Status of Singularity Resolution in Loop Quantum Cosmology

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Singularities of General Relativity and their Quantum Fate

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Based on work of: Ashtekar, Bojowald, Craig, Corichi, Diener, Gupt, Joe, Kaminski, Lewandowski, Montoya, Megevand, Pawlowski, PS, Saini, Vandersloot, Wilson-Ewing, ...

Outline

- Introduction and a brief overview of loop quantum cosmology
- Singularity resolution and the quantum bounce.
- Development of new and more efficient tools for numerical simulations. Results from numerical simulations with various kinds of initial states, in presence of a potential and anisotropies.
- Possible singularities in loop quantum cosmology
- Open issues and future directions

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- Rigorous construction of mathematically and physically consistent model quantum spacetimes.
- Develop and rigorously test different tools and techniques to extract reliable physics.
- Understand potential quantum gravity implications for singularity resolution, early universe and black hole physics.

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Main Caveat: Quantization of homogeneous spacetimes is "quantum mechanics of spacetime." Where as full quantum gravity is "QFT of spacetime." Assuming homogeneity of spacetime, various hurdles of the full quantum gravity can be bypassed. Hope is that some qualitative aspects captured. Though, tempting to use these qualitative aspects to guess a fuller picture, there can be many pitfalls.

Loop quantum gravity/cosmology

Non-perturbative background independent quantization based on Ashtekar variables: connection A_a^i and the triad E_i^a.

- High mathematical precision, thanks to the work of several people in 90's: **(Ashtekar, Baez, Barbero, Bombelli, Corichi, Isham, Gambini, Jacobson, Lewandowski, Marolf, Morau, Pullin, Rovelli, Smolin, Thiemann, Varadarajan ...)**
- At a kinematical level, classical differential geometry replaced by quantum discrete geometry.
- Uniqueness of quantum theory **(Lewandowski, Okolow, Sahlmann, Thiemann; Fleischhack (2005))**

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Loop quantum cosmology: Symmetry reduce connection and triads at classical level, and then quantize. Various kinematical features of LQG understood in LQC **(Bojowald; Ashtekar, Bojowald, Lewandowski (2001-03))**. (Indications of non-singular behavior).

First explicit evidence of singularity resolution in physical Hilbert space in isotropic models **(Ashtekar, Kaminski, Lewandowski, Pawlowski, PS, Szulc, Vandersloot (06-07))**. Many regularizations severly restricted **(Corichi, PS (08))**, leading to a unique choice in isotropic models **(Ashtekar, Pawlowski, PS (06))**.

Gravitational part of Hamiltonian constraint:

$$C_{\text{grav}} = - \int_{\mathcal{V}} d^3x \, N \, \varepsilon_{ijk} \, F^i_{ab} \, (E^{aj} E^{bk} / \sqrt{|\det E|})$$

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Curvature modifications from field strength/holonomies

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Inverse triad (or inverse volume) corrections

Many early results (pre-2006) based on this. Examples: black hole mass threshold **(Bojowald, Goswami, Maartens, PS (05))**, absence of naked singularities **(Goswami, Joshi, PS (05))**. Modification not tied to any curvature scale and does not dictate quantum dynamics unless intrinsic curvature is non-vanishing. But, can lead to singularity resolution by itself in latter spacetimes **(PS, Toporensky (04))**.

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- Extract robust predictions

Massless scalar in spatially flat FLRW spacetime

Classically singularity inevitable (disjoint expanding and contracting trajectories)

Quantum Hamiltonian constraint: $\partial_\phi^2 \Psi = -\Theta \Psi$ **(Ashtekar, Pawlowski, PS (06))**

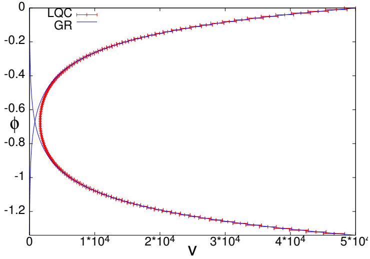
$$\Theta\Psi := -B(v)^{-1}[C^+(v)\Psi(v+4,\phi) + C^o(v)\Psi(v,\phi) + C^-(v)\Psi(v-4,\phi)]$$

$$\begin{aligned}
C^+(v) &= \frac{3\pi KG}{8}|v+2|\,||v+1|-|v+3||\,, \quad K = \frac{2}{3\sqrt{3\sqrt{3}}} \\
C^-(v) &= C^+(v-4) = \frac{3\pi KG}{8}|v-2|\,||v-3|-|v-1||\,, \\
C^0(v) &= -C^+(v) - C^-(v), \\
B(v) &= \frac{27K}{8}|v|\,\Big||v+1|^{1/3} - |v-1|^{1/3}\Big|^3 .
\end{aligned}$$

- Discreteness fixed by the underlying quantum geometry
- ϕ plays the role of internal time. Relational dynamics.
- Θ positive definite and self adjoint.
- Dirac Observables: p_ϕ, $|v|_{\phi_o}$

Quantum difference equation resulting from quantum geometry results in Wheeler-DeWitt differential equation at large volumes.

Loop quantum universes do not encounter big bang in the backward evolution. Big bang is replaced by a quantum bounce without any fine tuning.



Initial states sharply peaked in a macroscopic universe numerically found to bounce at $\rho_{\rm max} \approx 3/8\pi G\Delta \approx 0.41\rho_{\rm Planck}$.
(Δ: minimum eigenvalue of the area operator in LQG).

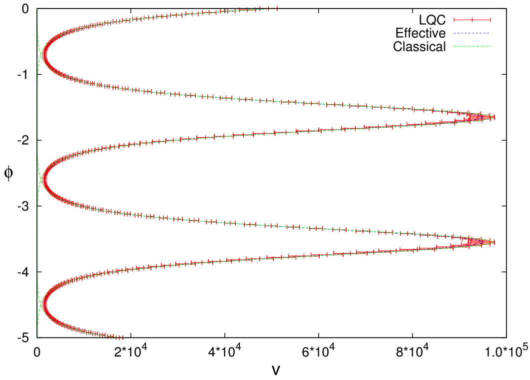
For such states, under certain assumptions, bounce captured by modified Friedmann equation on an effective continuum spacetime

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_{\rm max}}\right)$$

Massless scalar in $k = 1$ FLRW model

A small change in classical theory, but a significant change in quantization (Ashtekar, Pawlowski, PS, Vandersloot; Szulc, Kaminski, Lewandowski (2007)).

Quantization overcame difficulties noted by Green and Unruh (04) on viability of earlier loop quantization by Bojowald and Vandersloot (03).



Singularities avoided, and a very accurate test of recovery of GR from LQC when spacetime curvature is small.

Rigorous quantization and detailed physics analyzed for:

- Using a different lapse, quantum Hamiltonian constraint can be exactly solved for spatially flat FLRW with a massless scalar (sLQC) **(Ashtekar, Corichi, PS (08))**. Predicts minimum non-zero volume for all the states. Universal maximum of energy density in the physical Hilbert space ($\rho_{\max}$)

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- Cosmological constant **(Ashtekar, Bentivegna, Kaminski, Pawlowski (07-12))**
- Radiation **(Pawlowski, Pierini, Wilson-Ewing (15))**
- Potentials **(Ashtekar, Pawlowski, PS; Diener, Gupt, Megevand, PS (To appear))**
- Bianchi-I, II and IX spacetimes **(Ashtekar, Diener, Joe, Martin-Benito, Megevand, Mena-Marugan, Pawlowski, PS, Wilson-Ewing (09-16))**
- LRS Gowdy **(de Blas, Olmedo, Pawlowski (15))**; Hybrid quantization of polarized Gowdy models **(Garay, Martin-Benito, Mena-Marugan, ... (09-15))**

Effective spacetime description

Under appropriate conditions, quantum evolution can be approximated by a continuum effective spacetime description. Embedding method **(Willis' PhD Thesis at Penn State (2004), Taveras (2008), PS, Taveras (To appear))** and moment expansion method **(Bojowald's talk)**.

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Embedding method

Always derived from the quantum Hamiltonian constraint in LQC. Relationship with LQC always very transparent. Uses coherent states. Assumptions: (i) small relative fluctuations, (ii) state should not probe regions very close to the Planck volume.

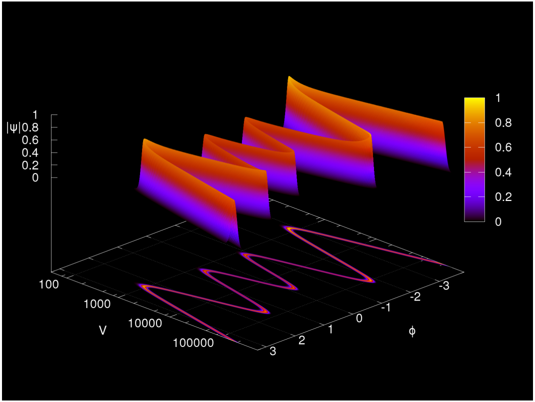
Rather tight assumptions and explicitely computed for limited models, but the method still works well.

For all the models in LQC where physical Hilbert space is known, numerical simulations show that it provides an excellent approximation for initial states which are sharply peaked at late times in a large macroscopic universe.

An example: bounce in presence of a potential

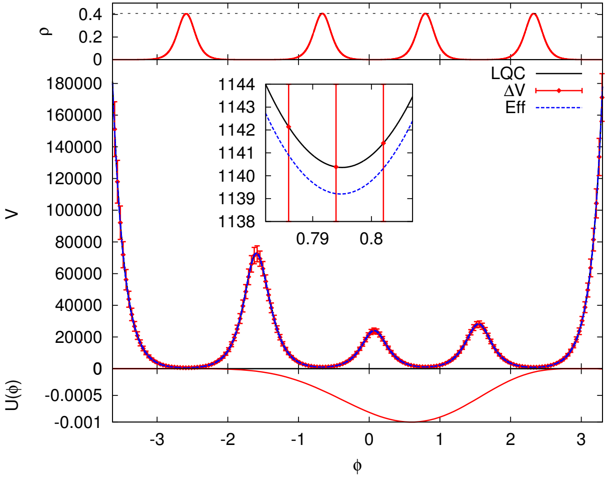
Cyclic model inspired potential: $U = U_o e^{-\phi^2}$

(Diener, Gupt, Megevand, PS (To appear))



Qualitative features of the bounce unaffected by the potential, for various choices of parameters and initial conditions.

Effective description in very good agreement in the presence of potential for sharply peaked initial states.



Quantum bounce and agreement with effective dynamics occurs also in non-kinetic domination regions.

Rich physics explored

Some examples:

- Singularity resolution in inflation for isotropic and anisotropic models, attractors and probability **(PS, Vereshchagin, Vandersloot (06); Ashtekar, Sloan (09,11); Corichi, Montoya (11); Ranken, PS (12); Gupt, PS (13); Bonga, Gupt (15))**
- Singularity resolution and onset of inflation in landscape scenarios **(Garriga, Vilenkin, Zhang; Gupt PS (2014))**
- Singularity resolution in pre-big bang cosmology, string inspired scenarios **(de Risi, Maartens, PS (07); Cailleteau, PS, Vandersloot (09))**
- Quantum Kasner transitions in Bianchi-I model with scalar field and perfect fluid **(Gupt, PS (2014))**. Interesting hierarchy found for different geometrical transitions across the bounce.
- Bianchi-IX spacetimes **(Corichi, Karami, Montoya (12-16))**
- Black hole interiors **(Ashtekar, Boehmer, Bojowald, Campiglia, Chiou, Corichi, Dadhich, Gambini, Joe, PS, Pullin (05-16))**

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- Is bounce an artifact of choosing special kinds of initial states? Does bounce occur if the initial state has very large quantum fluctuations?

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- Due to the heavy computational costs, many details of the anisotropic models in quantum theory partially explored (Martin-Benito, Mena Marugan, Pawlowski (2008)). Can we understand bounce(s) in anisotropic models with as much rigor as in the isotropic model?

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- Does quantum geometry always binds the curvature invariants? Are there any allowed singularities in LQC?

Numerical challenges for isotropic and anisotropic models

- Isotropic models:
  - For sharply peaked initial states simulations: $v_{\rm outer} \sim 10^5$, computational time $\sim$ 15 minutes on single core.
  - For widely spread states and those which can probe deep Planck regime, $v_{\rm outer} \sim 10^{12}$ (and higher). This requires 10^7 more spatial grid points. Since quantum grid is fixed, stability requirements lead to 10^7 finer time steps. Such a simulation would take 10^{10} years!

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- Anisotropic models:
  - Non-hyperbolicity encountered for Bianchi-I vacuum model when casted in relational observables. However, one can evaluate the entire physical wavefuntion by integration

$$\chi(b_1, v_2, v_3) = \int d\omega_2 d\omega_3 \tilde{\chi}(\omega_2, \omega_3) e_{\omega_1}(b_1) e_{\omega_2}(v_2) e_{\omega_3}(v_3)$$

  - For a state sharply peaked at $\omega_2 = \omega_3 = 10^3$, a typical simulations require 10^{14} floating point operations.
  - For wider states, and states probing deep quantum geometry, typical simulations require 10^{19} flop. Memory needed $\sim$ 5 Tb.

Probing deep Planck regime

Chimera scheme (Diener, Gupt, PS (2014)): Use an inner grid where the LQC difference equation is solved, and a carefully chosen outer grid at large volumes where the Wheeler-DeWitt theory is an excellent approximation. Choose logarithmic variable in outer grid. Makes characteristic speeds constant. Getting stable evolution easier at far less expense. With $v_{\rm int} = 10000$ and $v_{\rm outer} = 10^{12}$, evolution takes less than 10 minutes on a single core.

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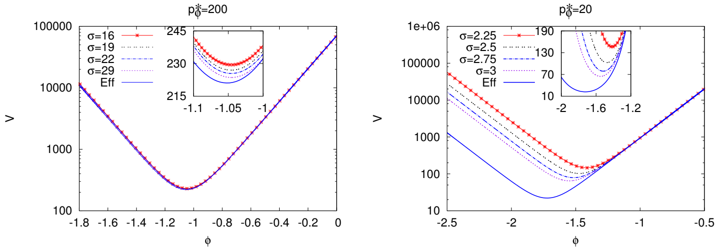
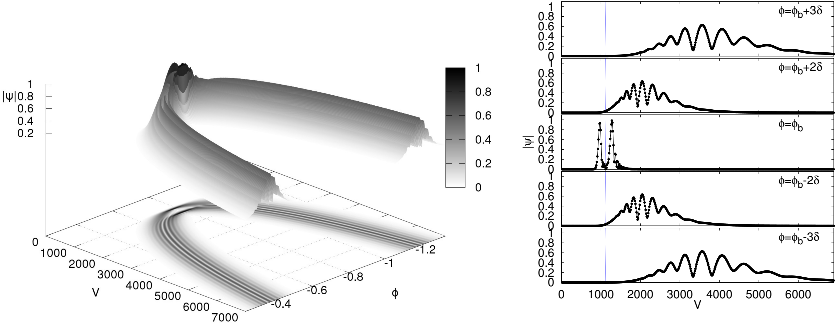


Widely spread quantum states bounce at smaller volume than predicted by effective theory (Diener, Gupt, PS (2014))

Quantum bounce for highly quantum states

Bounce not restricted to any special states. Even occurs for states which are highly non-Gaussian or squeezed.

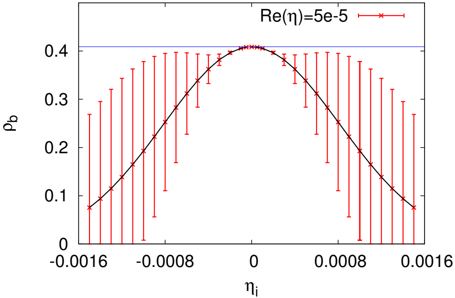
(Diener, Gupt, Megevand, PS (2014))



Tight constraints on the growth of the fluctuations across the bounce. State in the asymptotic future turns out to be very similar to the one in the asymptotic past. Results are in agreement with analytical estimates using sLQC **(Corichi, Kaminski, Montoya, Pawlowski, PS (2008-11))**

Effect of large quantum fluctuations on the bounce

In the isotropic model, quantum fluctuations are found to always lower the curvature scale at which the bounce occurs. Quantum fluctuations in the state enhance the “repulsive nature of gravity” in the quantum regime.



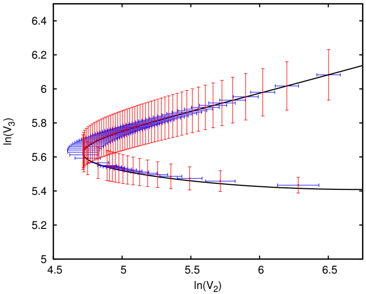
The bounce density can be much smaller than the estimate from sLQC $\rho_{\max} \approx 0.41\rho_{\rm Pl}$ depending on the initial state.

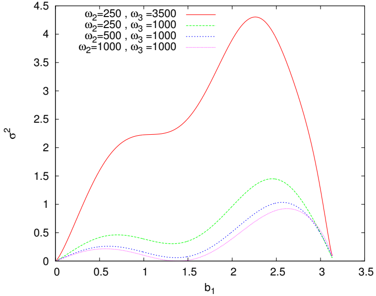
Anisotropic quantum bounce

Rigorous quantization of Bianchi-I vacuum model available. Singularity resolution found **(Martin-Benito, Mena Marugan, Pawlowski (2008))**.

Using high performance computing we can now rigorously understand the physics of quantum bounce in Bianchi-I vacuum

(Diener, Joe, Megevand, PS (To appear))





The effective theory of Bianchi-I spacetime is in an excellent agreement with the quantum evolution for various states. However, depending on the relative fluctuations in the state, departures exist.

Does LQC resolve all the singularities?

Assuming the validity of effective spacetime, spacetime curvature invariants can in principle diverge for isotropic, Bianchi-I and Kantowski-Sachs spacetimes (PS (09,11); Saini, PS (16))

Example: In the spatially flat isotropic model in LQC, spacetime curvature captured by

$$R = 6\left(H^2 + \frac{\ddot{a}}{a}\right) = 8\pi G\rho\left(1 - 3w + 2\frac{\rho}{\rho_{\max}}(1+3w)\right), \quad w = p/\rho$$

Even though energy density and Hubble rate have upper bound in LQC, pressure is not bounded.

If pressure diverges at a finite value of energy density, such as in sudden singularities (Barrow, Tsagas (04-05)), curvature invariants diverge in effective LQC.

Strength of possible singularities in LQC

Królak's criteria: The singularity at $\tau = \tau_o$ is strong if $\int_0^\tau d\tau' |R_{i\;\;4j4}|$ diverges as $\tau \to \tau_o$

Królak's conjecture: All relevant physical singularities which lead to geodesic incompleteness are strong curvature type.

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Strong curvature singularities are forbidden in effective LQC at least for isotropic, Bianchi-I and Kantowski-sachs spacetimes. No big bang/crunch, big rip, big freeze in finite proper time evolution **(PS (09,11); Saini, PS (16))**

Some of the most important open issues

Are predictions of LQC robust when inhomogeneous fluctuations of loop quantum gravity are switched on? **(Brunnemann, Thiemann (05))**

The inverse triad operators may not be bounded in LQG. (Recall they play little role in singularity resolution)

Does not affect results on bounce, but an important open issue to understand is whether holonomy modifications leading to bounce survive in top-down approach from loop quantum gravity.

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Rigorous relationship between loop quantum gravity and LQC: new insights and interesting results **(Beetle, Bodendorfer, Brunnemann, Engle, Fleishchack, Hogan, Mendonca (08-16)).**

Possible generalization of LQC results to infinite degrees of freedom in loop quantum gravity can be studied rigorously **(Domagala, Giesel, Kaminski, Lewandowski (10))**

Effective dynamics of bounce in LQC can be obtained from a gauge fixed version of loop quantum gravity **(Alesci, Cianfrani (14))**

Future directions and ongoing work

- Numerical simulations for various loop quantized Bianchi models can be performed rigorously (Pawlowski; Diener, PS (work in progress)). Promising avenue for quantum generalization of results in the classical theory (Berger, Garfinkle, Isenberg, Moncrief ...)
- Inclusion of inhomogenieities: Ongoing work in polarized Gowdy models suggests singularity resolution (Martin-Benito, Martin-de Blas, Mena Marugan, Olmedo, Pawlowski). Rich area for detailed explorations (Garfinkle's question).
- New avenues for cosmologies in spinfoams and group field theory (Bianchi, Gielen, Rovelli, Oriti, Sindoni, Sloan, Vidotto, Wilson-Ewing, ...).
- Schwarschild interior: Analytical studies indicate singularity resolution (Ashtekar, Bojowald, Campiglia, Chiou, Corichi, Gambini, Modesto, Pullin, PS). Detailed physics starting to be explored.