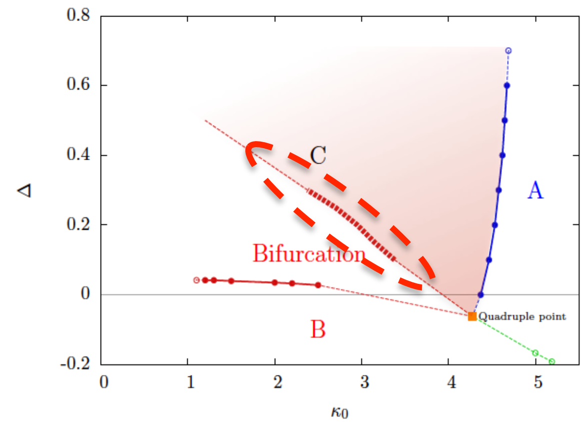

$$\langle n | M_C | m \rangle = N[n+m] \exp\left(-\frac{(m-n)^2}{\Gamma(n+m)} \right)$$

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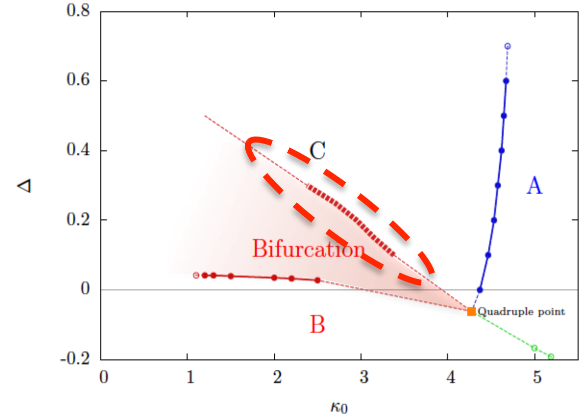

$$\langle n | M_B | m \rangle = N[n+m]\left[\exp\left(-\frac{\left(m-n-\left[c_0(n+m-s_b)\right]_+\right)^2}{\Gamma(n+m)}\right)+\exp\left(-\frac{\left(m-n+\left[c_0(n+m-s_b)\right]_+\right)^2}{\Gamma(n+m)}\right)\right]$$

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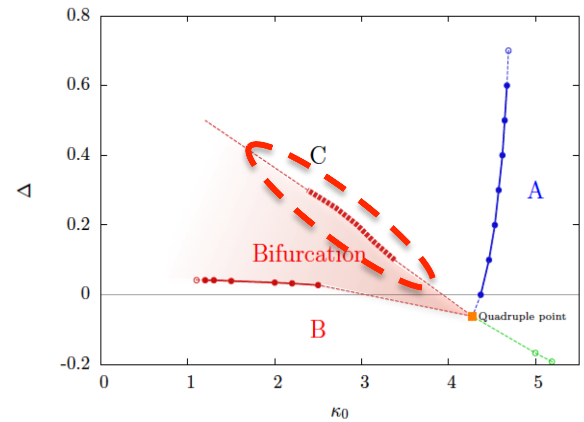

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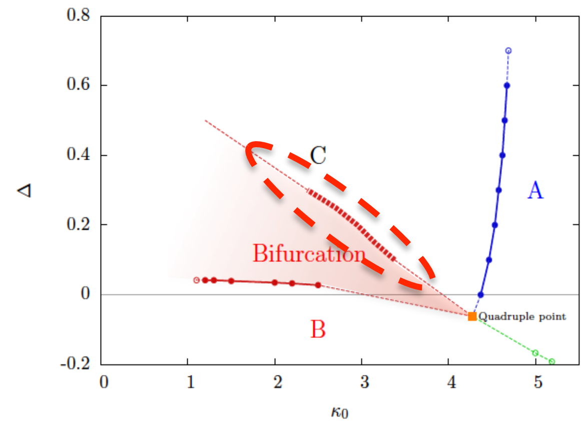

$$\langle n | M_B | m \rangle = N[n+m] \exp\left[-\frac{c_0^2}{\Gamma}(n+m)\right] \exp\left[-\left(1-\frac{2c_0^2(n+m)^2}{\Gamma}\right)\frac{1}{\Gamma}\frac{(m-n)^2}{(n+m)} - \frac{4}{3}\left(\frac{c_0(n-m)}{\Gamma}\right)^4 + \ldots\right]$$

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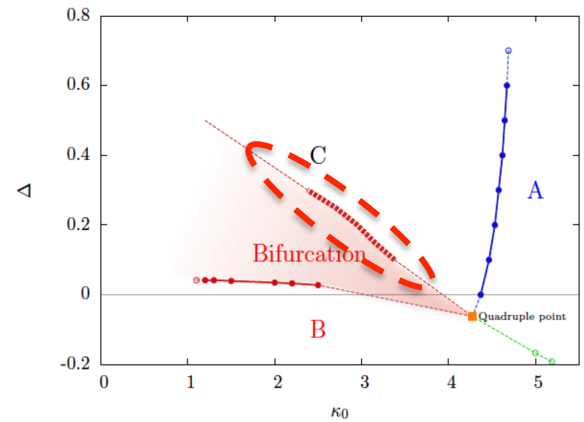

$$L_B = \left(1 - \frac{2c_0^2 (n+m)^2}{\Gamma}\right)\frac{1}{\Gamma}\frac{(n-m)^2}{n+m} + potential[n+m]$$

$$\langle n | M_B | m \rangle = N[n+m]\exp\left[-\frac{c_0^2}{\Gamma}(n+m)\right]\exp\left[-\left(1-\frac{2c_0^2(n+m)^2}{\Gamma}\right)\frac{1}{\Gamma}\frac{(m-n)^2}{(n+m)} - \frac{4}{3}\left(\cancel{\frac{c_0(n-m)}{\Gamma}}\right)^4 + \ldots\right]$$

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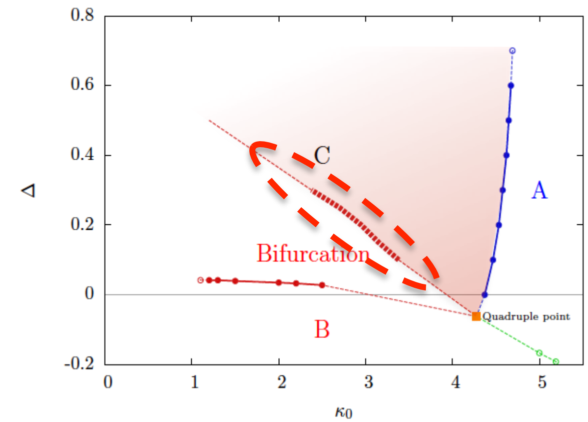

$$L_c = \frac{1}{\Gamma}\frac{(n-m)^2}{n+m} + potential[n+m]$$

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Conclusions

✧ *Using the transfer matrix method a new D (bifurcation) phase was discovered*
JHEP1406(2014)034

✧ *The geometry inside the new phase is very non-trivial and much different than inside the C (de Sitter) phase*
JHEP1508(2015)033

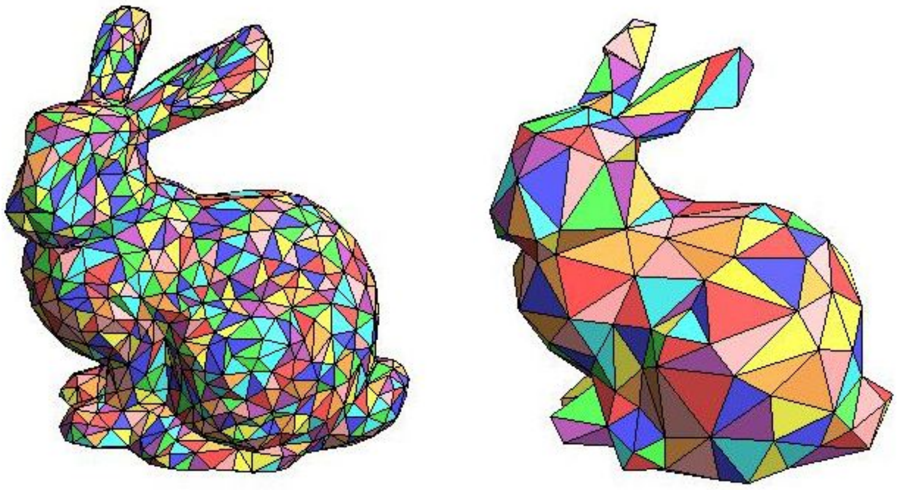
✧ *The transition seems to be 2nd order*
JHEP1602(2016)144

✧ *New phase transition may be related to signature change*
JHEP1508(2015)033

Prospects

✧ *Different topology of spatial slices ($S^3 \Rightarrow T^3$)*

✧ *Inclusion of matter (massless scalar fields)*



Thank You !

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JAGIELLOŃSKI

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