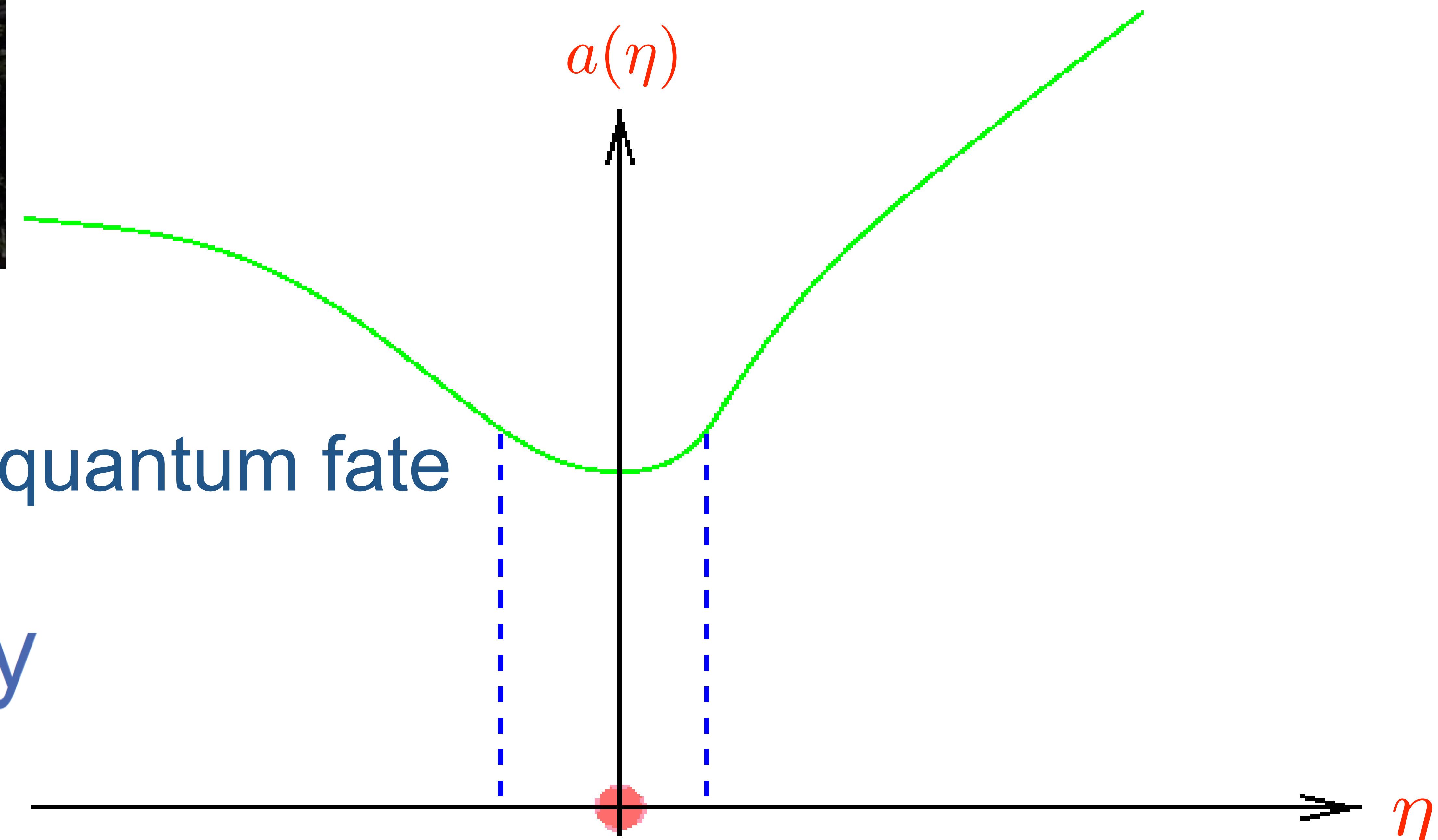
Patrick Peter (w/ S. Vitenti & A. Valentini)
Institut d'Astrophysique de Paris

Bouncing quantum cosmological solutions in the dBB approach

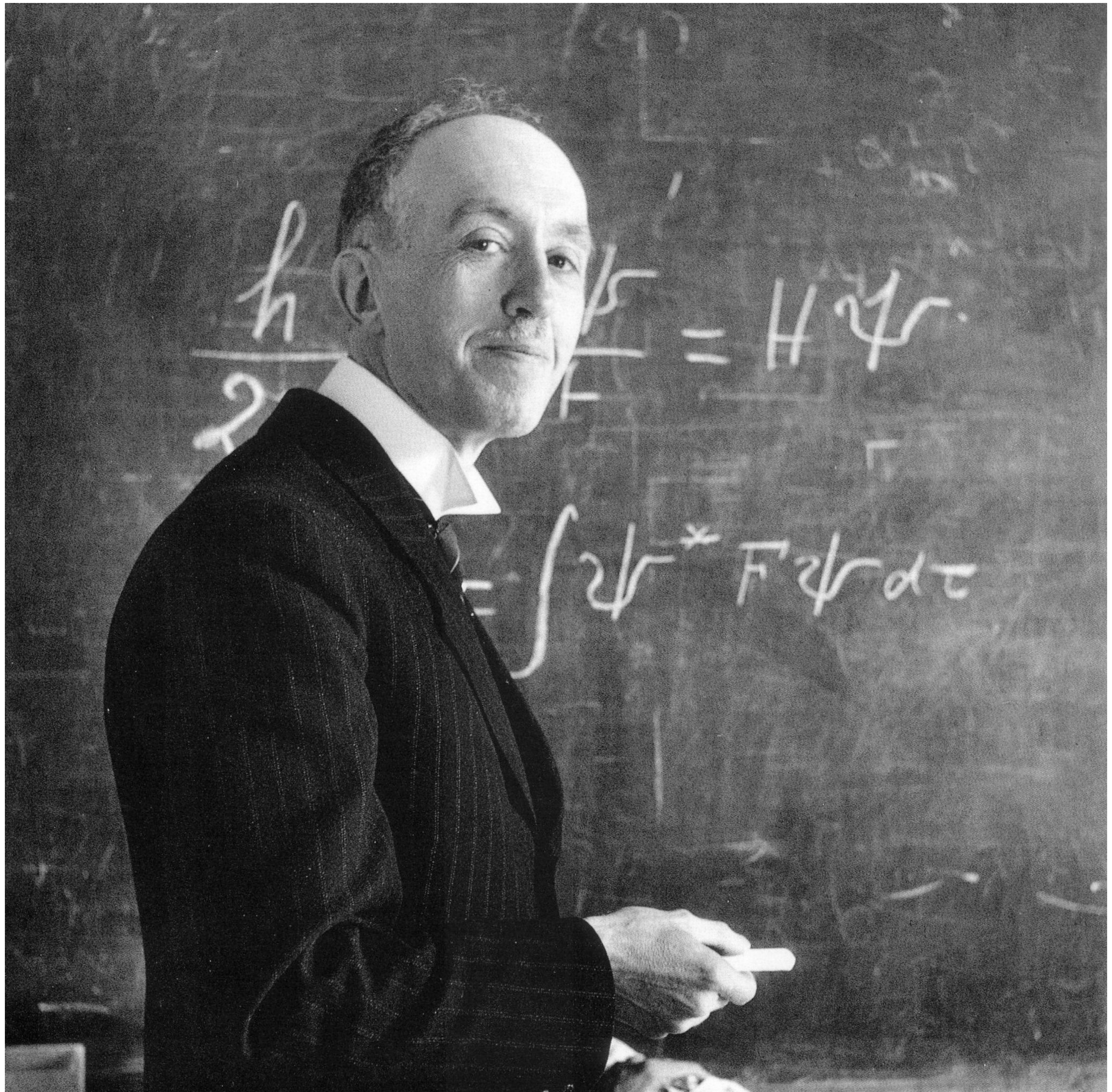


Singularities of general relativity and their quantum fate

Instytut Matematyczny
Polskiej Akademii Nauk

Banach center - Warsaw- June 30, 2016

Ontological interpretation (dBB)



Louis de Broglie

David Bohm

1927 Solvay meeting and von Neuman mistake ... ‘*In 1952, I saw the impossible done*’ (J. Bell)

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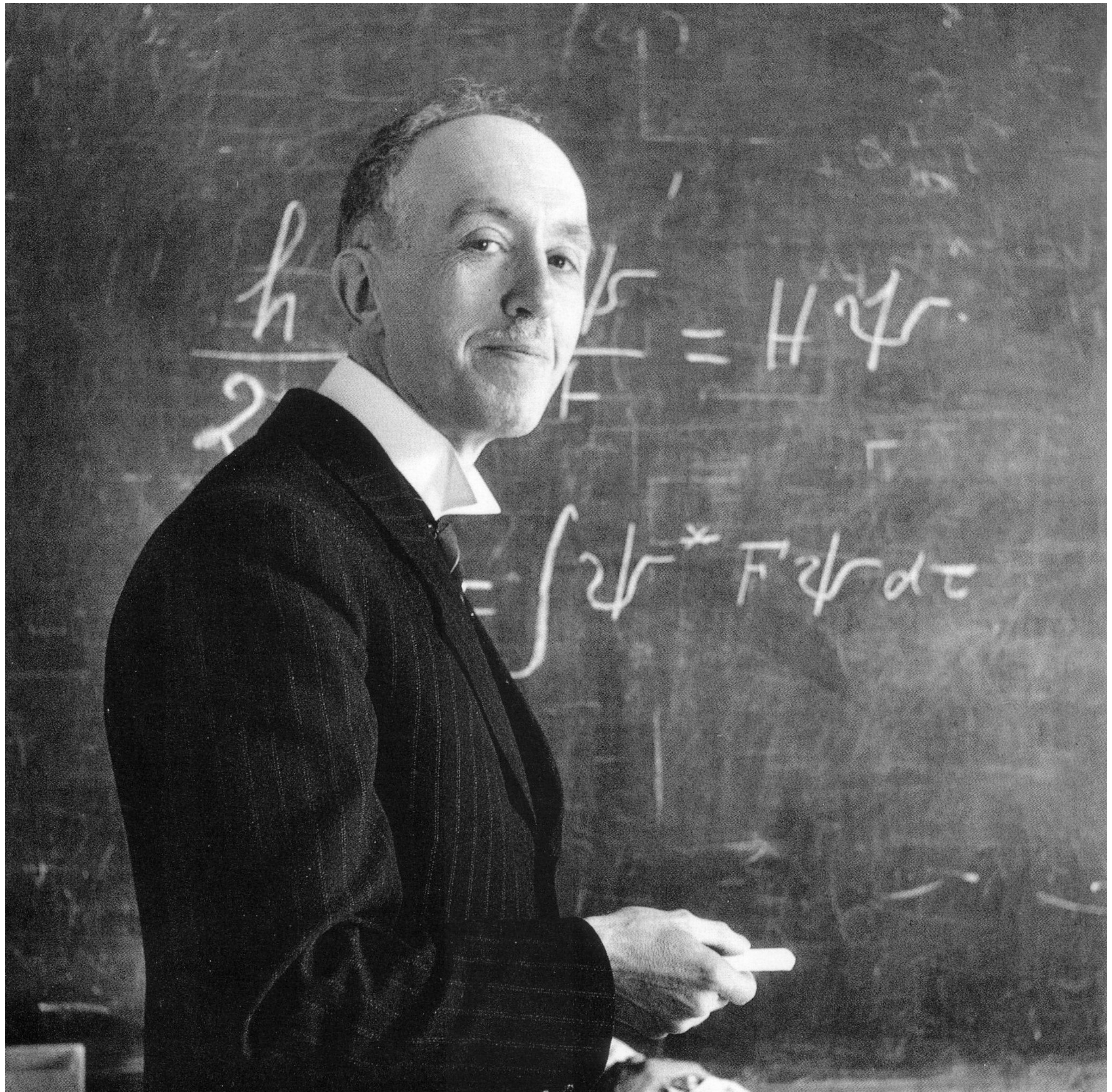

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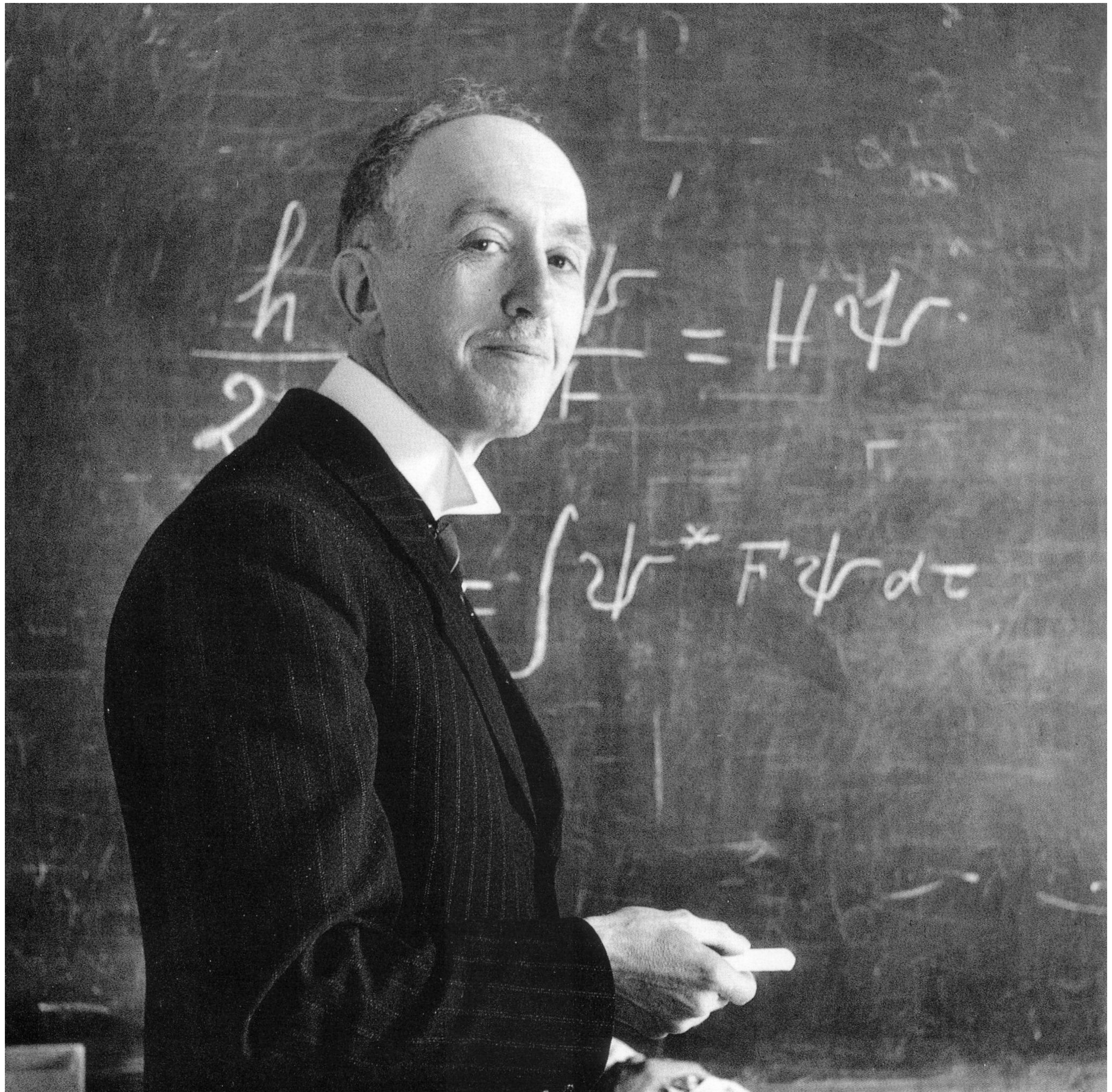

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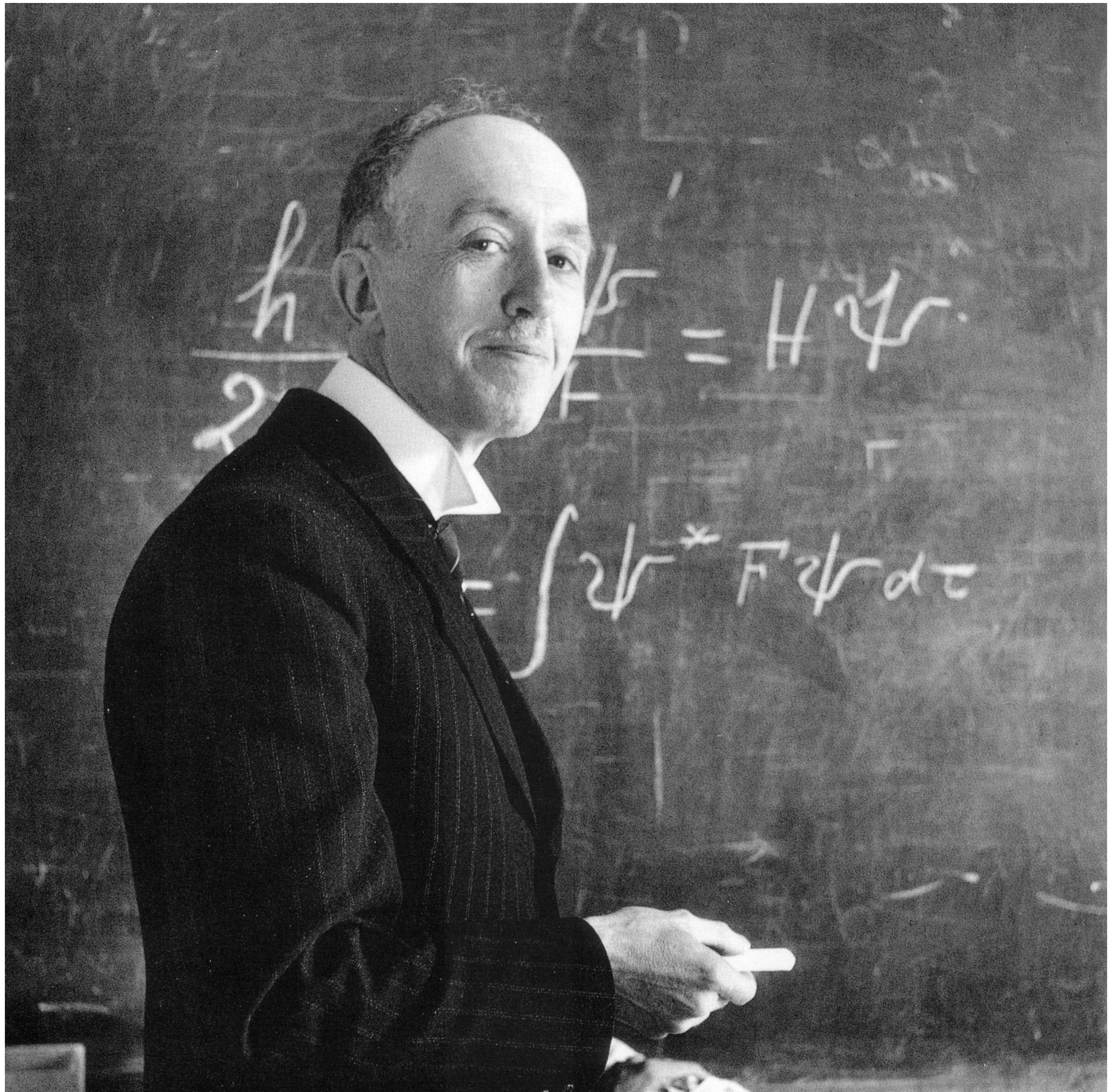
Ontological *formulation* (dBB)



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$$\Psi = A\,(\boldsymbol{r},t)\,\mathrm{e}^{iS(\boldsymbol{r},t)}$$

Ontological *formulation* (dBB) $\exists\, \boldsymbol{x}(t)$

Trajectories satisfy (de Broglie) $$m\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \Im\mathrm{m}\,\frac{\Psi^* \boldsymbol{\nabla}\Psi}{|\Psi(\boldsymbol{x},t)|^2} = \boldsymbol{\nabla} S$$

$$\Psi = A\left(\boldsymbol{r},t\right)\mathrm{e}^{iS(\boldsymbol{r},t)}$$

Ontological *formulation* (BdB) $\exists\,\boldsymbol{x}(t)$

Trajectories satisfy (Bohm) $$m\frac{\mathrm{d}^2\boldsymbol{x}}{\mathrm{d}t^2} = -\boldsymbol{\nabla}(V+Q)$$ $$Q \equiv -\frac{1}{2m}\frac{\boldsymbol{\nabla}^2|\Psi|}{|\Psi|}$$

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Properties:

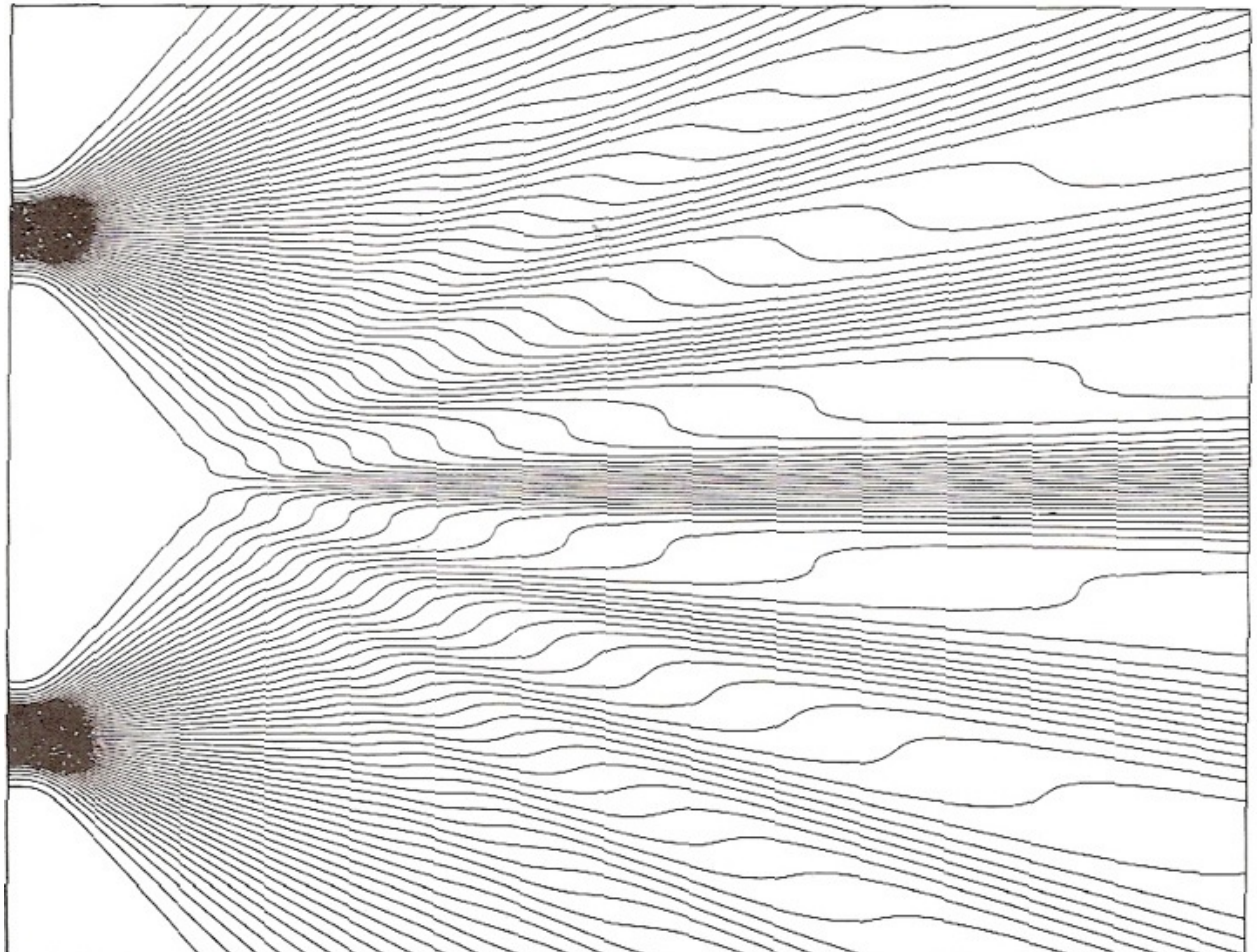
- ☺ strictly equivalent to Copenhagen QM
  - probability distribution (attractor)
    $\exists t_0; \rho(\boldsymbol{x},t_0) = |\Psi(\boldsymbol{x},t_0)|^2$

$$Q \equiv -\frac{1}{2m}\frac{\boldsymbol{\nabla}^2|\Psi|}{|\Psi|}$$

- ☺ classical limit well defined $Q \longrightarrow 0$
- ☺ state dependent
- ☺ $\exists$ intrinsic reality
  - non local …
- ☺ no need for external classical domain/observer!

The two-slit experiment:

Surrealistic trajectories?



Non straight in vacuum...

$$m\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} = -\nabla\left(V+Q\right)$$

... a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics.

R. P. Feynman (1961)

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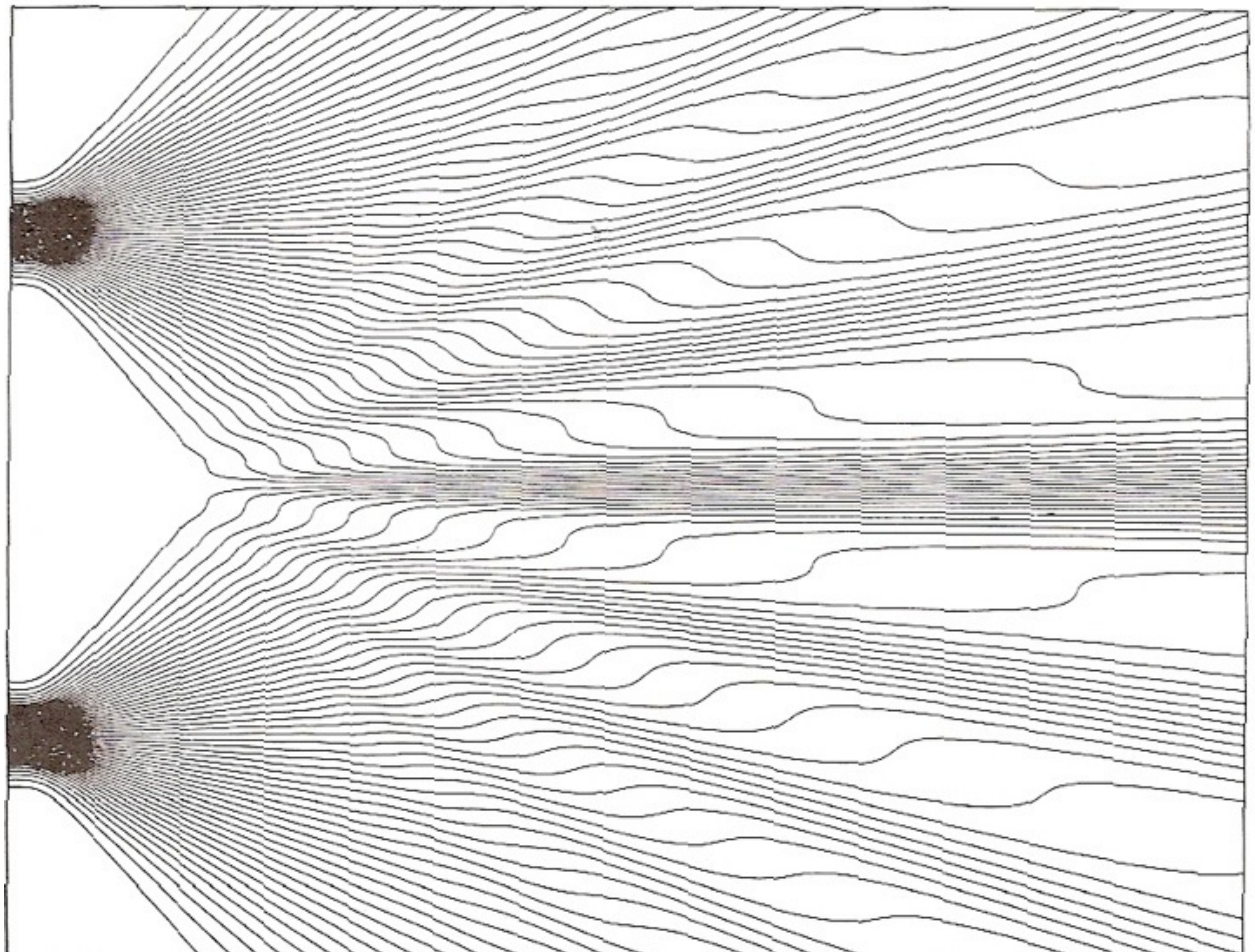

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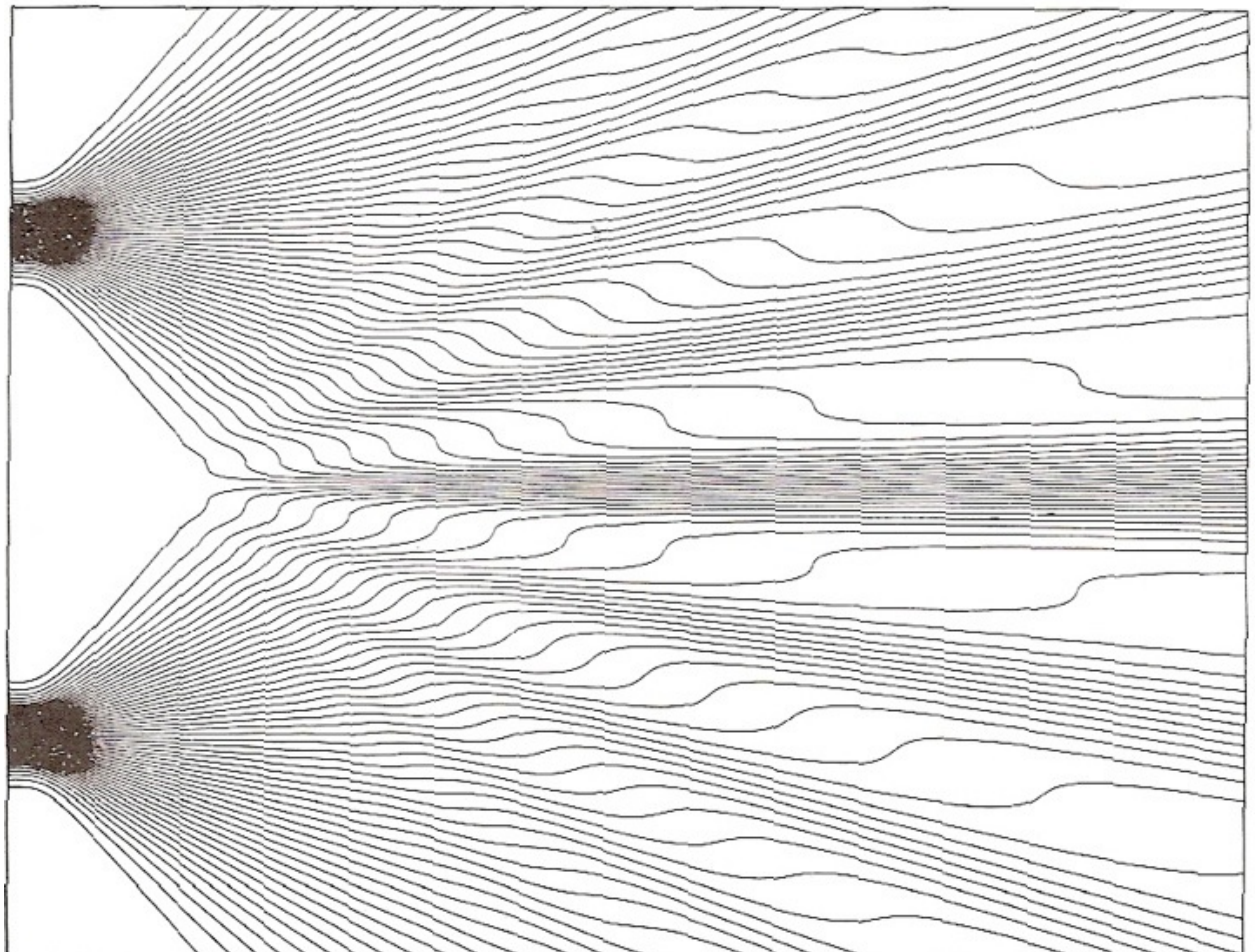

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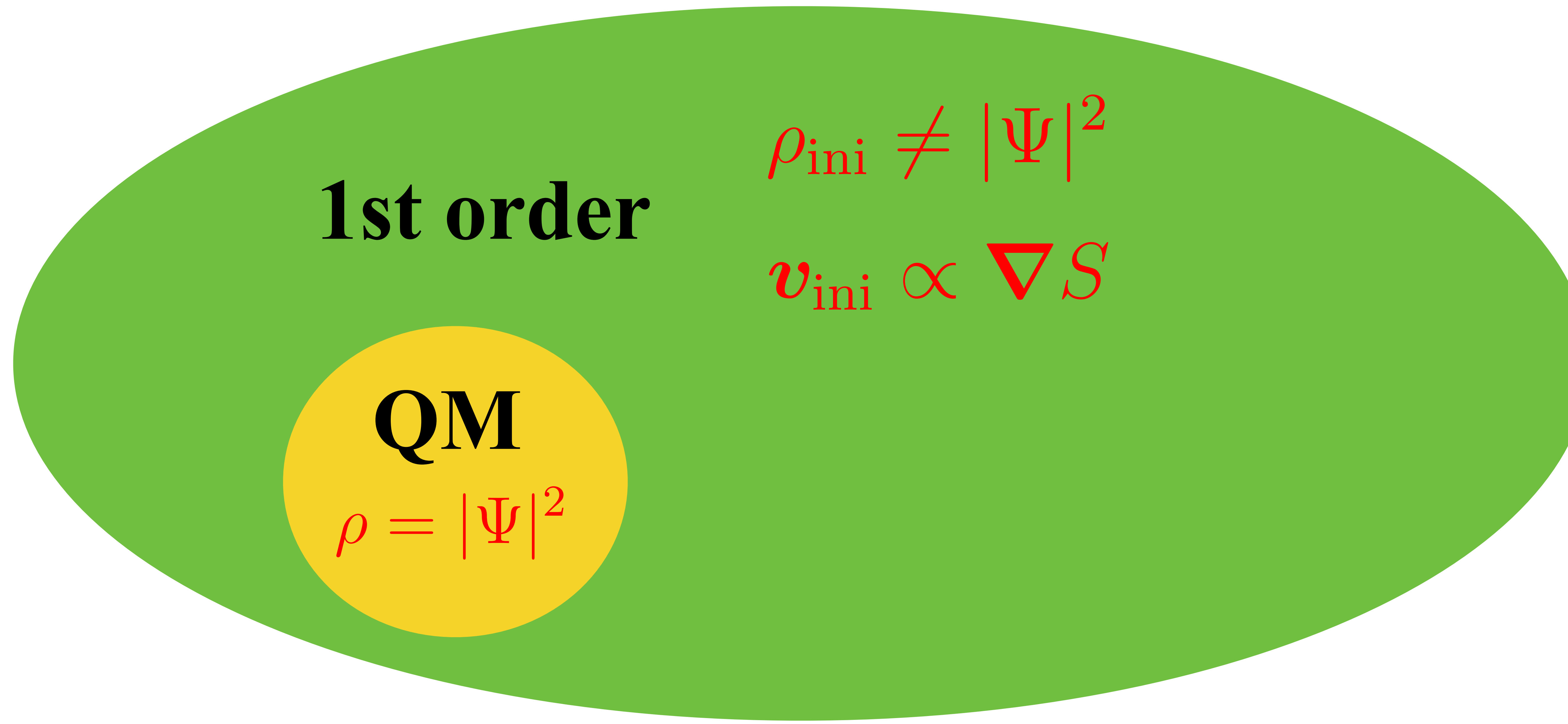
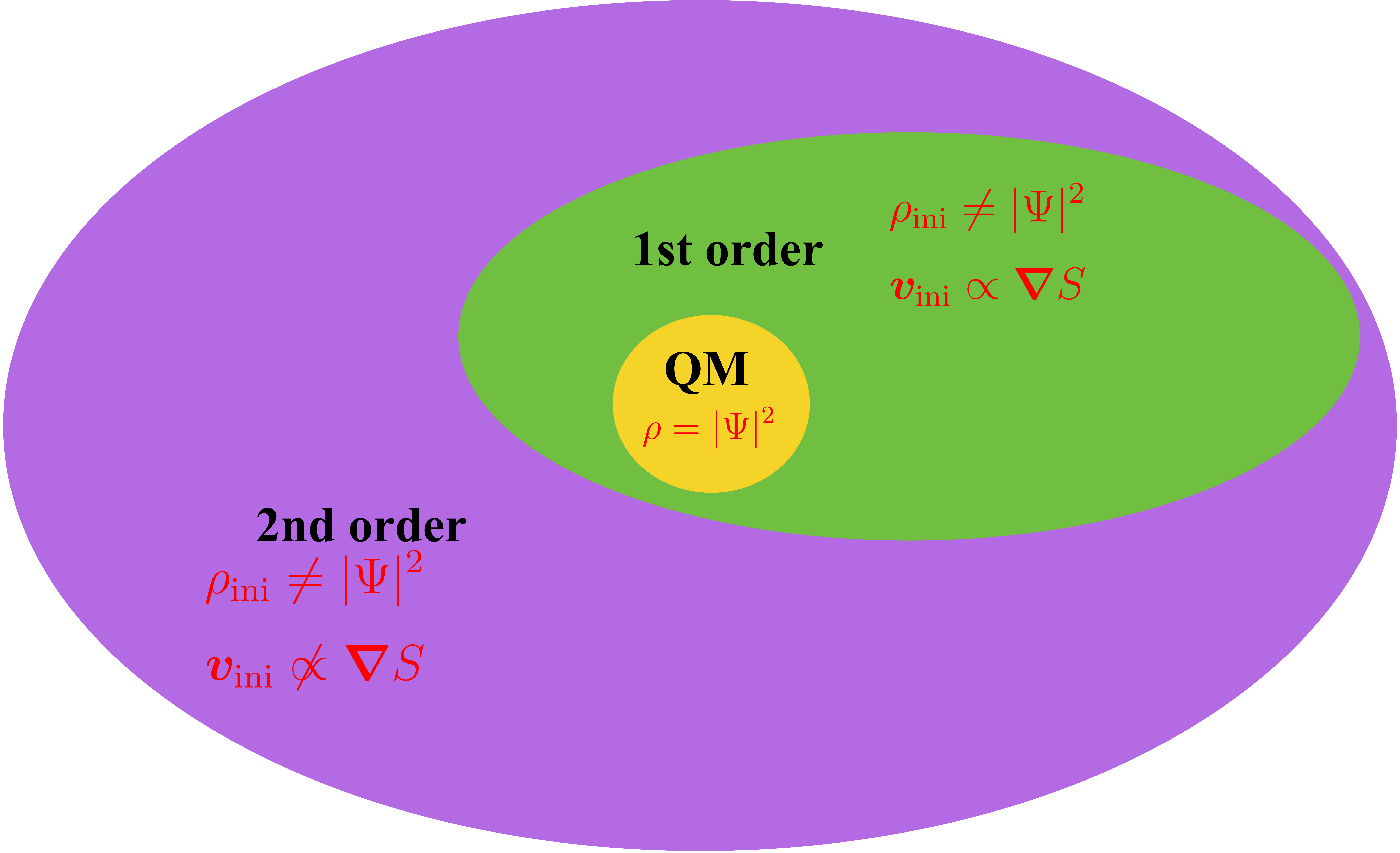
QM

$$\rho = |\Psi|^2$$


1st order
$\rho_{\mathrm{ini}} \neq |\Psi|^2$
$\boldsymbol{v}_{\mathrm{ini}} \propto \boldsymbol{\nabla} S$
QM
$\rho = |\Psi|^2$



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2nd order
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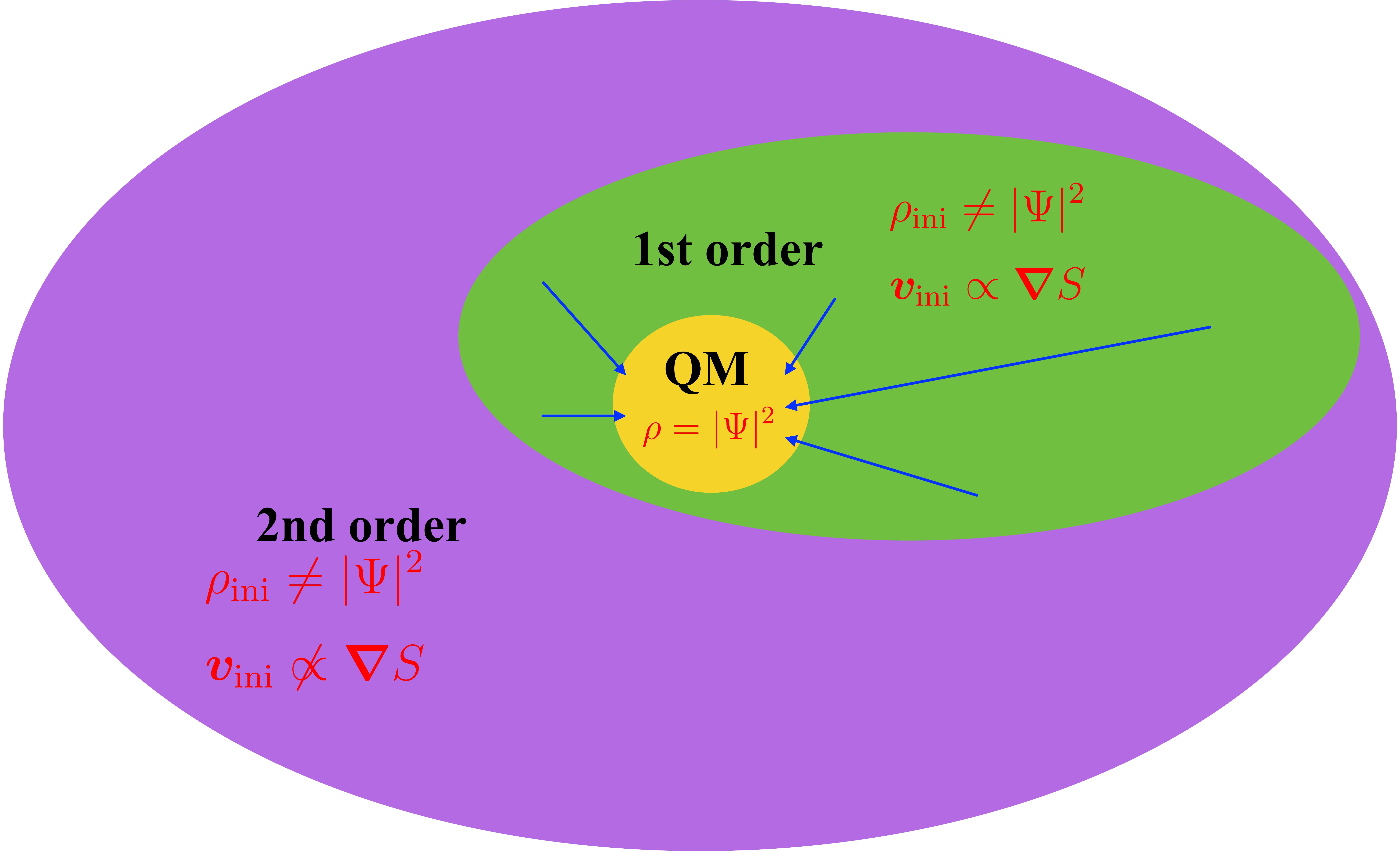

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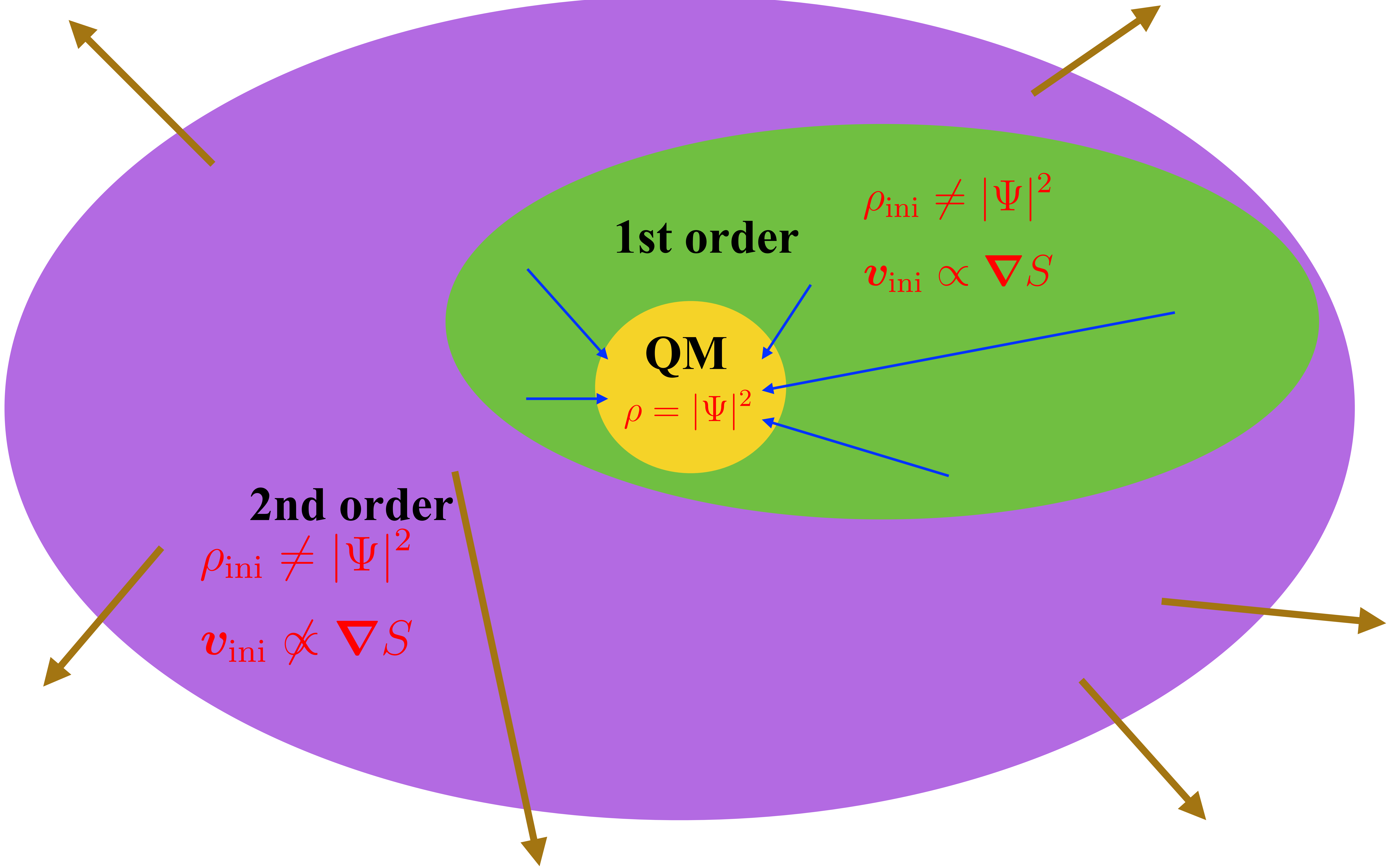

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2nd order: is unstable…

S. Colin & A. Valentini, Proc. R. Soc. A 470, 20140288 (2014)

ruled out!

1st order: can be tested?

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ruled out!

Primordial
Perturbation
Theory

Quantum equilibrium

(Valentini & Westman, 2005)

$$i\frac{\mathrm{d}}{\mathrm{d}t}|\Psi\rangle = \hat{H}|\Psi\rangle$$

Particle in a box - 2D

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\frac{\partial^2\psi}{\partial x^2} - \frac{1}{2}\frac{\partial^2\psi}{\partial y^2} + V\psi$$

infinite square well - size π

Density of actual configurations

$$\rho\,(x,y,t) \quad \Longrightarrow \frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}\left(\rho\dot{x}\right) + \frac{\partial}{\partial y}\left(\rho\dot{y}\right) = 0$$

continuity equation

Energy eigenfunctions $\phi_{mn}\left(x,y\right) = \frac{2}{\pi}\sin\left(mx\right)\sin\left(ny\right)$

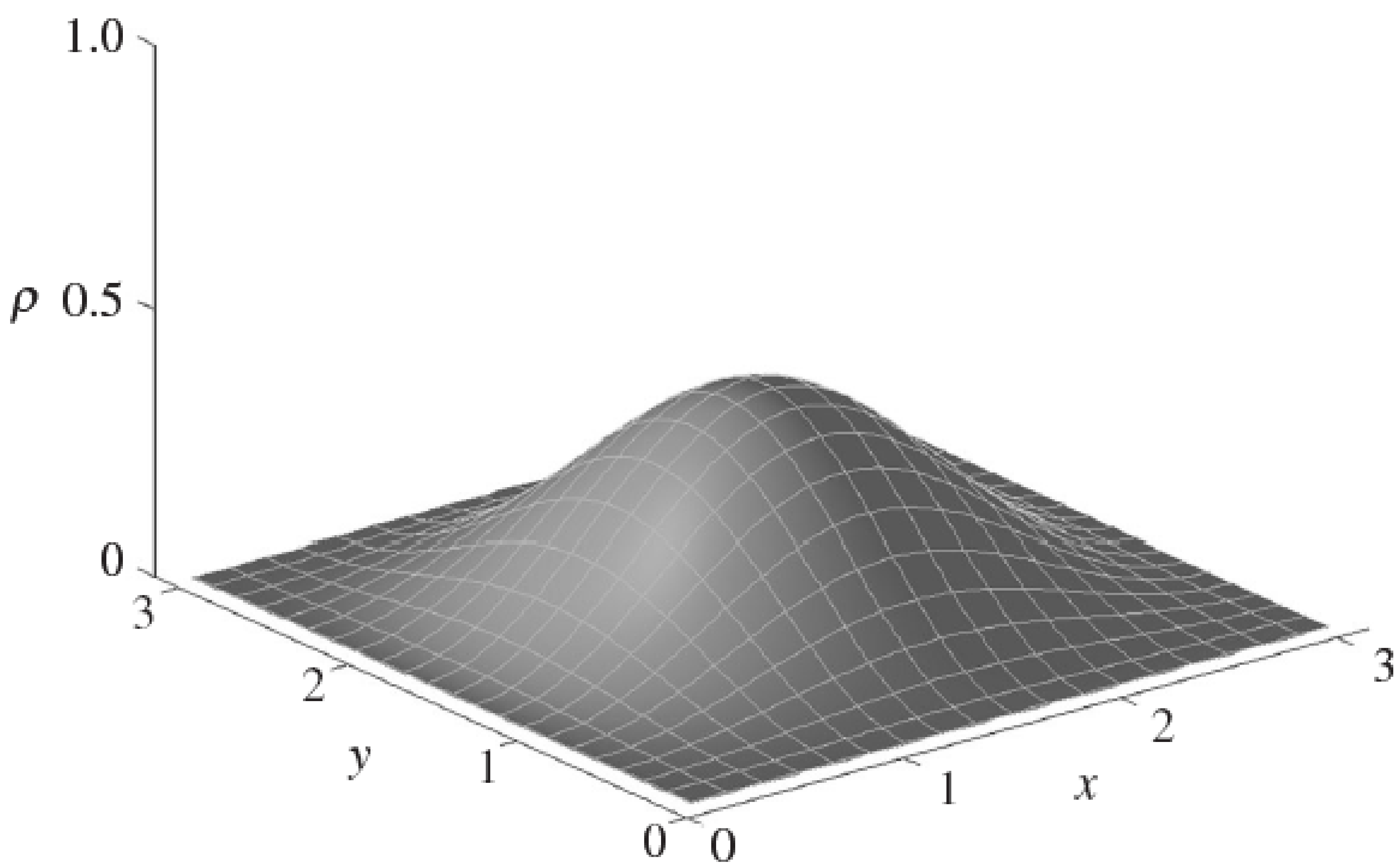
Energy levels $E_{mn} = \frac{1}{2}\left(m^2+n^2\right)$

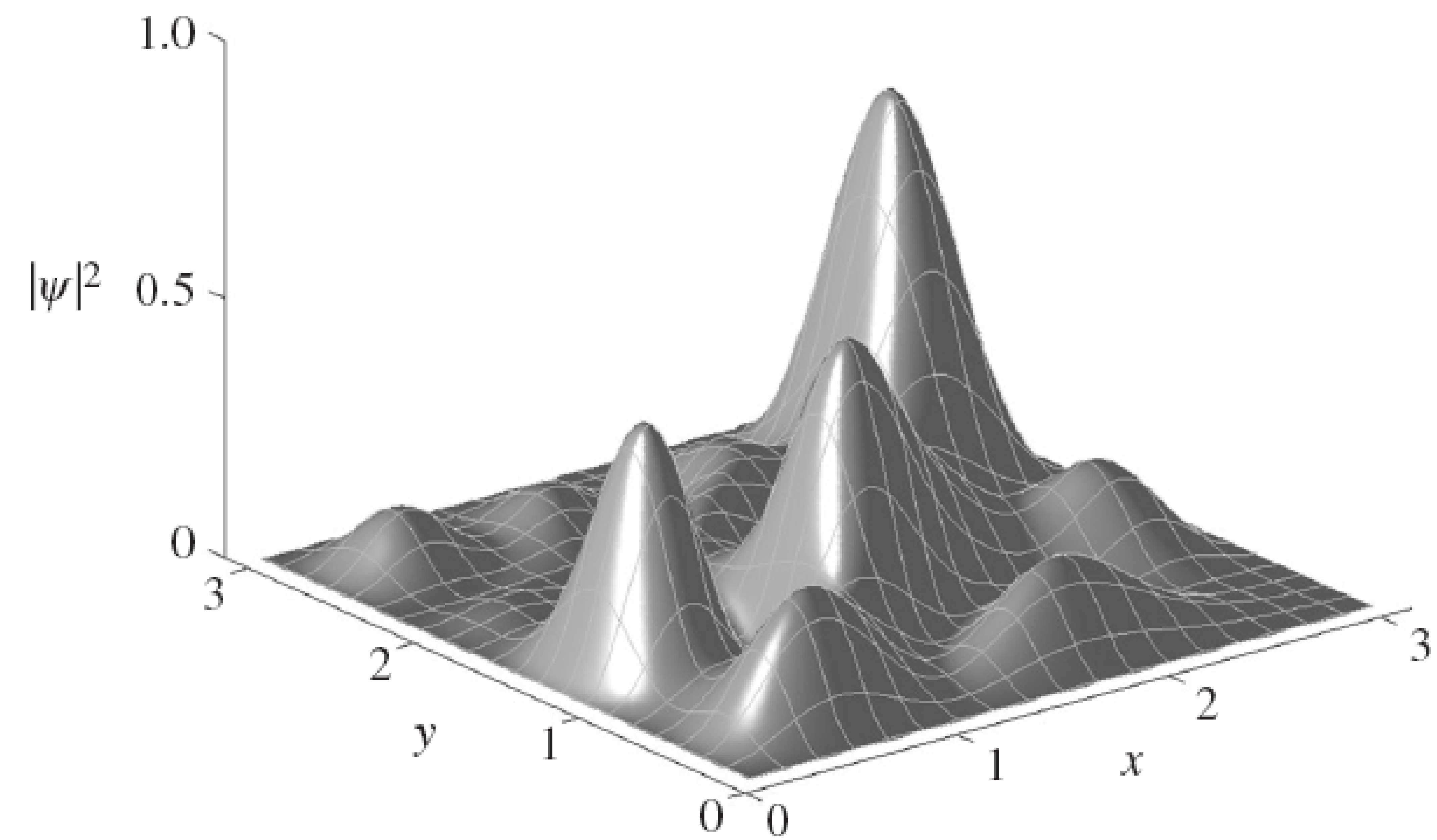
Initial configuration

$$\rho(x, y, 0) = |\phi_{11}(x, y)|^2$$

$$\psi(x, y, 0) = \sum_{m,n=1}^{4} \frac{1}{4} \phi_{mn}(x, y) \exp(i\theta_{mn})$$

$$\psi(x, y, t) = \sum_{m,n=1}^{4} \frac{1}{4} \phi_{mn}(x, y) \exp i(\theta_{mn} - E_{mn} t)$$

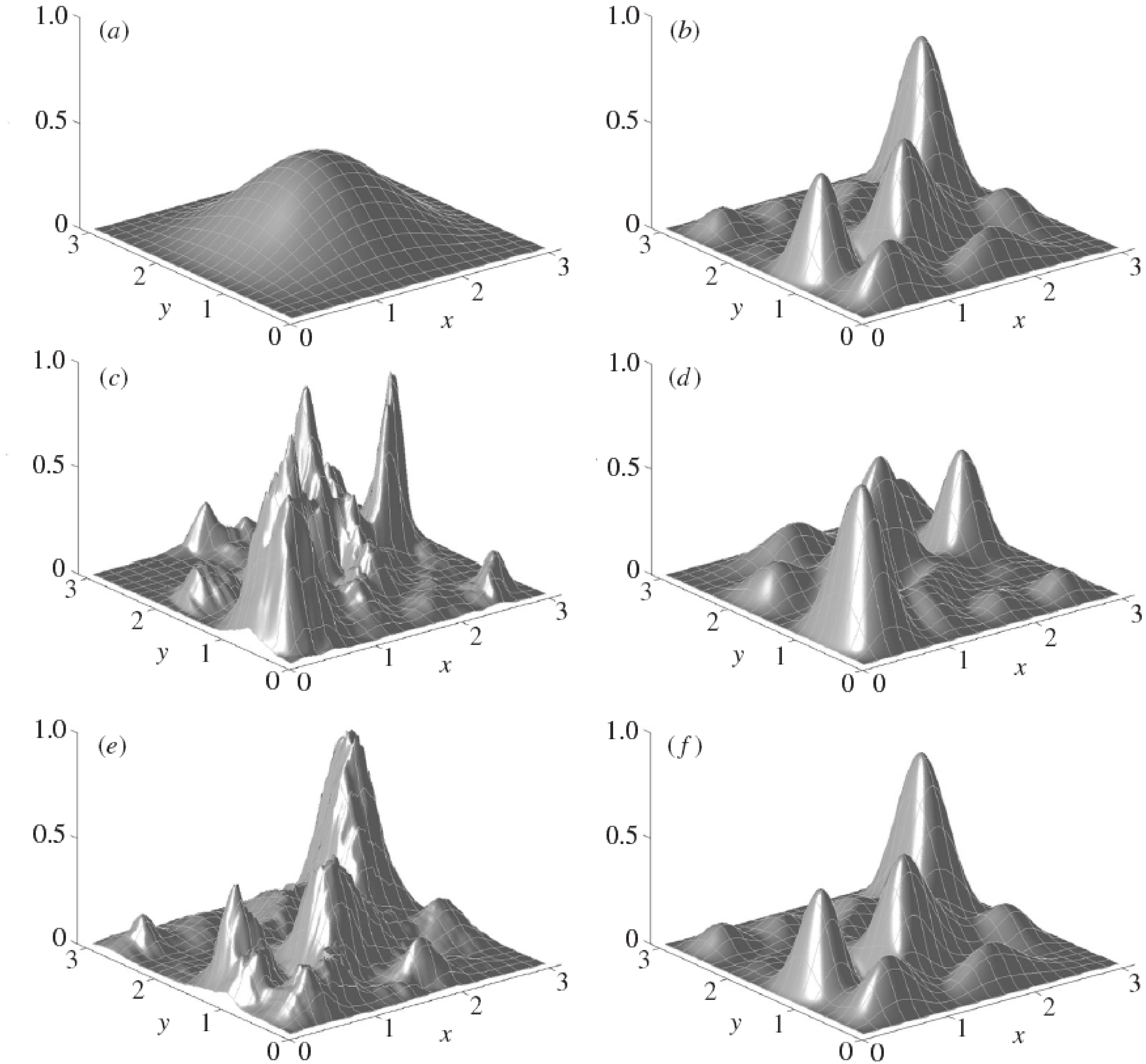




Dynamical evolutions

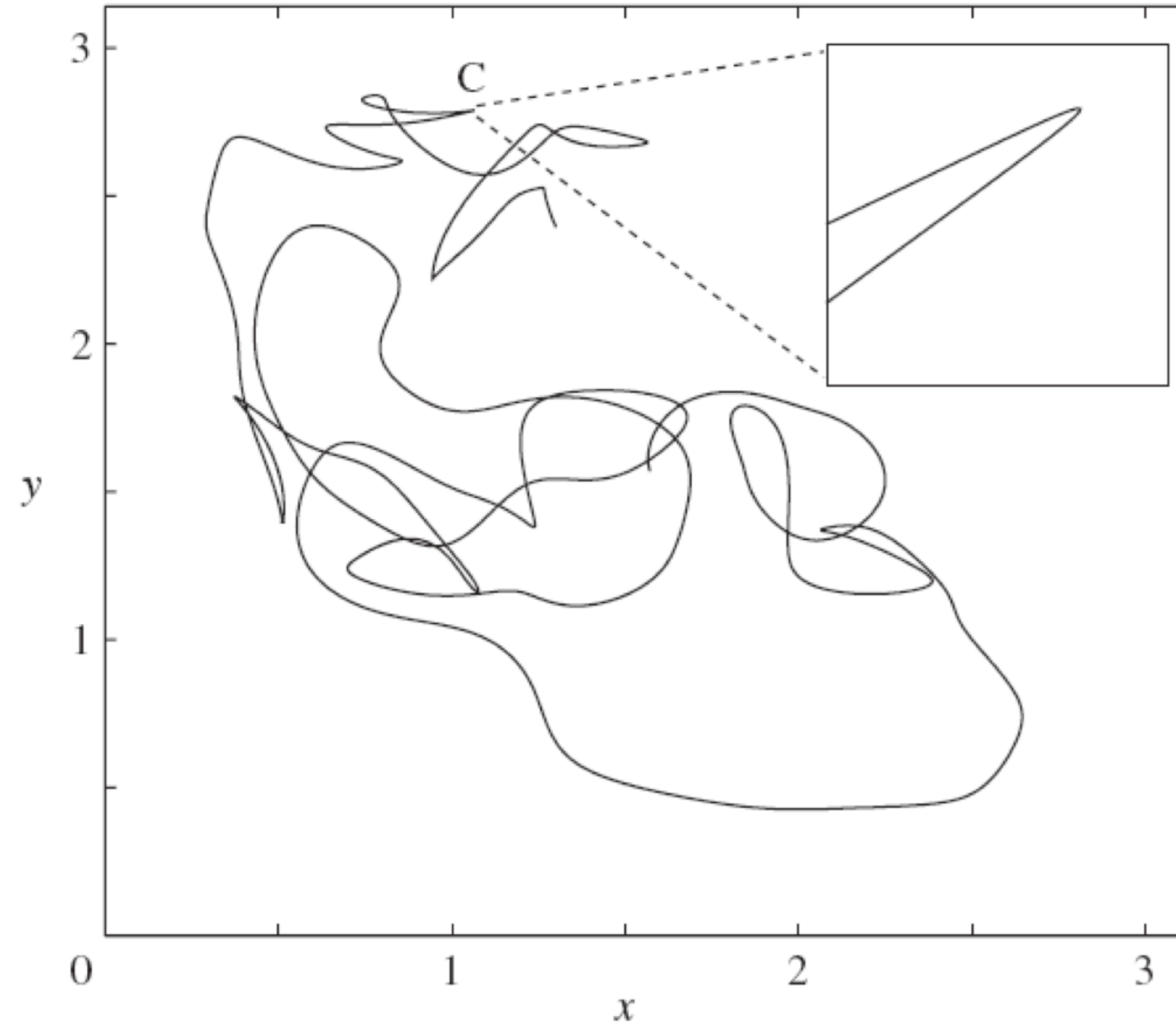
time

ρ

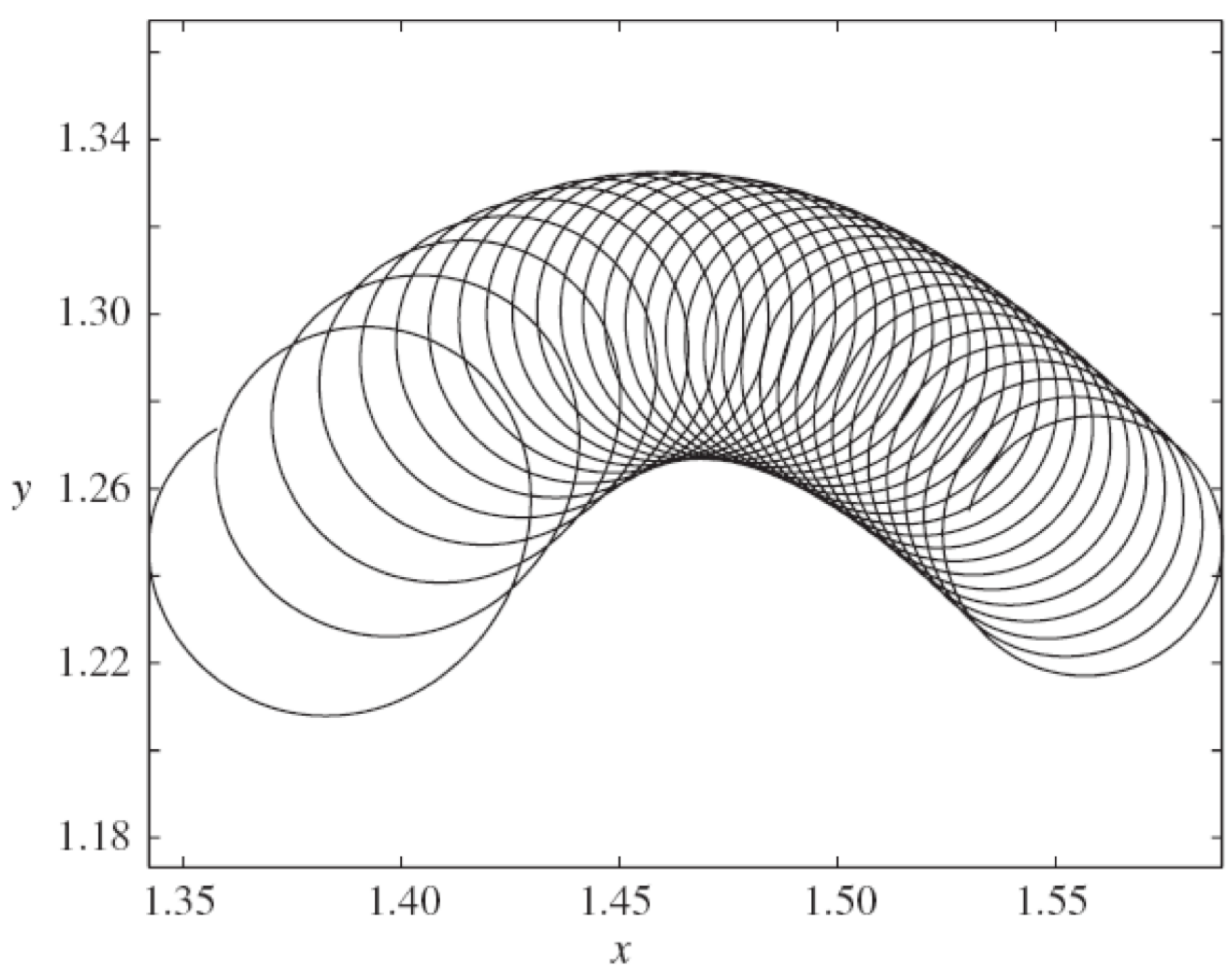


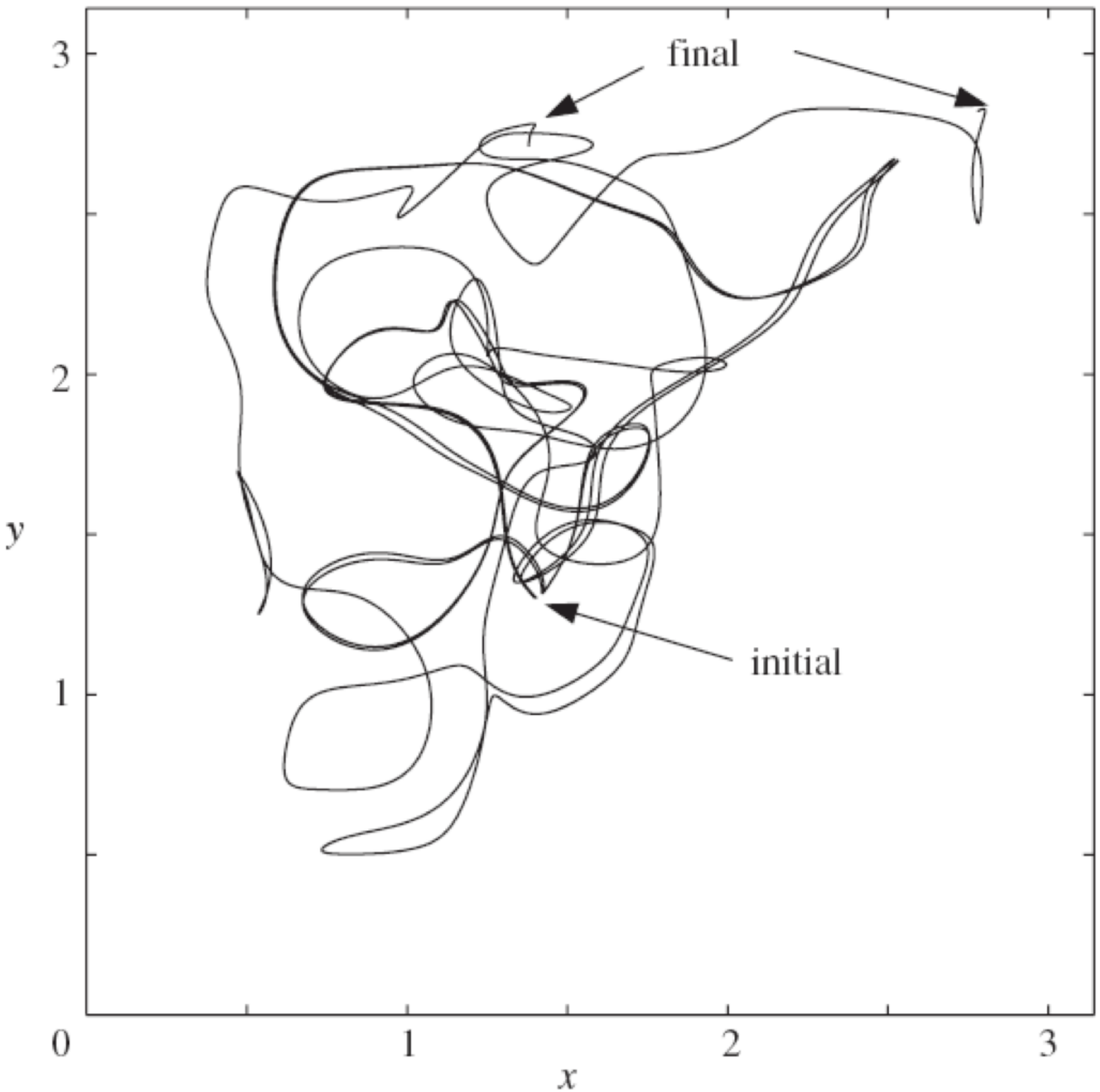
$|\Psi|^2$

Typical quantum trajectory…

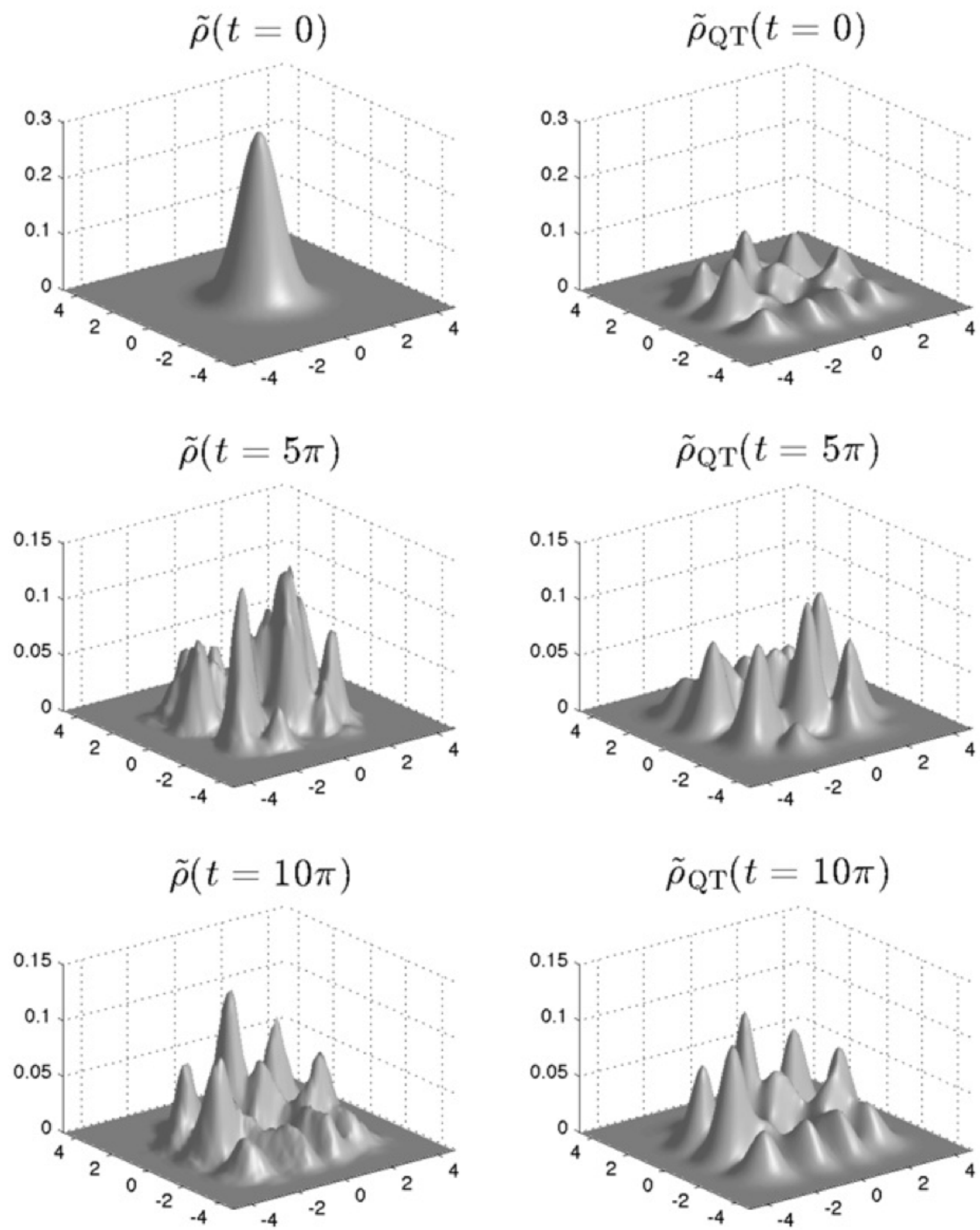


Close-up of a trajectory near a node





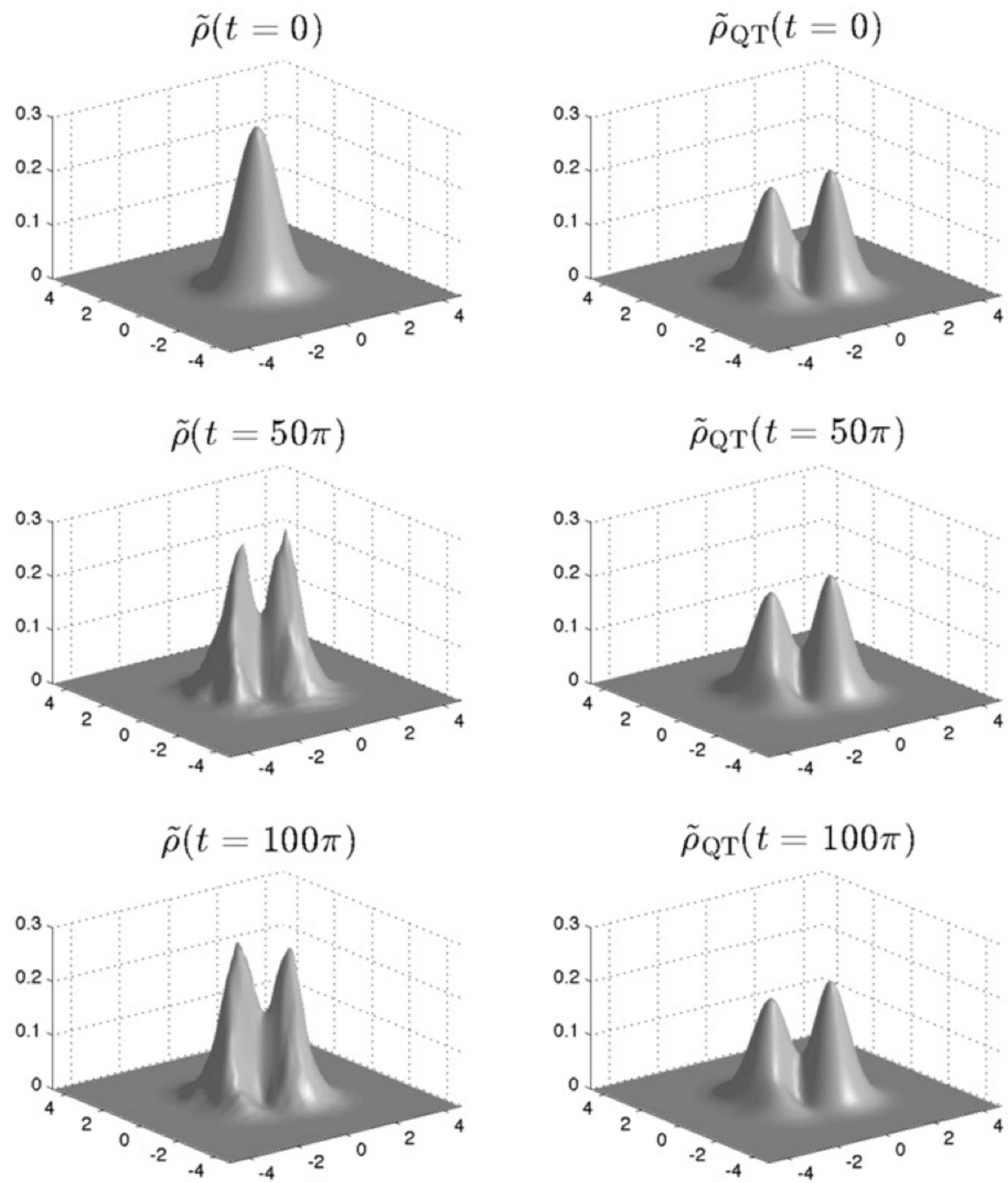
chaotic mixing…



chaotic mixing…

relaxation towards equilibrium

just like ordinary thermal equilibrium



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Quantum cosmology

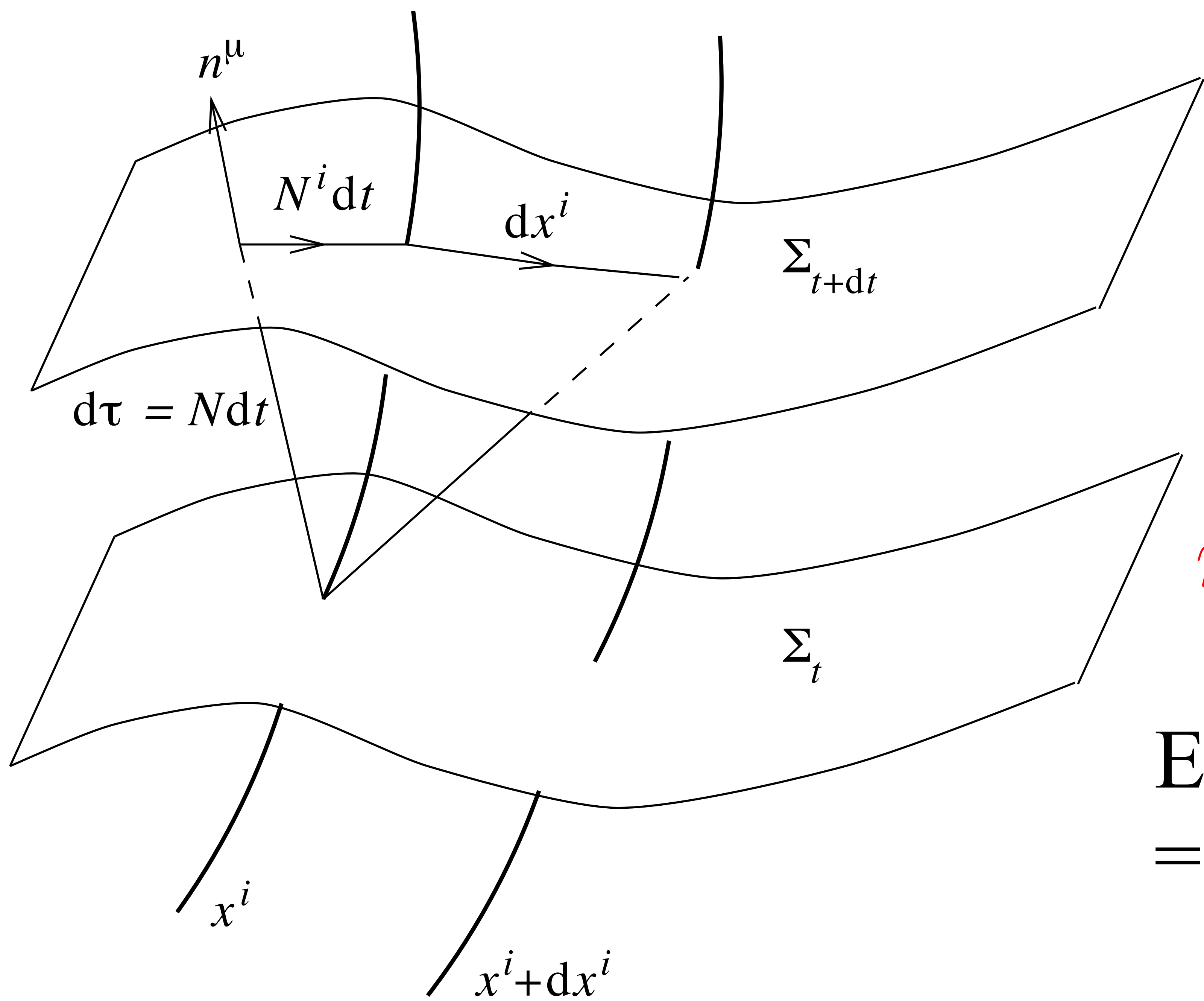
- Hamiltonian GR

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathcal{N}^2\mathrm{d}t^2 + h_{ij}\left(\mathrm{d}x^i + \mathcal{N}^i\mathrm{d}t\right)\left(\mathrm{d}x^j + \mathcal{N}^j\mathrm{d}t\right)$$

Shift vector

Lapse function

Intrinsic metric
= first fundamental form



n^μ Normal to Σ_t Intrinsic curvature tensor ${}^3R^i{}_{jkl}(h)$

Extrinsic curvature
= second fundamental form

$$K_{ij} \equiv -\nabla_j n_i = -\Gamma^0{}_{ij} n_0 = \frac{1}{2\mathcal{N}}\left(\nabla_j \mathcal{N}_i + \nabla_i \mathcal{N}_j - \frac{\partial h_{ij}}{\partial t}\right)$$

Action: $$\mathcal{S} = \frac{1}{16\pi G_{\mathrm{N}}}\left[\int_{\mathcal{M}} \mathrm{d}^4x\sqrt{-g}\left({}^4R - 2\Lambda\right) + 2\int_{\partial\mathcal{M}} \mathrm{d}^3x\sqrt{h}K^i{}_i\right] + \mathcal{S}_{\mathrm{matter}}$$

- Superspace & canonical quantisation

Relevant configuration space?

matter fields

$$\mathrm{Riem}(\Sigma) \equiv \left\{ h_{ij}\,(x^\mu), \Phi\,(x^\mu)\ \mid x \in \Sigma \right\}$$

parameters

GR $\Longrightarrow$ invariance / diffeomorphisms $\Longrightarrow$ $\mathrm{Conf} = \dfrac{\mathrm{Riem}(\Sigma)}{\mathrm{Diff}_0(\Sigma)}$ superspace

Wave functional $\Psi\left[h_{ij}(x), \Phi(x)\right]$

Dirac canonical quantisation

$$\pi^{ij} \to -i\frac{\delta}{\delta h_{ij}} \qquad \pi_\Phi \to -i\frac{\delta}{\delta \Phi} \qquad \pi^0 \to -i\frac{\delta}{\delta \mathcal{N}} \qquad \pi^i \to -i\frac{\delta}{\delta \mathcal{N}_i}$$

- Minisuperspace

Restrict attention from an infinite dimensional configuration space to 2 dimensional space = mini-superspace

$$h_{ij}\mathrm{d}x^i\mathrm{d}x^j = a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right)\right] \qquad \Phi(x) = \phi(t)$$

WDW equation becomes Schrödinger-like for $\Psi\left[a(t),\phi(t)\right]$

Conceptual and technical problems:

Infinite number of dof $\longrightarrow$ a few: mathematical consistency?
Freeze momenta? Heisenberg uncertainties?
QM = minisuperspace of QFT

- Minisuperspace

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WDW equation becomes Schrödinger-like for $\Psi\left[a(t), \phi(t)\right]$

However, one can actually make calculations!

Exemple : Quantum cosmology of a perfect fluid

$$\mathrm{d}s^2 = N^2(\tau)\mathrm{d}\tau - a^2(\tau)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

Perfect fluid: Schutz formalism ('70) $$p = p_0\left[\frac{\dot{\varphi}+\theta\dot{s}}{N(1+\omega)}\right]^{\frac{1+\omega}{\omega}}$$

$(\varphi, \theta, s) =$ Velocity potentials

canonical transformation: $T = -p_s \mathrm{e}^{-s/s_0} p_\varphi^{-(1+\omega)} s_0 \rho_0^{-\omega}$ ...

+ rescaling (volume…) + units… : simple Hamiltonian:

$$H = \left(-\frac{p_a^2}{4a} - \mathcal{K}a + \frac{p_T}{a^{3\omega}}\right) N$$

$a^{3\omega}$

Wheeler-De Witt $H\Psi = 0$

$$\mathcal{K} = 0 \Longrightarrow \chi \equiv \frac{2a^{3(1-\omega)/2}}{3(1-\omega)} \Longrightarrow \boxed{i\frac{\partial \Psi}{\partial T} = \frac{1}{4}\frac{\partial^2 \Psi}{\partial \chi^2}}$$

space defined by $\chi > 0$ $\longrightarrow$ constraint $\bar{\Psi}\dfrac{\partial \Psi}{\partial \chi} = \Psi \dfrac{\partial \bar{\Psi}}{\partial \chi}$

Gaussian wave packet

$$\Psi = \left[\frac{8T_0}{\pi\left(T_0^2 + T^2\right)^2}\right]^{\frac{1}{4}} \exp\left(-\frac{T_0 \chi^2}{T_0^2 + T^2}\right) \mathrm{e}^{-iS(\chi,T)}$$

phase $S = \dfrac{T\chi^2}{T_0^2 + T^2} + \dfrac{1}{2}\arctan\dfrac{T_0}{T} - \dfrac{\pi}{4}$

What do we do with the wave function of the Universe???

Gaussian wave packet

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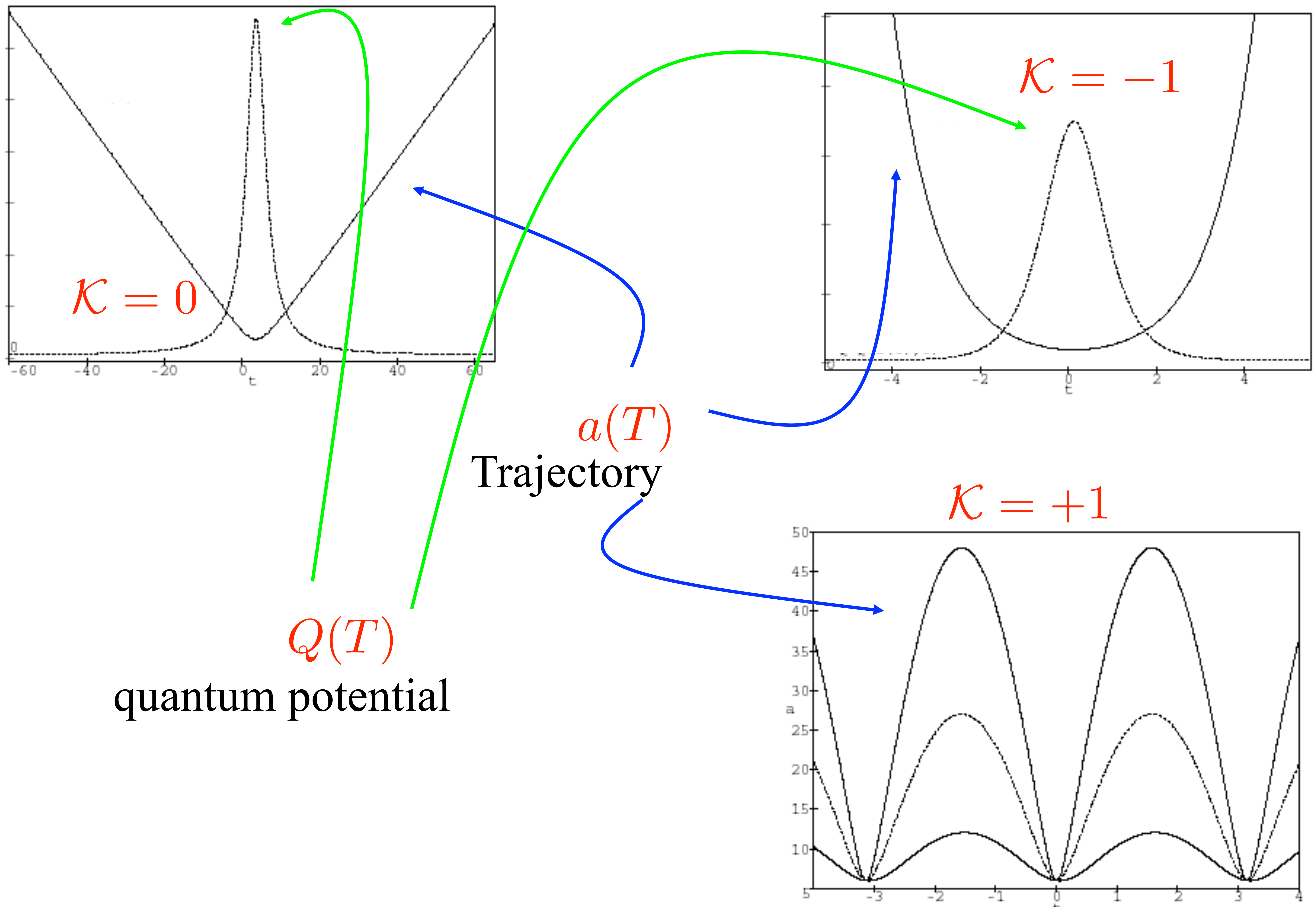
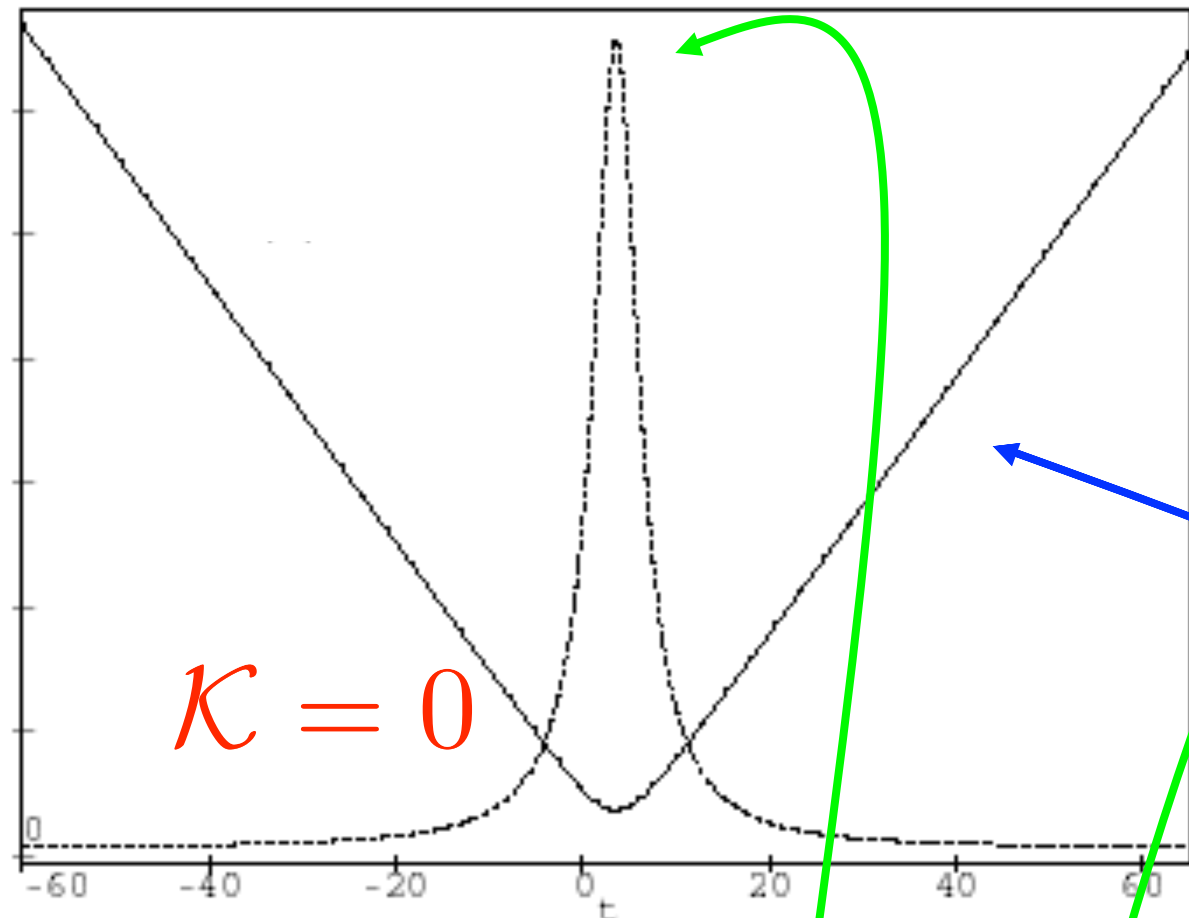
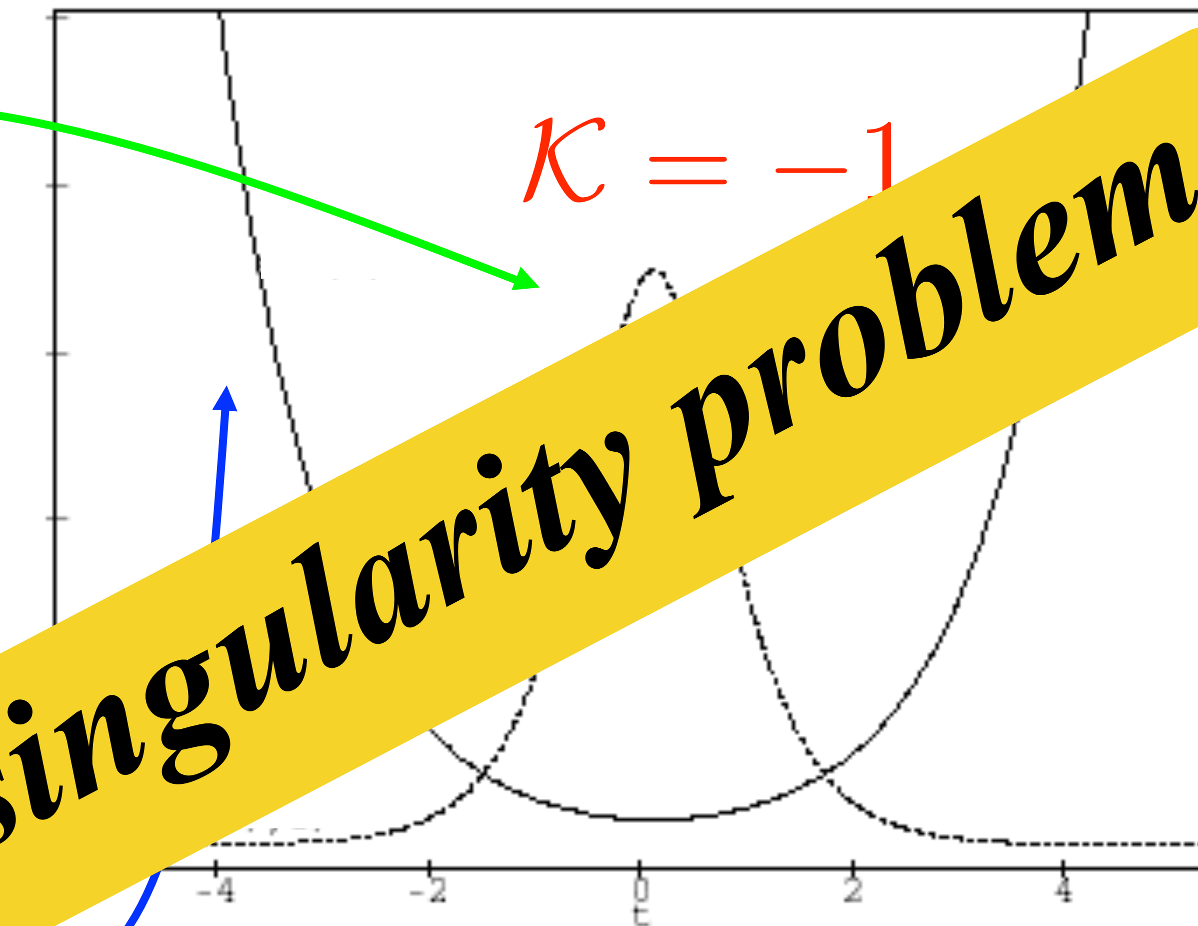
dB trajectory $$a = a_0\left[1+\left(\frac{T}{T_0}\right)^2\right]^{\frac{1}{3(1-\omega)}}$$

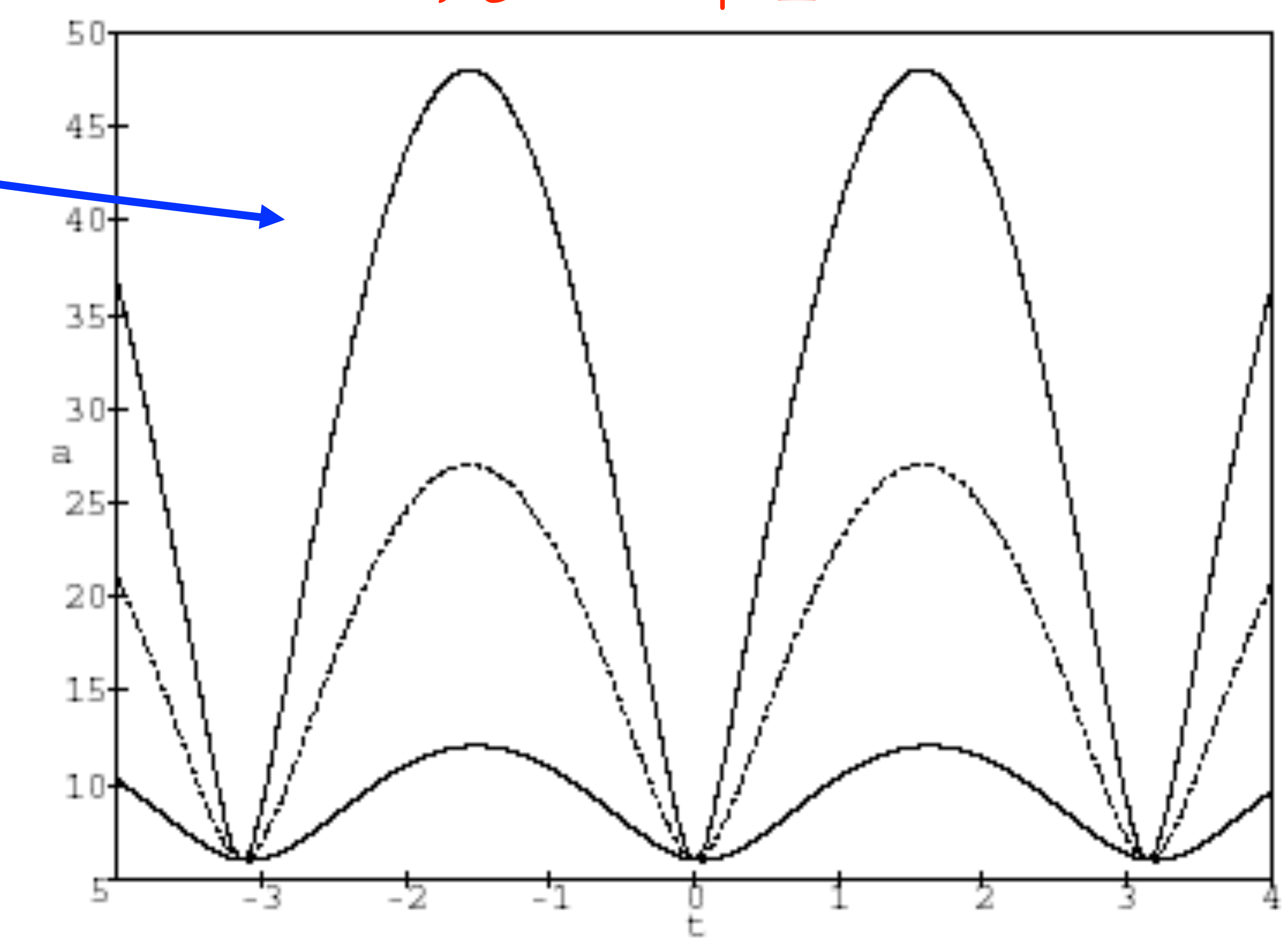


J. Acacio de Barros, N. Pinto-Neto & M. A. Sagorio-Leal,
Phys. Lett. **A241**, 229 (1998)





$\mathcal{K} = +1$



Natural quantum solution to the singularity problem!

(T)

potential

J. Acacio de Barros, N. Pinto-Neto & M. A. Sagorio-Leal, *Phys. Lett.* **A241**, 229 (1998)

A simple Bianchi I model $\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + \sum_{i=1}^{3} a_i^2(t)\left(\mathrm{d}x^i\right)^2$

+ (radiation) fluid / constant equation of state $w \equiv p/\rho = \frac{1}{3}$

conformal time choice $N \to a$

$t \to \eta$

GR Hamiltonian $H = \frac{\Pi_a^2}{24} - \frac{p_-^2 + p_+^2}{24a^2}$

$a \equiv (a_1 a_2 a_3)^{\frac{1}{3}}$

$\beta_- \equiv \frac{1}{2\sqrt{3}} \ln (a_1/a_2)$

$\beta_+ \equiv \frac{1}{6} \ln \left(a_1 a_2/a_3^2\right)$

Canonical commutation relations

$[\hat{a}, \hat{\Pi}_a] = [\hat{\beta}_\pm, \hat{p}_\pm] = i$

Rescaling:

$$\boxed{\hat{H} = \hat{\Pi}_a^2 - \left(\hat{p}_-^2 + \hat{p}_+^2\right)\hat{a}^{-2}}$$

mixed representation for the wave function

$$\left\{\begin{array}{rcl} \hat{a}\Psi & = & a\Psi \\ \hat{p}_\pm\Psi & = & p_\pm\Psi \\ \hat{\Pi}_a & = & -i\partial/\partial a \\ \hat{\beta}_\pm & = & i\partial/\partial p_\pm \end{array}\right.$$

Hilbert space $\mathbb{H}$

$$\mathbb{H} \subset \left\{ f\left(a, p_+, p_-\right) \in \mathbb{C} \,\middle|\, \int_0^\infty \mathrm{d}a \int_{-\infty}^{\infty} \mathrm{d}p_+ \int_{-\infty}^{\infty} \mathrm{d}p_- \left| f\left(a, p_+, p_-\right)\right|^2 < \infty \right\}$$

eigenvalue equation $\hat{H}\Psi = \ell^2\Psi$ $\longrightarrow$ $-\dfrac{\partial^2 \mathcal{U}_\ell^{(k)}}{\partial a^2} - \dfrac{k^2}{4a^2}\mathcal{U}_\ell^{(k)} = \ell^2 \mathcal{U}_\ell^{(k)}$

mixed representation for the wave function

$$\begin{cases} \hat{a}\Psi &= a\Psi \\ \hat{p}_\pm\Psi &= p_\pm\Psi \\ \hat{\Pi}_a &= -i\partial/\partial a \\ \hat{\beta}_\pm &= i\partial/\partial p_\pm \end{cases}$$

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$$k^2 \equiv 4(p_+^2 + p_-^2)$$

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$$k^2 \equiv 4(p_+^2 + p_-^2)$$

Wave function

$$\Psi(a, p_\pm) = \int_0^\infty \mathrm{d}\ell \int_{-\infty}^\infty \mathrm{d}\beta_+ \int_{-\infty}^\infty \mathrm{d}\beta_- \tilde{\Psi}(\ell, \beta_\pm)\, \mathrm{e}^{i[\beta_+ p_+ + \beta_- p_-]} \mathcal{U}_\ell^{(k)}(a)$$

Self-adjoint Hamiltonian $$\int \mathrm{d}a\,\mathrm{d}^2p\,(H\Psi)^*\,\Psi = \int \mathrm{d}a\,\mathrm{d}^2p\,\Psi^*\,(H\Psi)$$

automatically satisfied if $$\int_0^\infty \mathrm{d}a\,\mathcal{U}_\ell^{(k)*}(a)\mathcal{U}_{\ell'}^{(k)}(a) = \delta(\ell-\ell')$$

$$\int_0^\infty \mathrm{d}\ell \int_{-\infty}^{\infty} \mathrm{d}\beta_+ \int_{-\infty}^{\infty} \mathrm{d}\beta_- |\tilde{\Psi}(\ell,\beta_\pm)|^2 \ell^2 < \infty$$

$\nu = \frac{1}{2}\sqrt{1-k^2}$ general solution for the energy eigenmodes

$$\mathcal{U}_\ell^{(k)}(a) = c_+\sqrt{a\ell}\,J_\nu(a\ell) + c_-\sqrt{a\ell}\,J_{-\nu}(a\ell) \quad \begin{cases} c_+ = 1 \text{ and } c_- = 0 \\ c_+ = 0 \text{ and } c_- = 1 \end{cases}$$

Linear fluid momentum

$$\hat{P}_{\mathrm{fluid}} = -i\partial_\eta$$

Schödinger → Evolution operator $$i\frac{\partial U}{\partial \eta} = \hat{H}U$$

Initial gaussian wave function

$$\Psi_0(a) = \langle a, p_\pm | \Psi_0 \rangle = \frac{2^{(1-2\alpha)/4}\; a^\alpha}{\sigma^{\alpha+1/2}\sqrt{\Gamma\left(\alpha+\frac{1}{2}\right)}} \exp\left[-\frac{1}{2}a^2\left(\frac{1}{2\sigma^2} - i\mathcal{H}_{\rm ini}\right)\right]$$

Propagator $\quad G(a, p_\pm, a_0, p_\pm^0) \equiv \langle a, p_\pm | U | a_0, p_\pm^0 \rangle$

$$= \delta^{(2)}(p_\pm - p'_\pm) \int_0^\infty \mathrm{d}\ell\, \mathrm{e}^{-i\ell^2 \Delta\eta} \mathcal{U}_\ell^{(k)}(a) \mathcal{U}_\ell^{(k)*}(a')$$

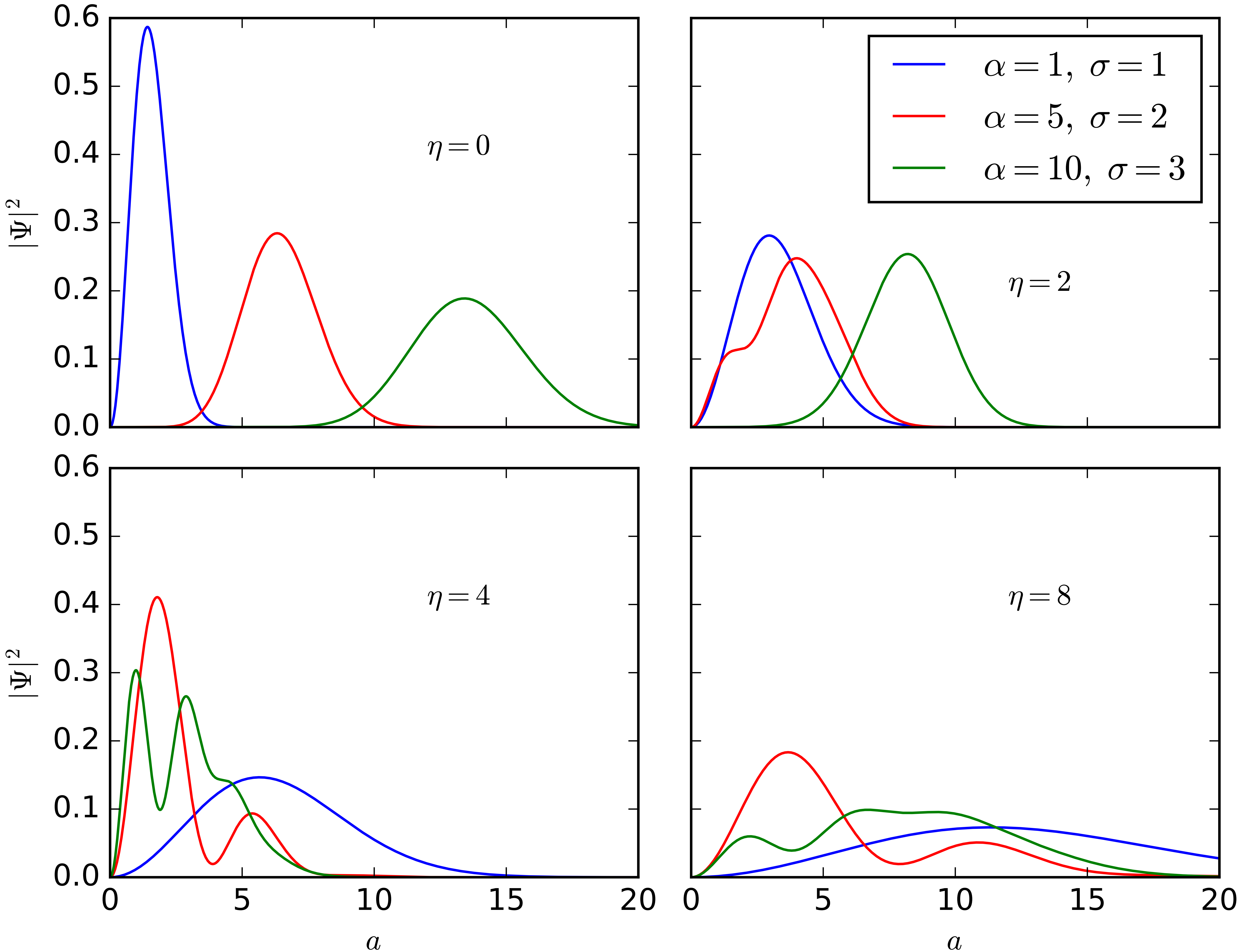
+ regularisation $\widetilde{\Delta\eta} = \Delta\eta(1+i\epsilon)$

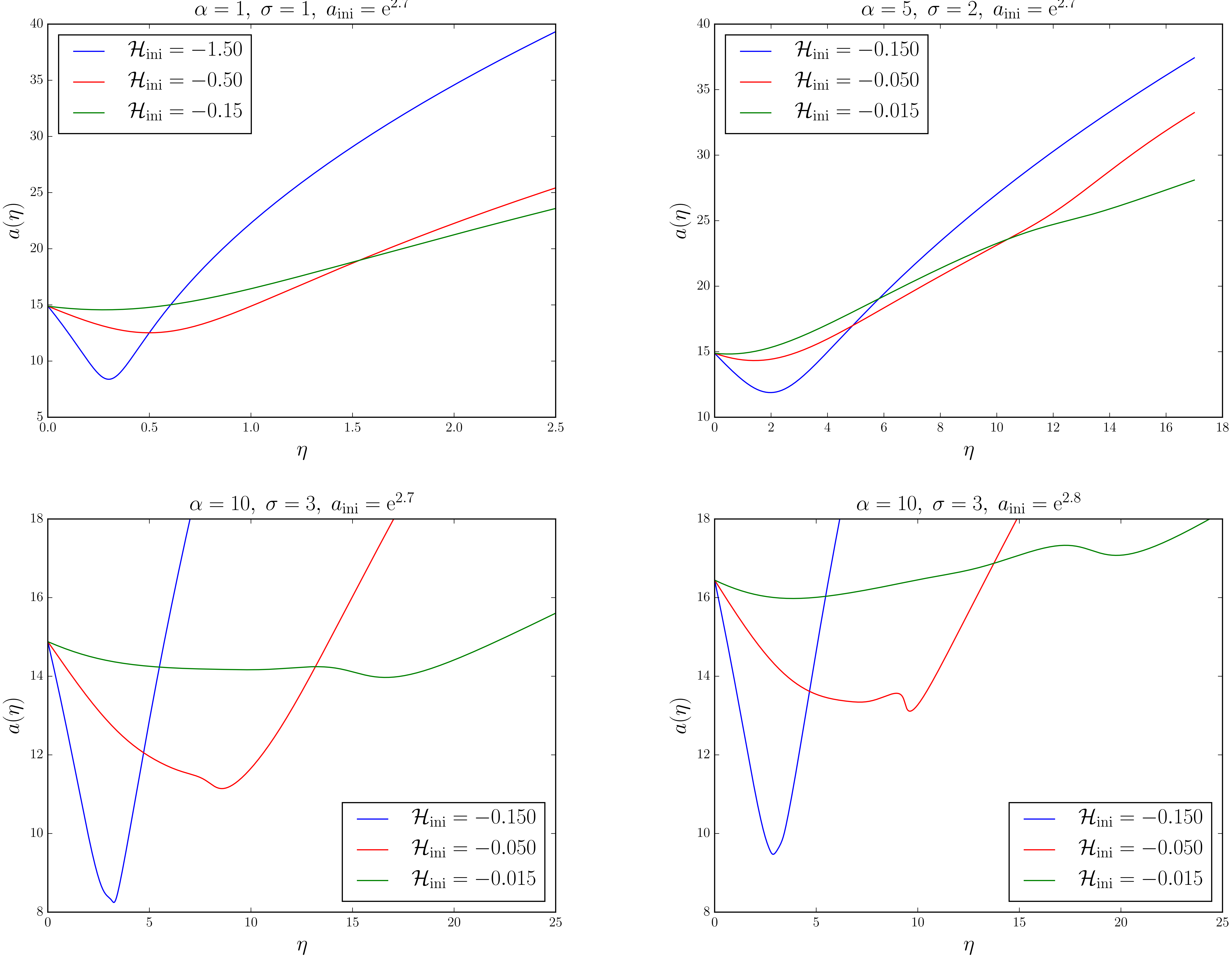
$$G(a, a_0; \eta) = -\frac{i\sqrt{a a_0}}{2\widetilde{\Delta\eta}} \mathrm{e}^{\frac{i}{4}(a^2+a_0^2)/\widetilde{\Delta\eta} - i\alpha\pi/2} J_\nu\left(\frac{a a_0}{2\widetilde{\Delta\eta}}\right)$$

dBB trajectory $\quad \frac{\mathrm{d}a}{\mathrm{d}\eta} = \frac{\partial S}{\partial a} = \frac{i}{2|\Psi|^2}\left(\Psi \frac{\partial \Psi^*}{\partial a} - \frac{\partial \Psi}{\partial a}\Psi^*\right)$

$\nu = \frac{1}{2}$

(FLRW)





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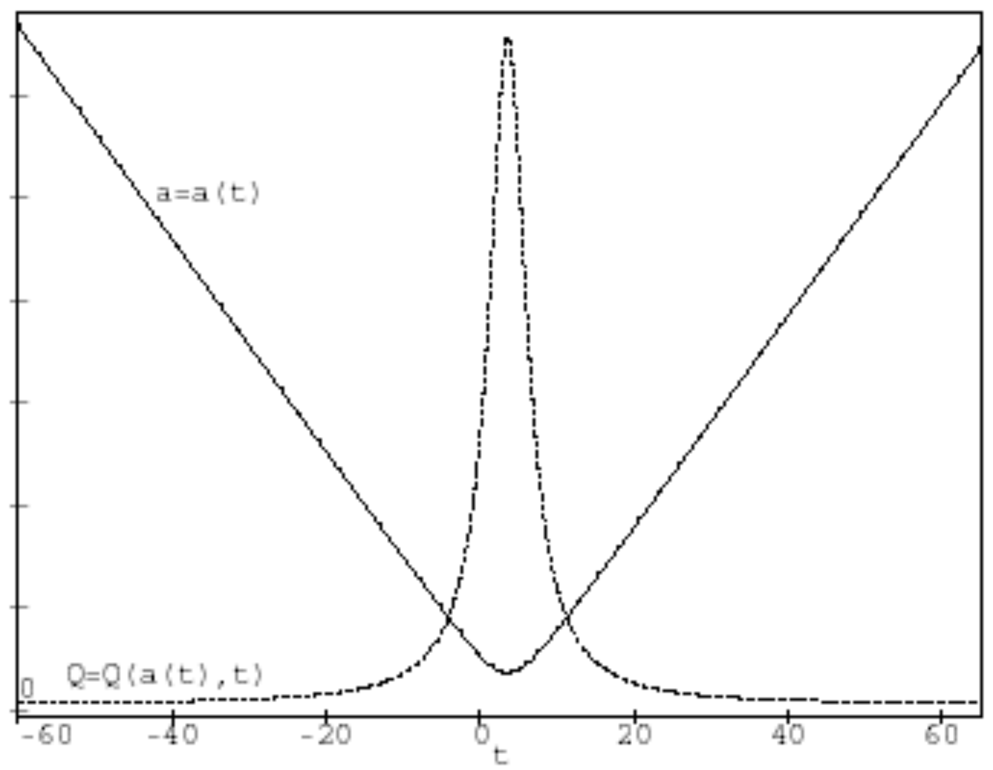
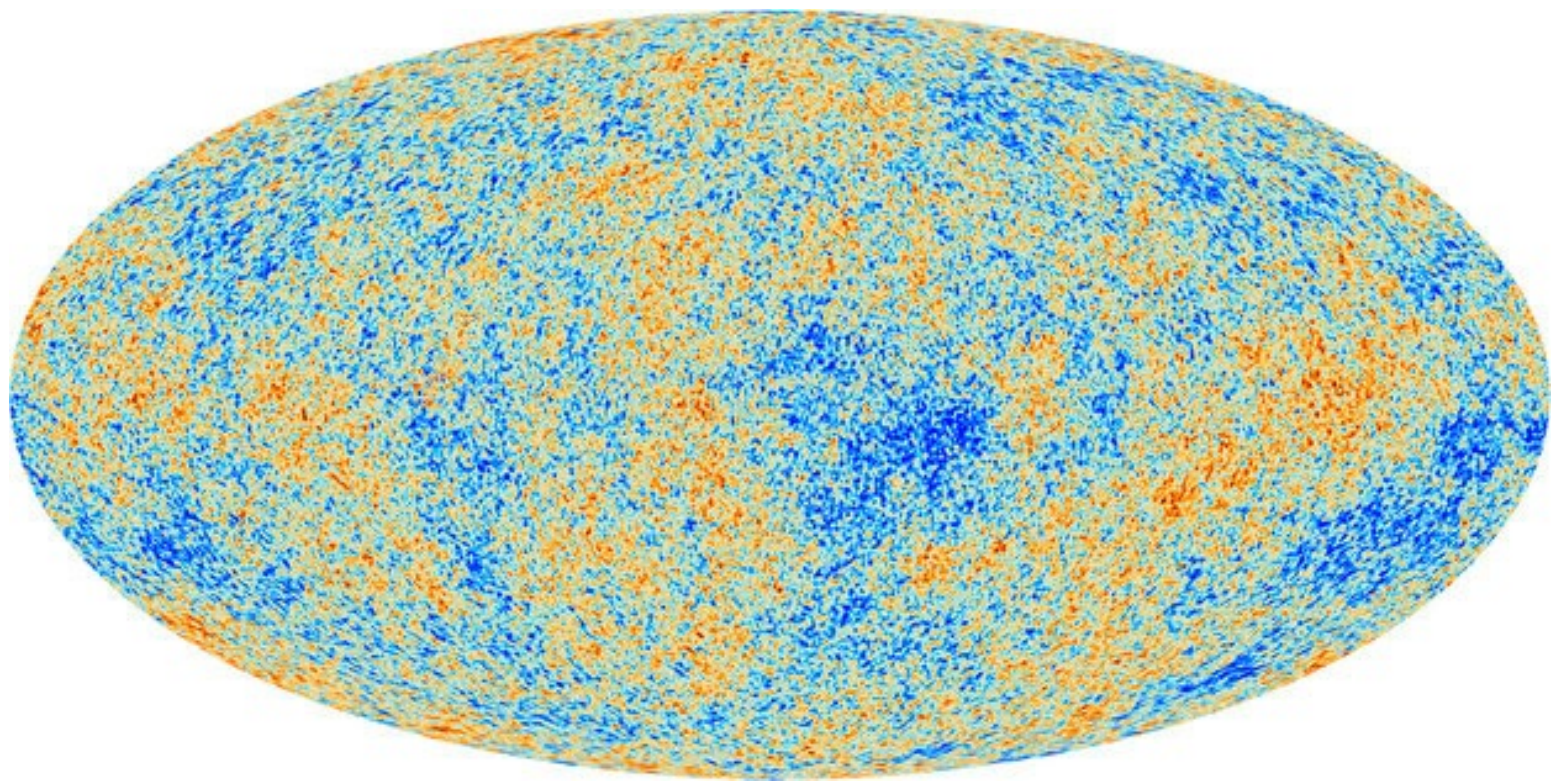


$$\mathrm{d}s^2 = a^2(\eta)\left\{(1+2\Phi)\,\mathrm{d}\eta^2 - \left[(1-2\Phi)\,\gamma_{ij} + h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$$

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$$



Inflationary perturbations: quantum fluctuations / expanding background

Classical temperature fluctuations

$$\frac{\Delta T}{T} \propto v \;\sim \Phi \sim \delta g_{00}$$

Inflationary perturbations: quantum fluctuations / expanding background

Classical temperature fluctuations promoted to quantum operators

$$\frac{\widehat{\Delta T}}{T} \propto \hat{v} \ \sim \Phi \sim \delta g_{00}$$

Inflationary perturbations: quantum fluctuations / expanding background

Classical temperature fluctuations promoted to quantum operators

$$\widehat{\frac{\Delta T}{T}} \propto \hat{v} \sim \Phi \sim \delta g_{00}$$

second order perturbed Einstein action $\quad {}^{(2)}\delta S = \frac{1}{2}\int \mathrm{d}^4 x \left[(v')^2 - \delta^{ij}\partial_i v \partial_j v + \frac{(a\sqrt{\epsilon_1})''}{a\sqrt{\epsilon_1}} v^2 \right]$

variable-mass scalar field in Minkowski spacetime

$\epsilon_1 = 1 - \mathcal{H}'/\mathcal{H}^2$

slow-roll parameter

+ Fourier transform $\quad v(\eta, \boldsymbol{x}) = \frac{1}{(2\pi)^{3/2}} \int_{\mathbb{R}^3} \mathrm{d}^3 \boldsymbol{k}\, v_{\boldsymbol{k}}(\eta) \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}}$

$$\Longrightarrow \quad {}^{(2)}\delta S = \int \mathrm{d}\eta \int \mathrm{d}^3 \boldsymbol{k} \left\{ v'_{\boldsymbol{k}} v^{*\,\prime}_{\boldsymbol{k}} + v_{\boldsymbol{k}} v^*_{\boldsymbol{k}} \left[\frac{(a\sqrt{\epsilon_1})''}{a\sqrt{\epsilon_1}} - k^2 \right] \right\}$$

Lagrangian formulation...

Hamiltonian

$$H = \int \mathrm{d}^3\boldsymbol{k} \left\{ p_{\boldsymbol{k}} p_{\boldsymbol{k}}^* + v_{\boldsymbol{k}} v_{\boldsymbol{k}}^* \left[\overbrace{k^2 - \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}}}^{\omega^2(\eta, \boldsymbol{k})} \right] \right\}$$

collection of parametric oscillators with time dependent frequency

factorization of the full wave function

real and imaginary parts

$$\Psi\left[v(\eta, \boldsymbol{x})\right] = \prod_{\boldsymbol{k}} \Psi_{\boldsymbol{k}}\left(v_{\boldsymbol{k}}^{\mathrm{R}}, v_{\boldsymbol{k}}^{\mathrm{I}}\right) = \prod_{\boldsymbol{k}} \Psi_{\boldsymbol{k}}^{\mathrm{R}}\left(v_{\boldsymbol{k}}^{\mathrm{R}}\right) \Psi_{\boldsymbol{k}}^{\mathrm{I}}\left(v_{\boldsymbol{k}}^{\mathrm{I}}\right)$$

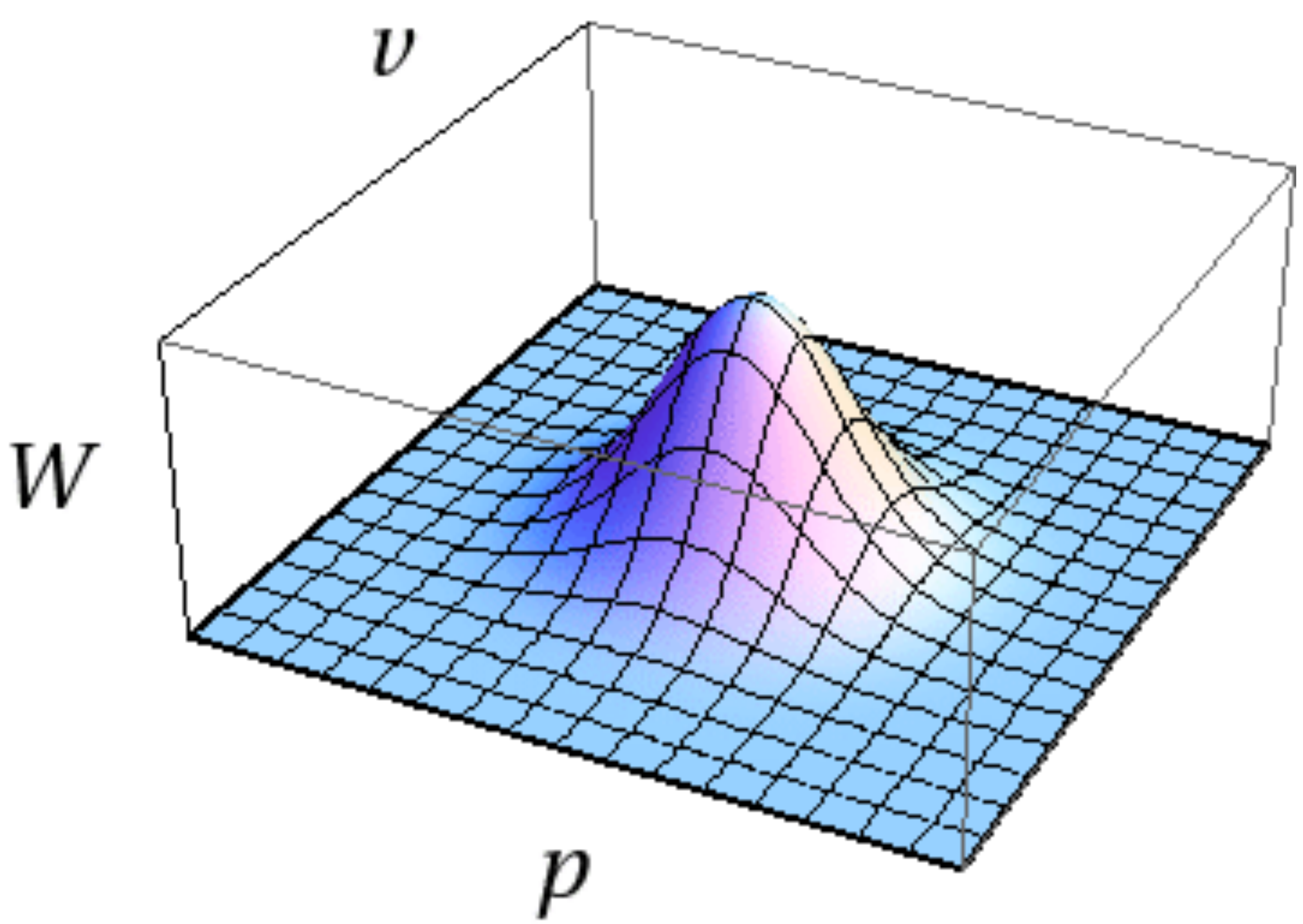
$$i \frac{\Psi_{\boldsymbol{k}}^{\mathrm{R,I}}}{\partial \eta} = \hat{\mathcal{H}}_{\boldsymbol{k}}^{\mathrm{R,I}} \Psi_{\boldsymbol{k}}^{\mathrm{R,I}}$$

$$\hat{\mathcal{H}}_{\boldsymbol{k}}^{\mathrm{R,I}} = -\frac{1}{2} \frac{\partial^2}{\partial \left(v_{\boldsymbol{k}}^{\mathrm{R,I}}\right)^2} + \frac{1}{2} \omega^2(\eta, \boldsymbol{k}) \left(\hat{v}_{\boldsymbol{k}}^{\mathrm{R,I}}\right)^2$$

Gaussian state solution $\Psi\left(\eta, v_{\boldsymbol{k}}\right)=\left[\frac{2 \Re \mathrm{e}\, \Omega_{\boldsymbol{k}}(\eta)}{\pi}\right]^{1 / 4} \mathrm{e}^{-\Omega_{\boldsymbol{k}}(\eta) v_{\boldsymbol{k}}^{2}}$

Wigner function $W\left(v_{\boldsymbol{k}}, p_{\boldsymbol{k}}\right)=\int \frac{\mathrm{d} x}{2 \pi^{2}} \Psi^{*}\left(v_{\boldsymbol{k}}-\frac{x}{2}\right) \mathrm{e}^{-i p_{\boldsymbol{k}} x} \Psi\left(v_{\boldsymbol{k}}+\frac{x}{2}\right)$

large squeezing limit $\Rightarrow$ $W \propto \delta\left(p_{\boldsymbol{k}}+k \tan \phi_{\boldsymbol{k}} v_{\boldsymbol{k}}\right)$



Stochastic distribution of classical processes

Ergodicity

$$\left\langle\frac{\Delta T(\xi, \boldsymbol{e})}{T}\right\rangle_{\xi} \simeq\left\langle\frac{\Delta T(\xi, \boldsymbol{e})}{T}\right\rangle_{\boldsymbol{e}}$$

realization

spatial direction

Gaussian state solution $\Psi\left(\eta, v_{\boldsymbol{k}}\right)=\left[\frac{2 \Re\mathrm{e}\, \Omega_{\boldsymbol{k}}(\eta)}{\pi}\right]^{1/4} \mathrm{e}^{-\Omega_{\boldsymbol{k}}(\eta) v_{\boldsymbol{k}}^{2}}$

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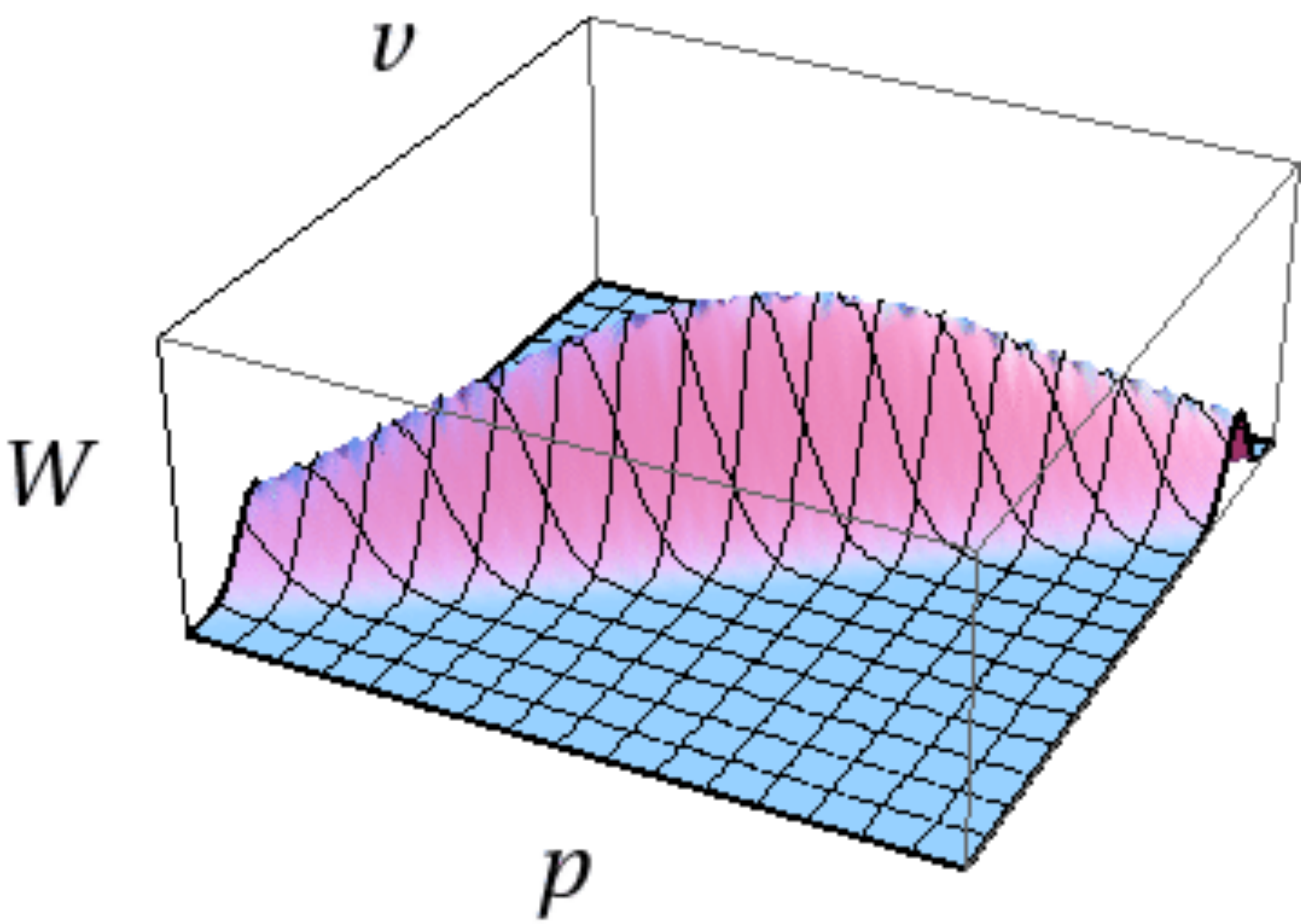


Animation provided by V. Vennin

Stochastic distribution of classical processes

Ergodicity $\left\langle\frac{\Delta T(\xi, \boldsymbol{e})}{T}\right\rangle_{\xi} \simeq\left\langle\frac{\Delta T(\xi, \boldsymbol{e})}{T}\right\rangle_{\boldsymbol{e}}$

realization (ξ) — spatial direction ($\boldsymbol{e}$)

Primordial Power Spectrum

Standard case

Quantization in the Schrödinger picture
(functional representation)

$$\Psi_{\boldsymbol{k}}(\eta, v_{\boldsymbol{k}}) = \left[\frac{2\,\Re\mathrm{e}\Omega_{\boldsymbol{k}}(\eta)}{\pi}\right]^{1/4} \mathrm{e}^{-\Omega_{\boldsymbol{k}}(\eta)\, v_{\boldsymbol{k}}^2}$$

$$i\frac{\mathrm{d}|\Psi_{\boldsymbol{k}}\rangle}{\mathrm{d}\eta} = \hat{\mathcal{H}}_{\boldsymbol{k}}\,|\Psi_{\boldsymbol{k}}\rangle$$

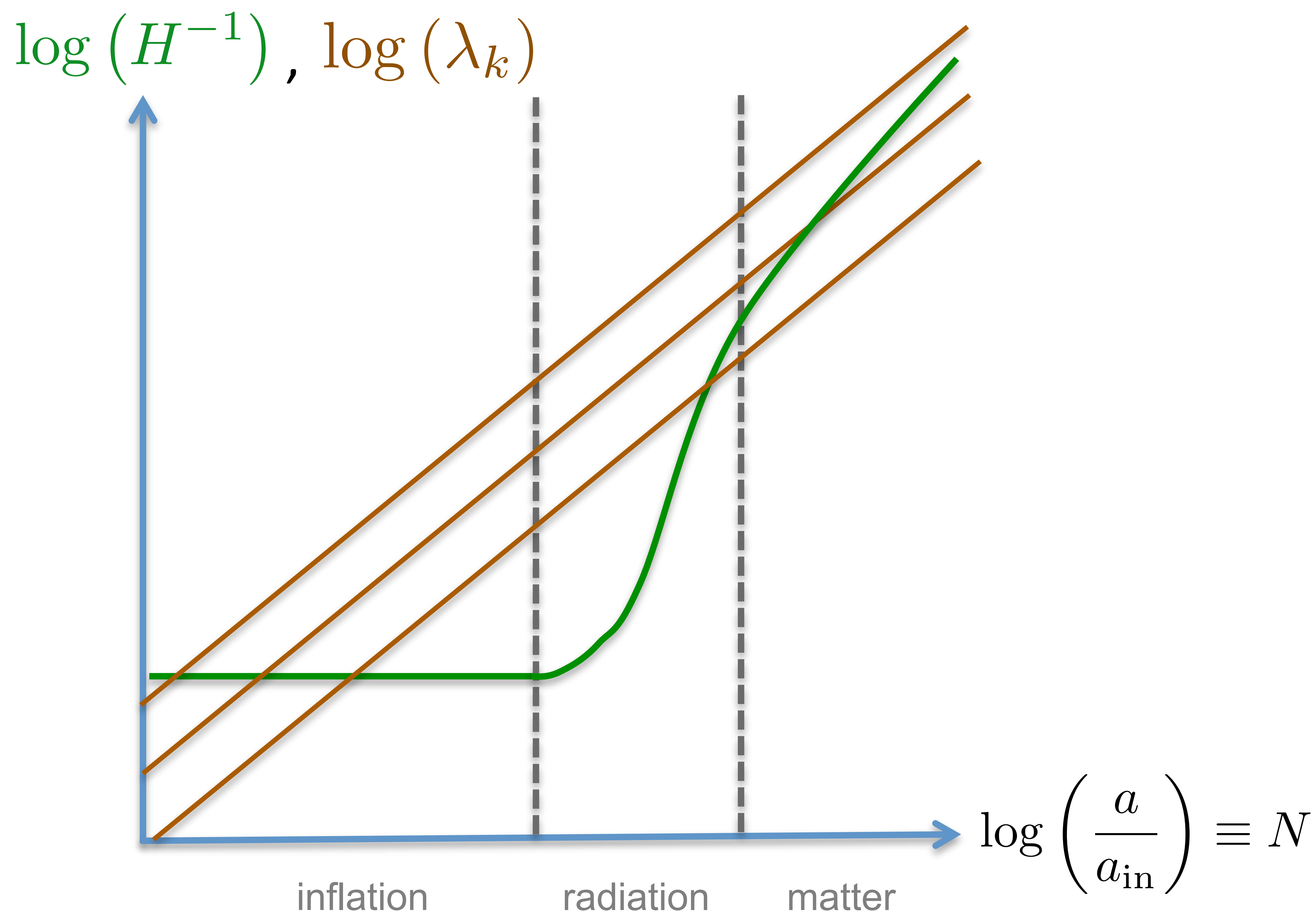
with

$$\hat{\mathcal{H}}_{\boldsymbol{k}} = \frac{\hat{p}_{\boldsymbol{k}}^2}{2} + \omega^2(\boldsymbol{k},\eta)\hat{v}_{\boldsymbol{k}}^2$$

$$\Omega_{\boldsymbol{k}}' = -2i\Omega_{\boldsymbol{k}}^2 + \frac{i}{2}\omega^2(\eta,\boldsymbol{k})$$

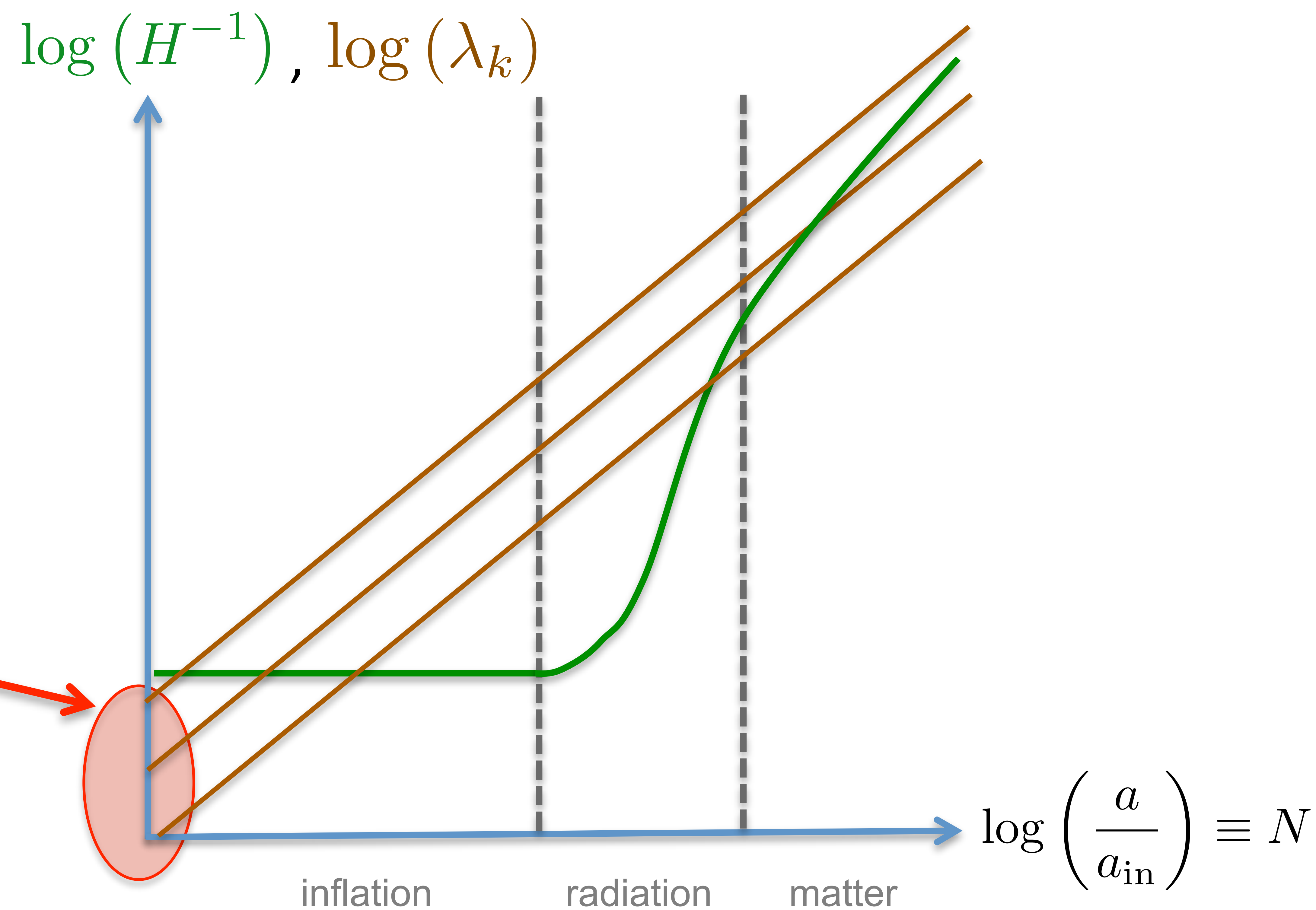
$$\Omega_{\boldsymbol{k}} = -\frac{i}{2}\frac{f_{\boldsymbol{k}}'}{f_{\boldsymbol{k}}}$$

$$\boxed{f_{\boldsymbol{k}}'' + \omega^2(\boldsymbol{k},\eta)f_{\boldsymbol{k}} = 0}$$


$\log\left(H^{-1}\right)$, $\log\left(\lambda_k\right)$
$\log\left(\frac{a}{a_{\rm in}}\right) \equiv N$
inflation
radiation
matter

Harmonic oscillator fundamental state

$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2}v_{\boldsymbol{k}}^2}$$



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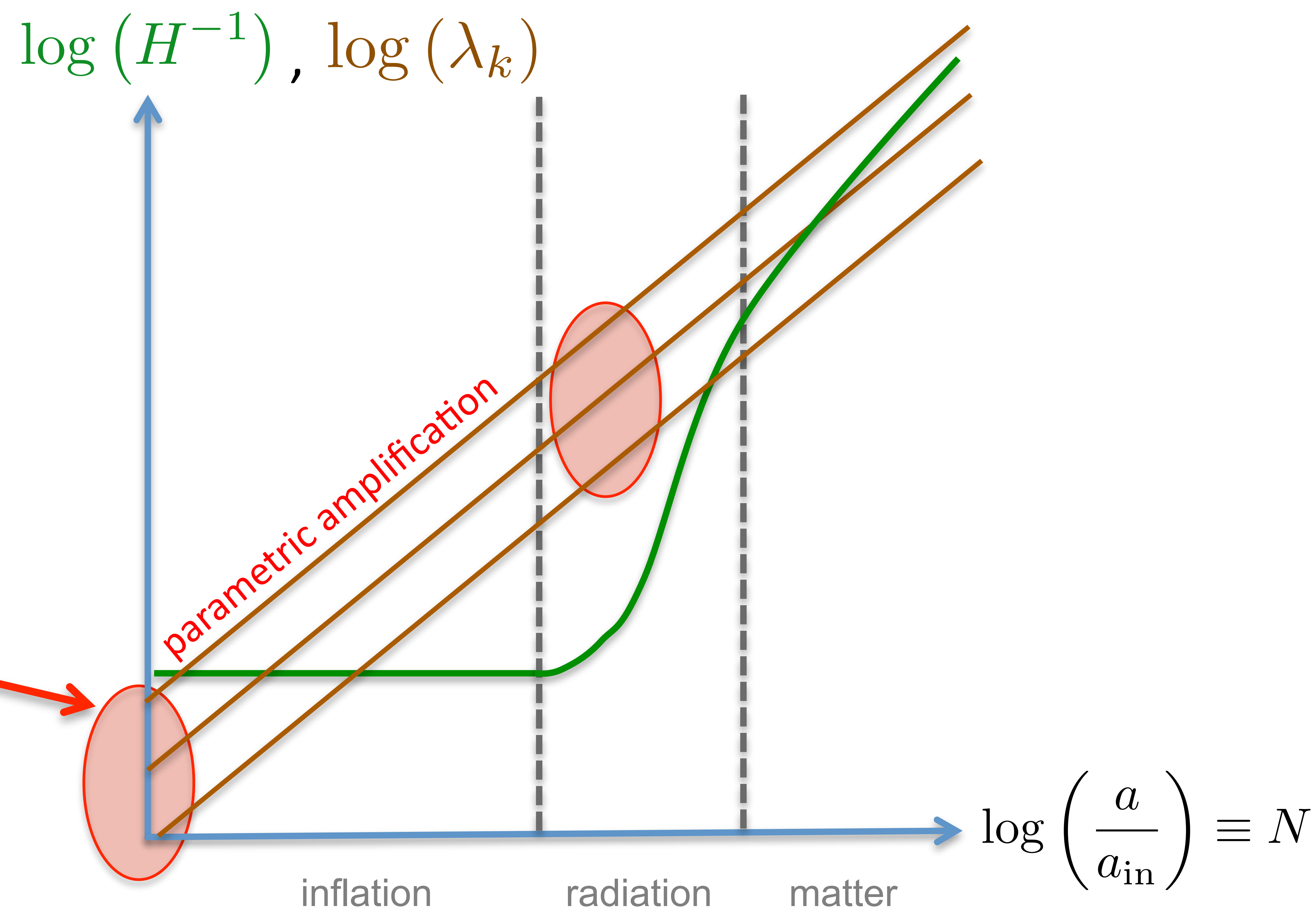

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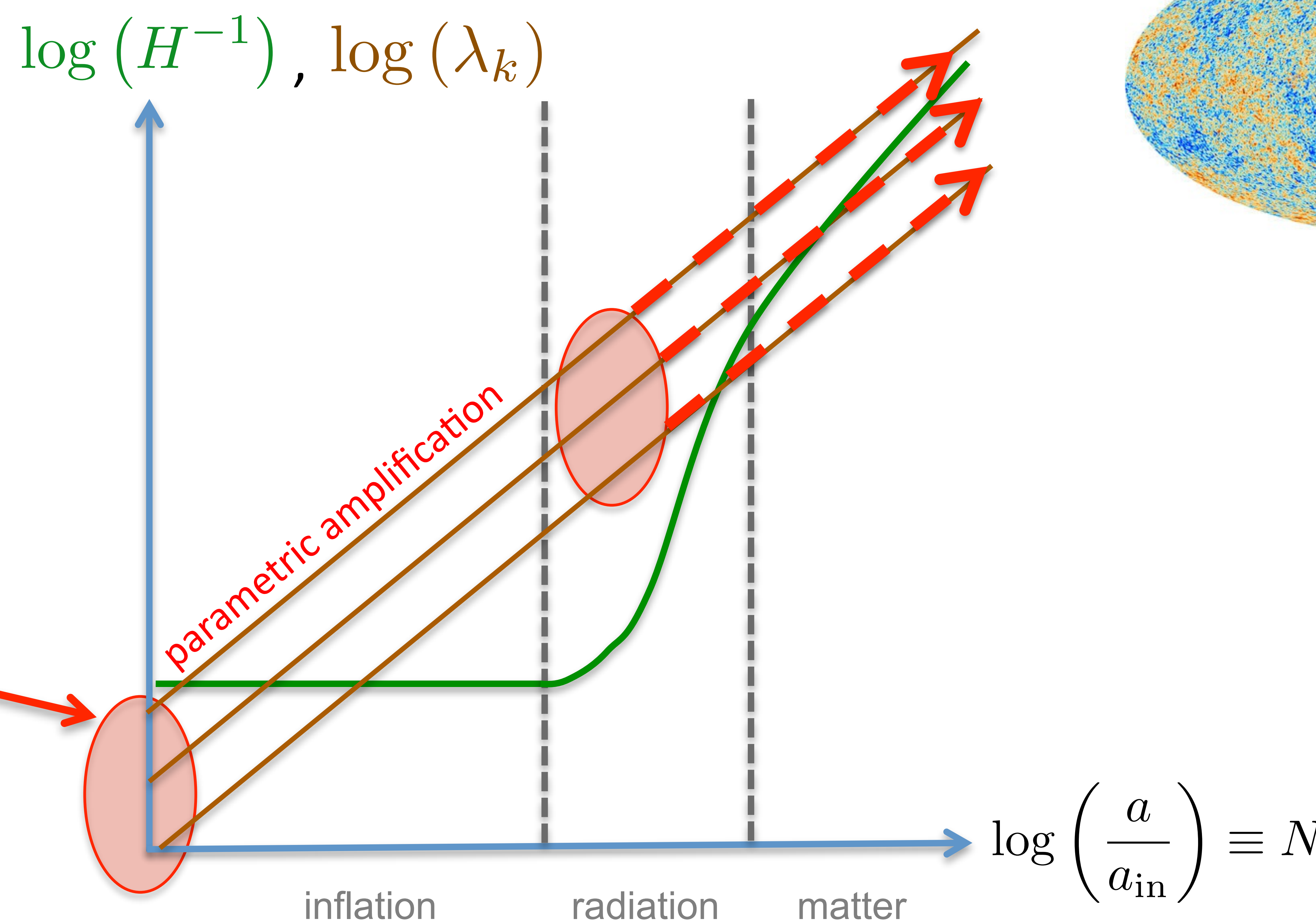
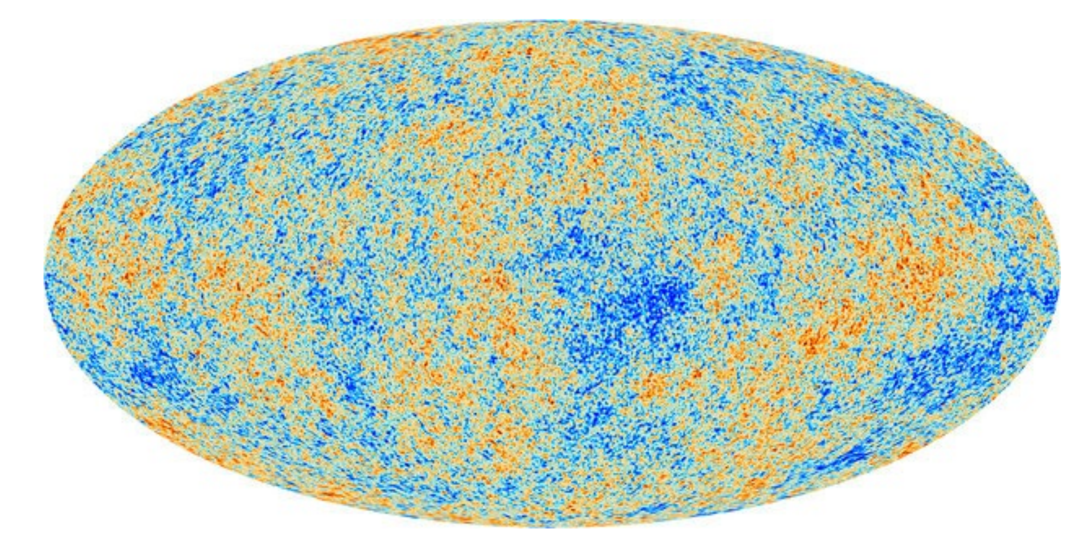




Primordial Power Spectrum

Standard case

Two physical scales

Hubble radius $H^{-1} = \frac{a^2}{a'} \underset{\beta \sim -2}{\simeq} \ell_0$

wavelength $\lambda = \frac{a}{k} \underset{\beta \sim -2}{\simeq} \frac{\ell_0}{-k\eta}$

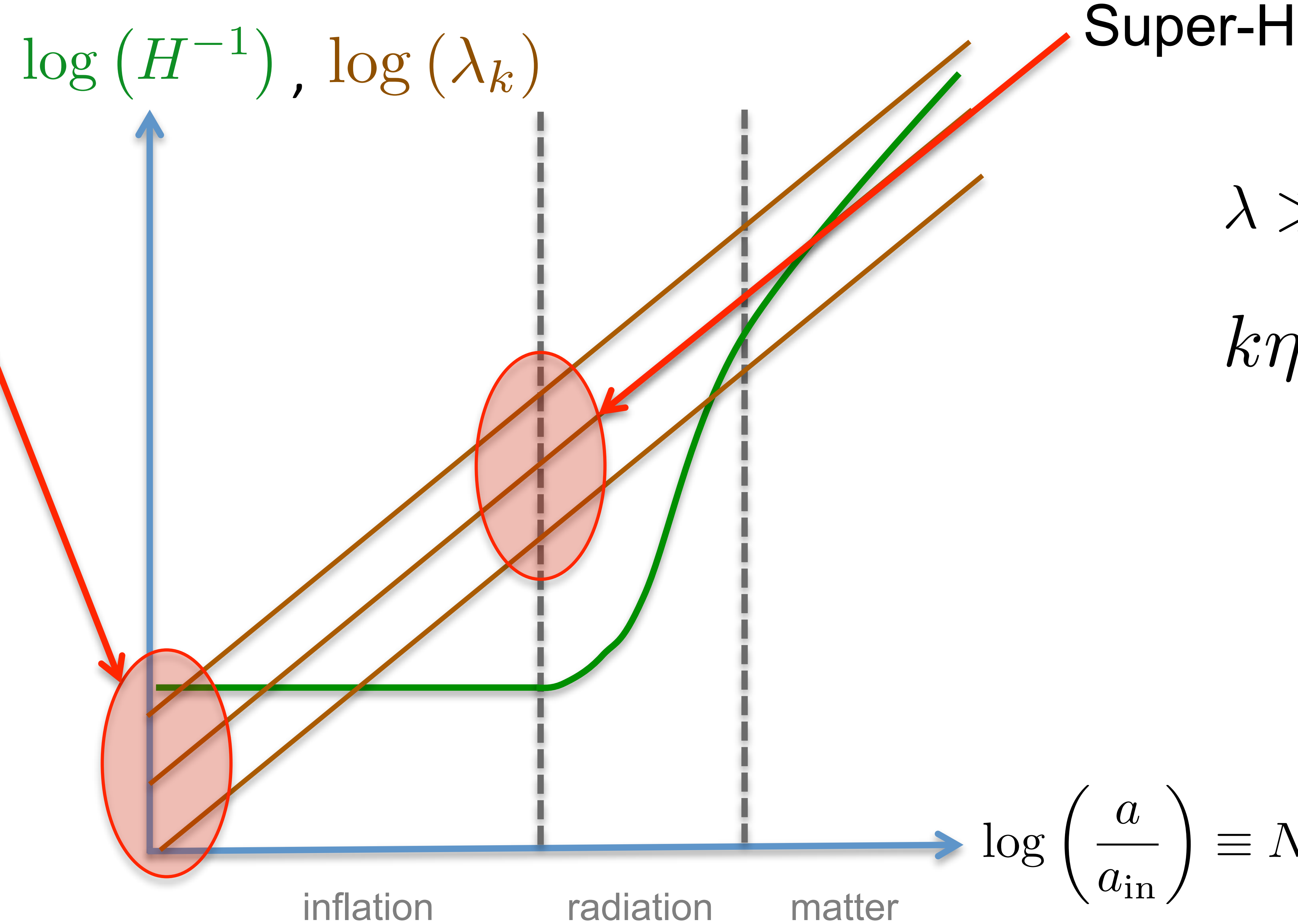
Sub-Hubble regime

$\lambda \ll H^{-1}$

$k\eta \to -\infty$

$\omega \simeq k$

harmonic oscillator



Super-Hubble regime

$\lambda \gg H^{-1}$

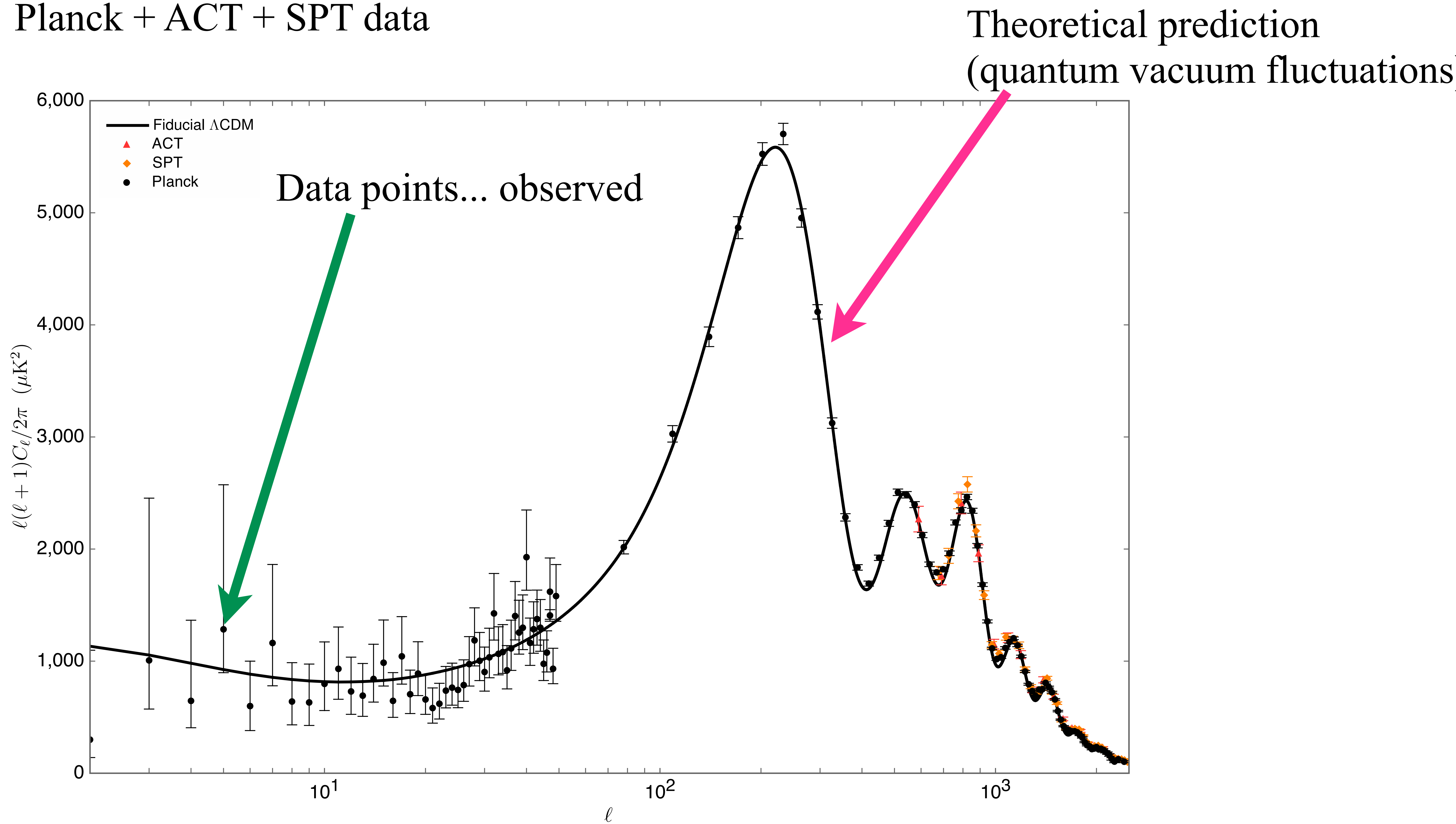
$k\eta \to 0^-$

$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2} v_{\boldsymbol{k}}^2}$$

Bunch Davis vacuum

sets initial conditions $f_{\boldsymbol{k}}(k\eta \to -\infty) = \mathrm{e}^{ik\eta}/\sqrt{2k}$

Planck + ACT + SPT data



- Both background and perturbations are quantum

Usual treatment of the perturbations?

Einstein-Hilbert action up to 2nd order $$\mathcal{S}_{\mathrm{E-H}} = \int \mathrm{d}^4 x \, \left[R^{(0)} + \delta^{(2)} R \right]$$

Bardeen (Newton) gravitational potential

$$\mathrm{d}s^2 = a^2(\eta) \left\{ (1+2\Phi)\,\mathrm{d}\eta^2 - \left[(1-2\Phi)\,\gamma_{ij} + h_{ij} \right] \mathrm{d}x^i \mathrm{d}x^j \right\}$$

conformal time $\mathrm{d}\eta = a(t)^{-1}\mathrm{d}t$

$$\Delta\Phi = -\frac{3\ell_{\mathrm{Pl}}^2}{2}\sqrt{\frac{\rho+p}{\omega}}\, a \frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{v}{a}\right)$$

Mukhanov-Sasaki variable

$$\int \mathrm{d}^4 x \, \delta^{(2)} \mathcal{L} = \frac{1}{2} \int \sqrt{\gamma}\, \mathrm{d}^3 \boldsymbol{x} \, \mathrm{d}\eta \left[(\partial_\eta v)^2 - \gamma^{ij} \partial_i v \partial_j v + \frac{z''}{z} v^2 \right]$$

V. F. Mukhanov, H. A. Feldman & R. H. Brandenberger, *Phys. Rep.* **215**, 203 (1992)

Simple scalar field with varying mass in Minkowski space!!! $z = z[a(\eta)]$

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Classical

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Quantum

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Hamiltonian up to 2nd order $H = H_{(0)} + H_{(2)} + \cdots$

$$\Delta\Phi = -\frac{3\ell_{\rm Pl}^2}{2}\sqrt{\frac{\rho+p}{\omega}}\,a\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{v}{a}\right)$$

factorization of the wave function

$$\Psi = \Psi_{(0)}\left(a,T\right)\Psi_{(2)}\left[v,T;a\left(T\right)\right]$$

comes from 0th order

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Use dBB...

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factorization of the wave function

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comes from 0th order

Use dBB...

Question: what if initial perturbation out of quantum equilibrium?

Recall: Hamiltonian

$$H = \int \mathrm{d}^3\boldsymbol{k} \left\{ p_{\boldsymbol{k}} p_{\boldsymbol{k}}^* + v_{\boldsymbol{k}} v_{\boldsymbol{k}}^* \left[\overbrace{k^2 - \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}}}^{\omega^2(\eta, \boldsymbol{k})} \right] \right\}$$

collection of parametric oscillators with time dependent frequency

Simpler model: spectator scalar field in an expanding and finite size Universe

$$\phi_{\boldsymbol{k}} = \frac{\sqrt{V}}{(2\pi)^{3/2}} \left(q_{\boldsymbol{k}1} + i q_{\boldsymbol{k}2}\right) \qquad H = \sum_{\boldsymbol{k},\ r=1,2} \frac{1}{2a^3} \pi_{\boldsymbol{k}r}^2 + \frac{1}{2} a k^2 q_{\boldsymbol{k}r}^2$$

$$a^3 \to m$$

$$k/a \to \omega$$

$$i \frac{\partial \psi}{\partial t} = \sum_{r=1}^{2} \left(-\frac{1}{2m} \frac{\partial^2}{\partial q_r^2} + \frac{1}{2} m \omega^2 q_r^2 \right) \psi$$

Recall: Hamiltonian

$$H = \int \mathrm{d}^3\boldsymbol{k} \left\{ p_{\boldsymbol{k}} p^*_{\boldsymbol{k}} + v_{\boldsymbol{k}} v^*_{\boldsymbol{k}} \left[\overbrace{k^2 - \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}}}^{\omega^2(\eta,\boldsymbol{k})} \right] \right\}$$

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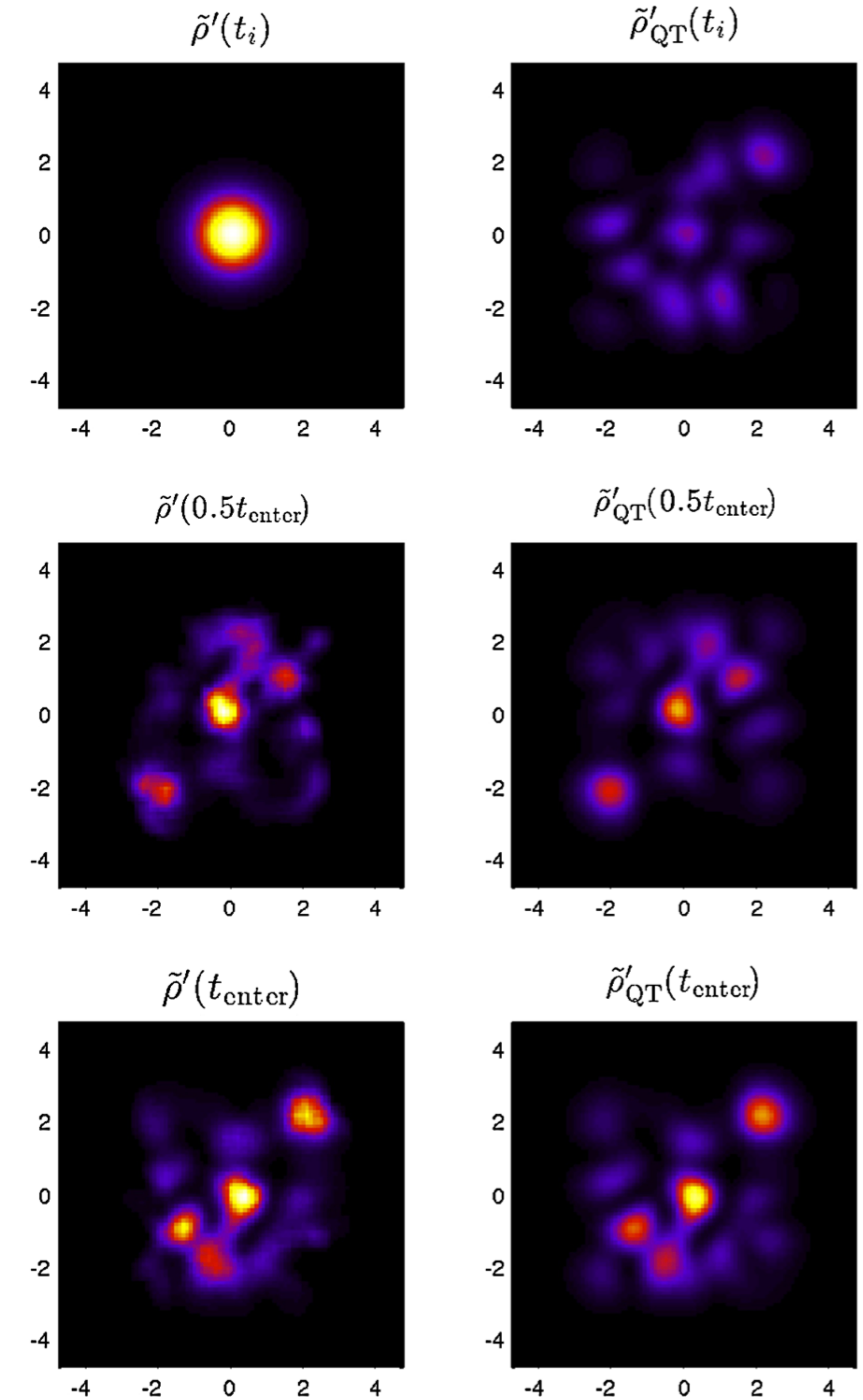
dBB trajectory of the field component $\dot{q}_r = m^{-1} \Im\mathrm{m} \dfrac{\partial_r \psi}{\psi}$

Statistical distribution $\dfrac{\partial \rho}{\partial t} + \sum_r \partial_r \left(\dfrac{\rho}{m} \Im\mathrm{m} \dfrac{\partial_r \psi}{\psi} \right) = 0$

$$i \frac{\partial \psi}{\partial t} = \sum_{r=1}^{2} \left(-\frac{1}{2m} \frac{\partial^2}{\partial q_r^2} + \frac{1}{2} m \omega^2 q_r^2 \right) \psi$$

Relaxation of a 2D harmonic oscillator

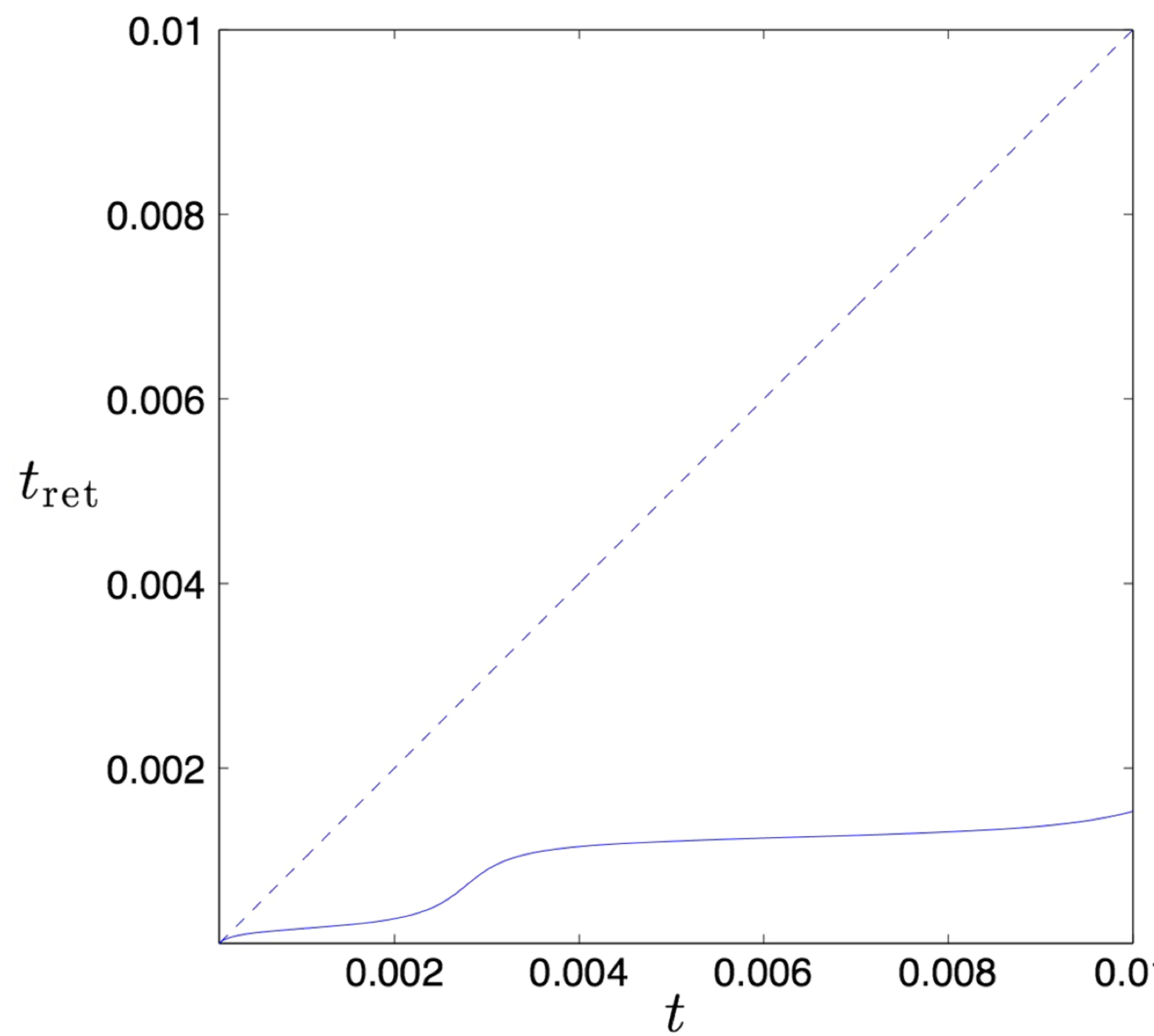
(time dependent mass & frequency)



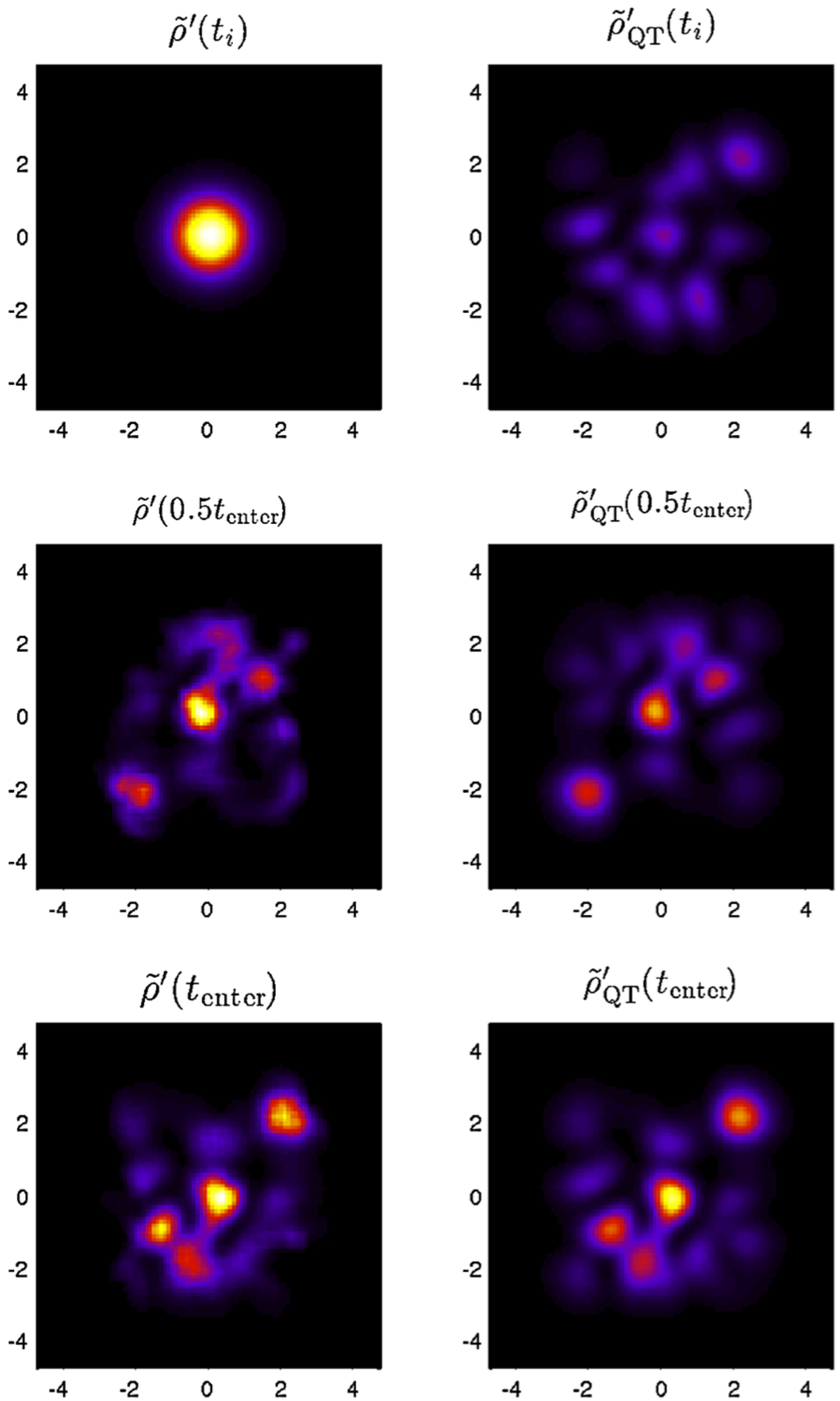
(constant mass & frequency)

Out-of-equilibrium time evolution

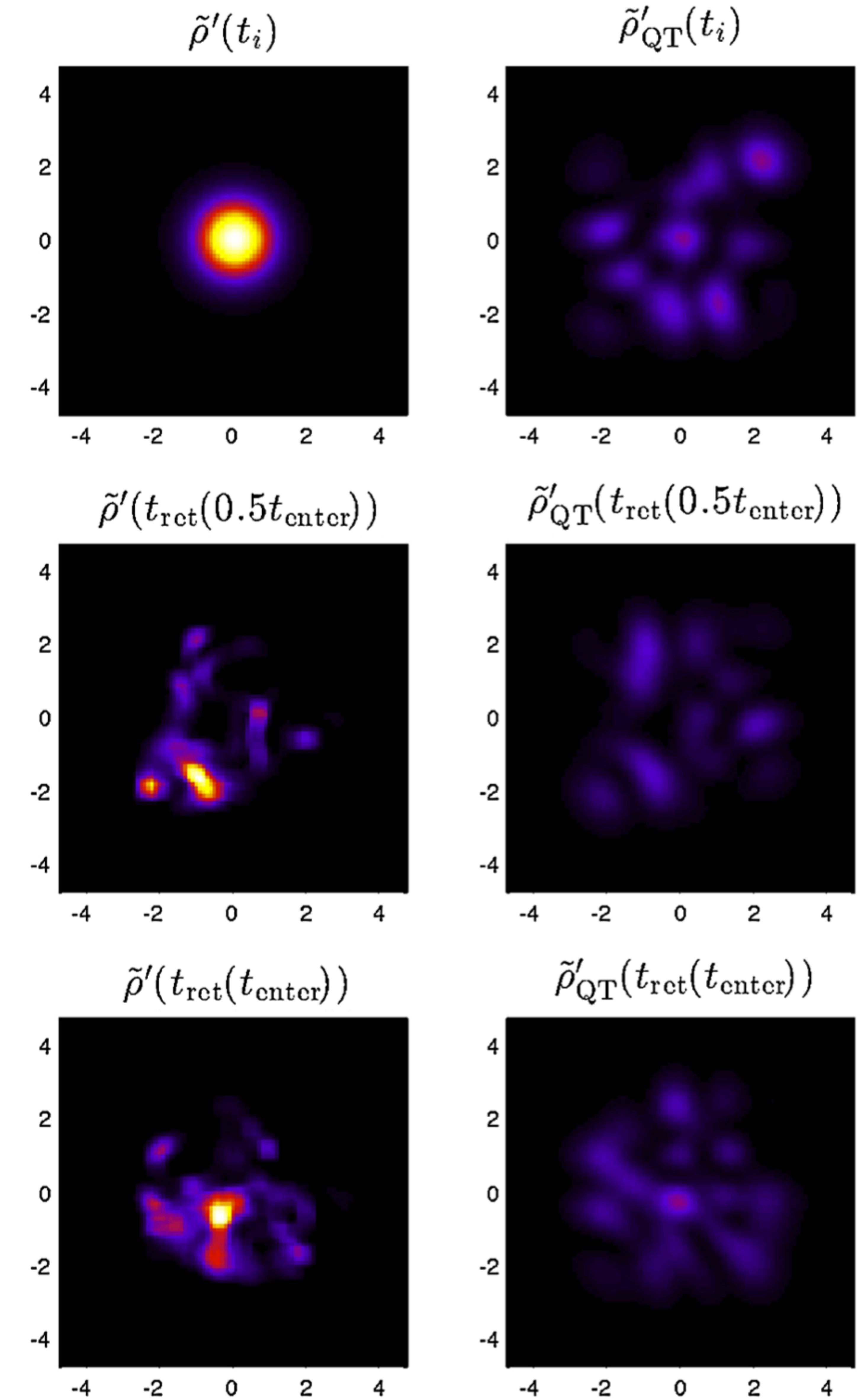
- Usual behaviour = evolves towards equilibrium (Minkowski or slowly expanding Universe)
- expansion: there is a retarded time…



Freezing the pdf out of equilibrium



without expansion

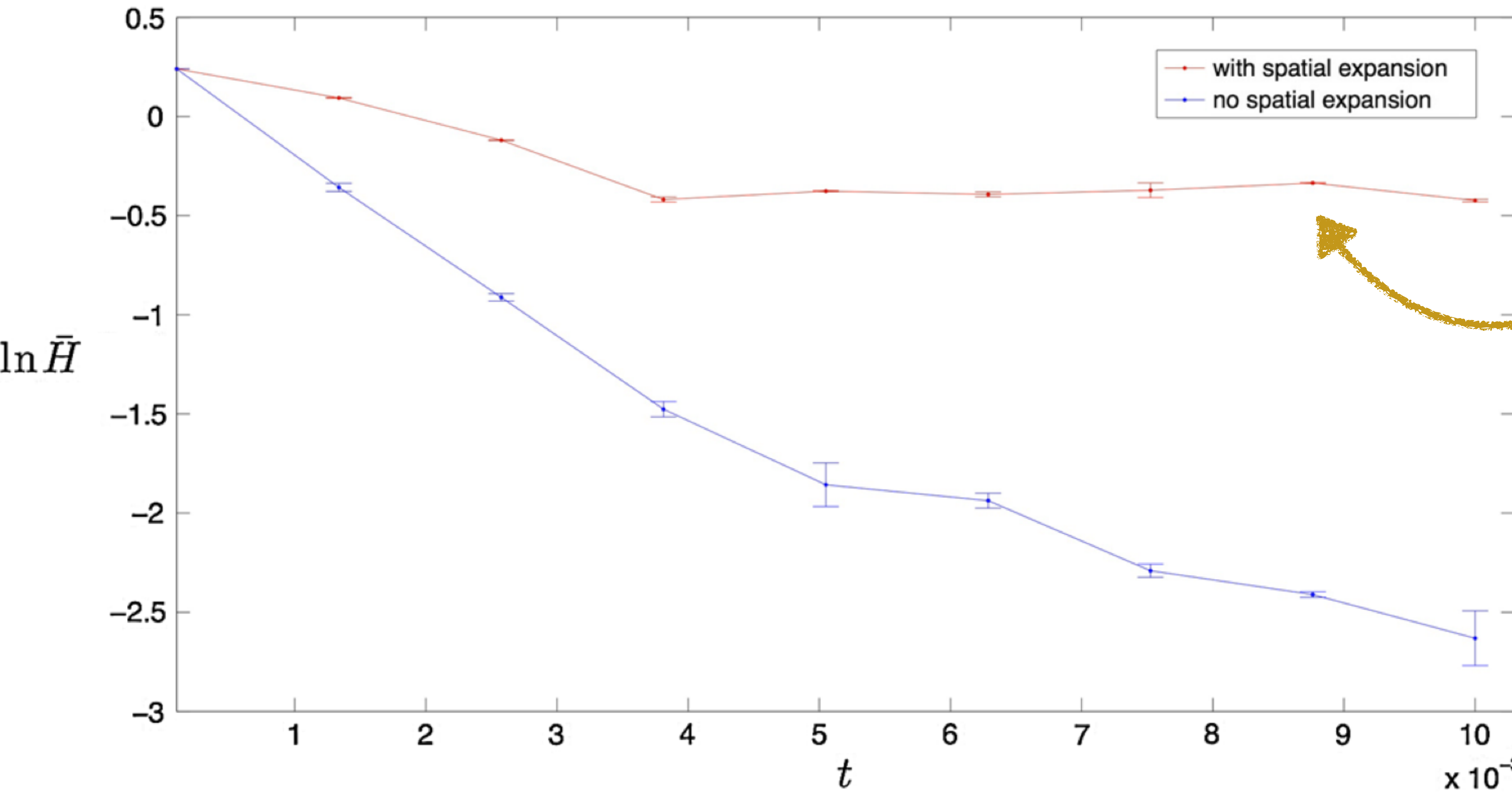


with expansion

S. Colin & A. Valentini,
Phys. Rev. **D88** 103515 (2013)

$$H \equiv \int \mathrm{d}q\, \rho \ln \left(\frac{\rho}{|\Psi|^2} \right)$$

measures "out-of-equilibrium-ness"



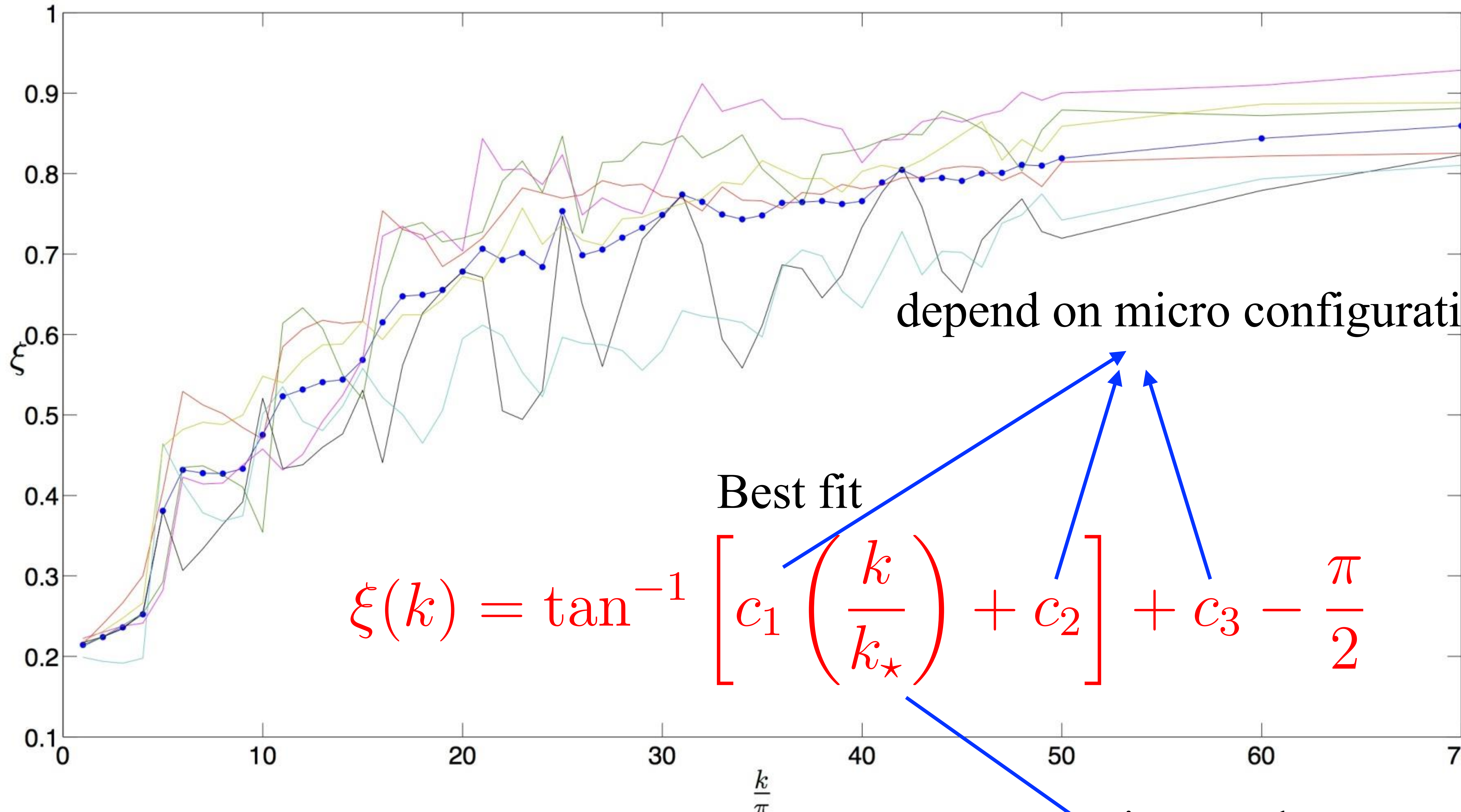
freezing of out-of-equilibrium modes

Initial out-of-equilibrium conditions

S. Colin & A. Valentini, Int. J. Mod. Phys. D 25, 1650068 (2016)

$$\mathcal{P}(k) = \mathcal{P}(k)_{\mathrm{QE}}\,\xi(k)$$

width deficit



depend on micro configuration

Best fit

$$\xi(k) = \tan^{-1}\left[c_1\left(\frac{k}{k_\star}\right) + c_2\right] + c_3 - \frac{\pi}{2}$$

pivot scale

Harmonic oscillator fundamental state

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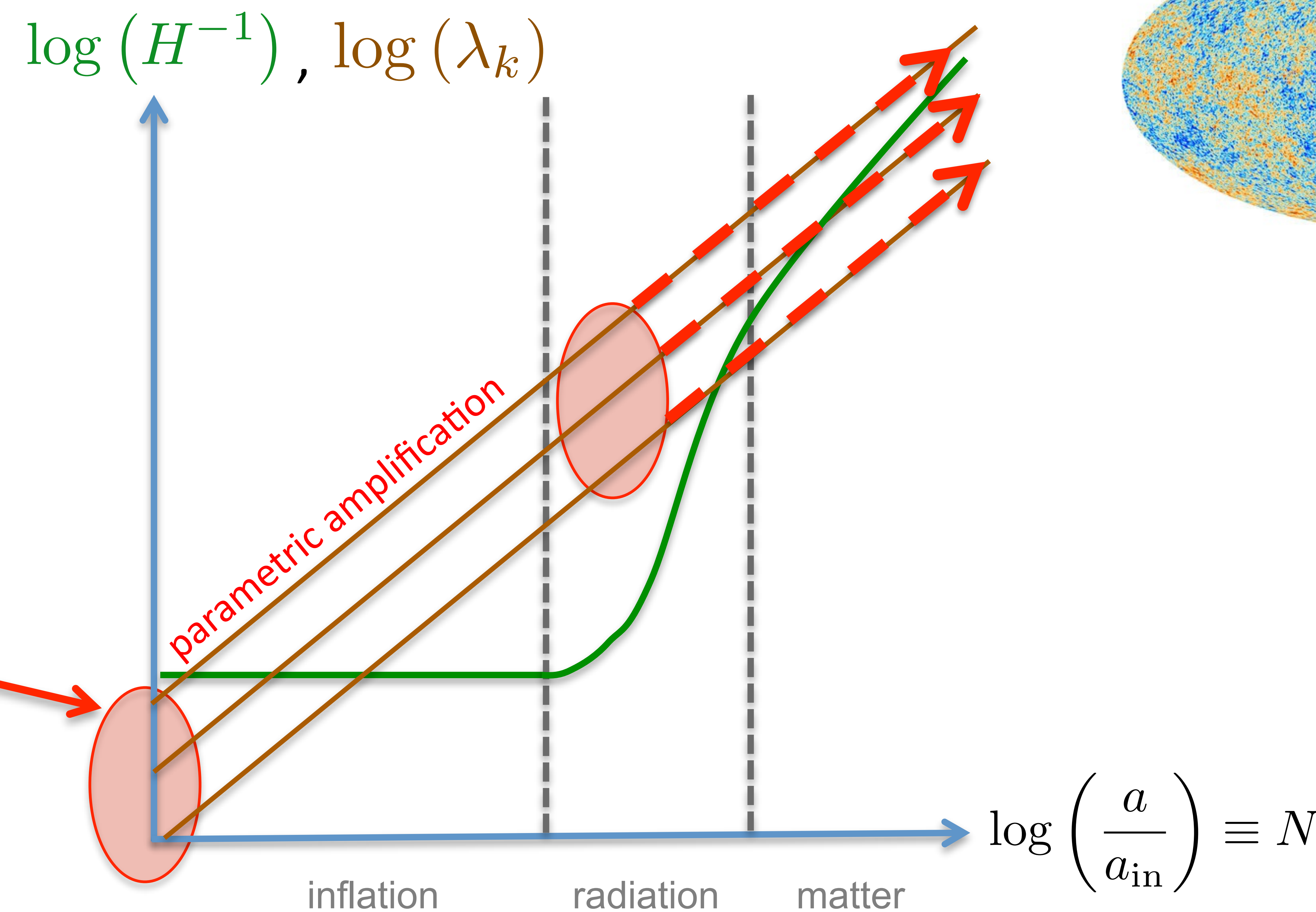

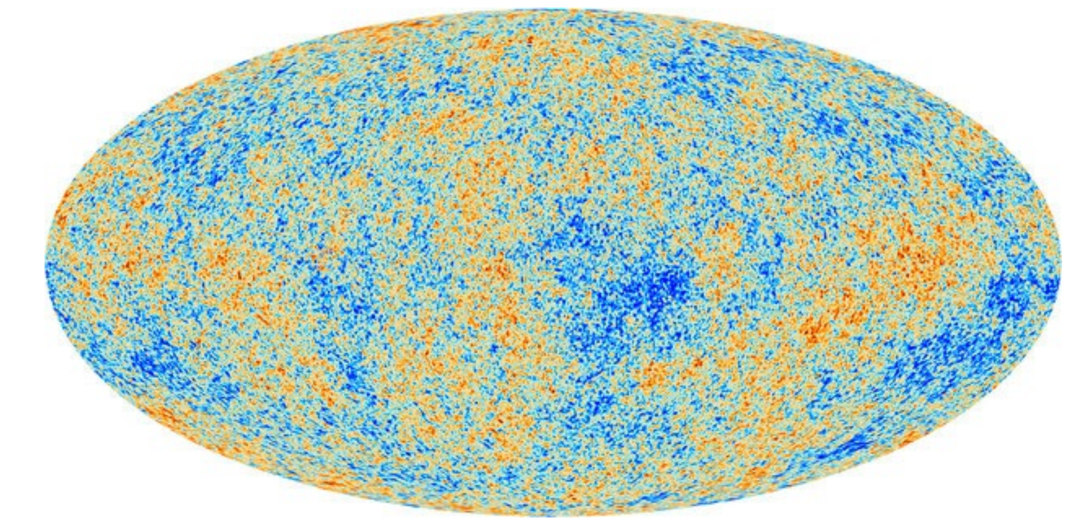

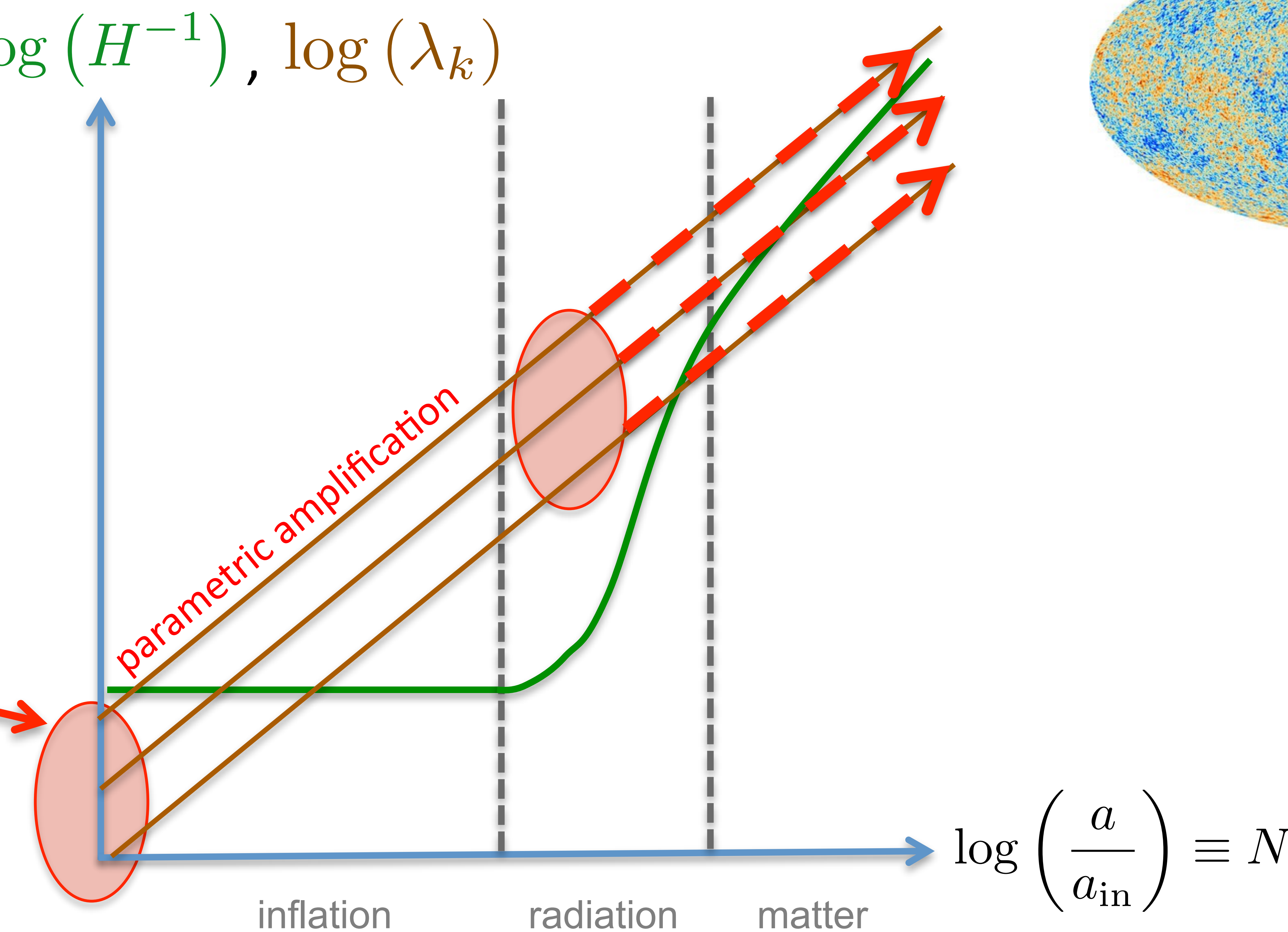

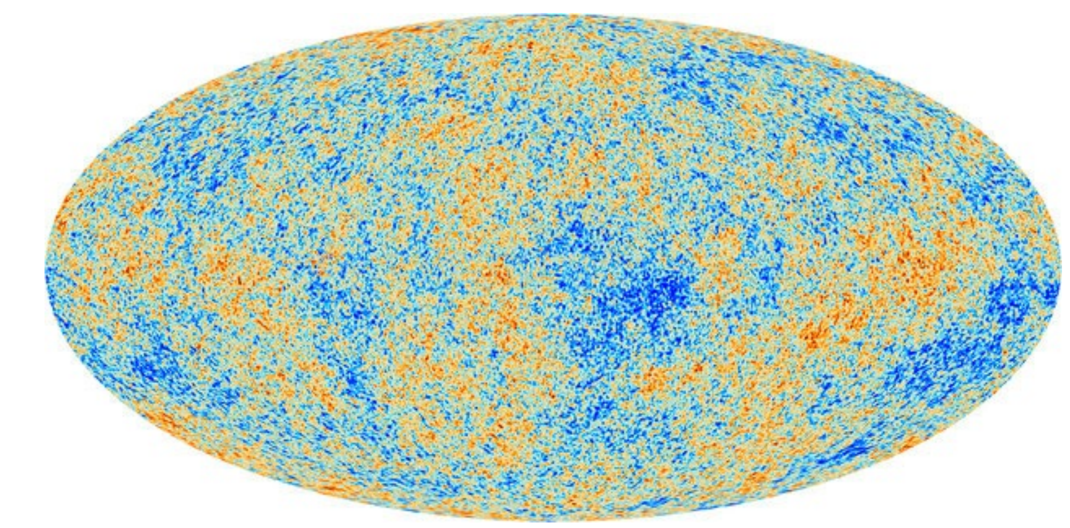

Harmonic oscillator fundamental state

$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2}v_{\boldsymbol{k}}^2}$$

Out-of-Equilibrium
initial density:
less quantum noise

Harmonic oscillator
fundamental state

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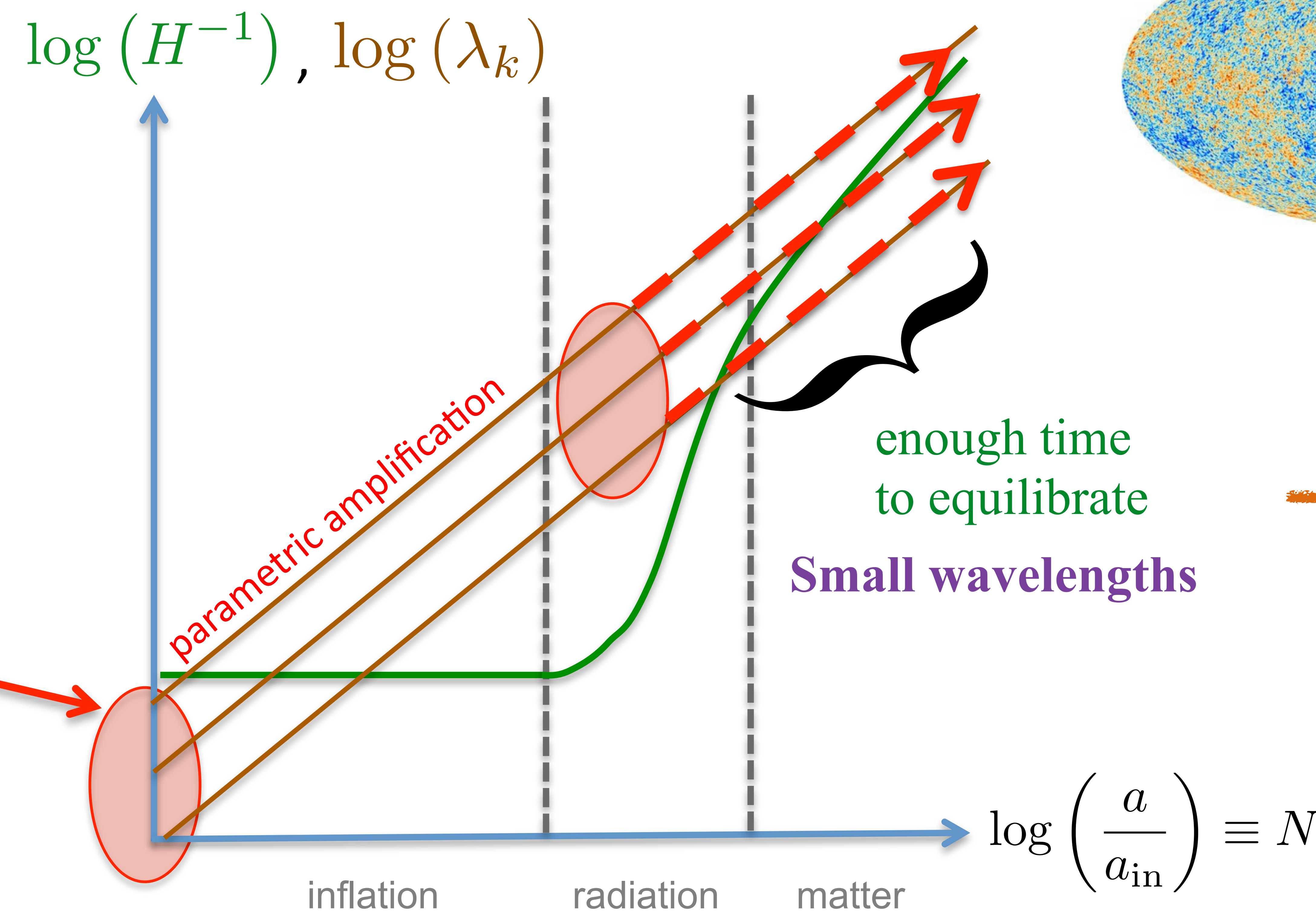
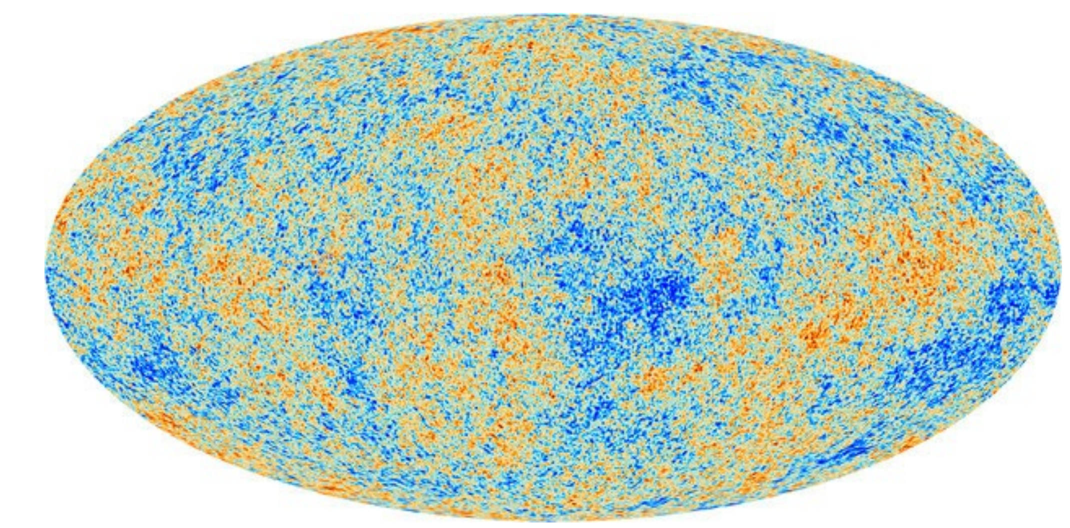
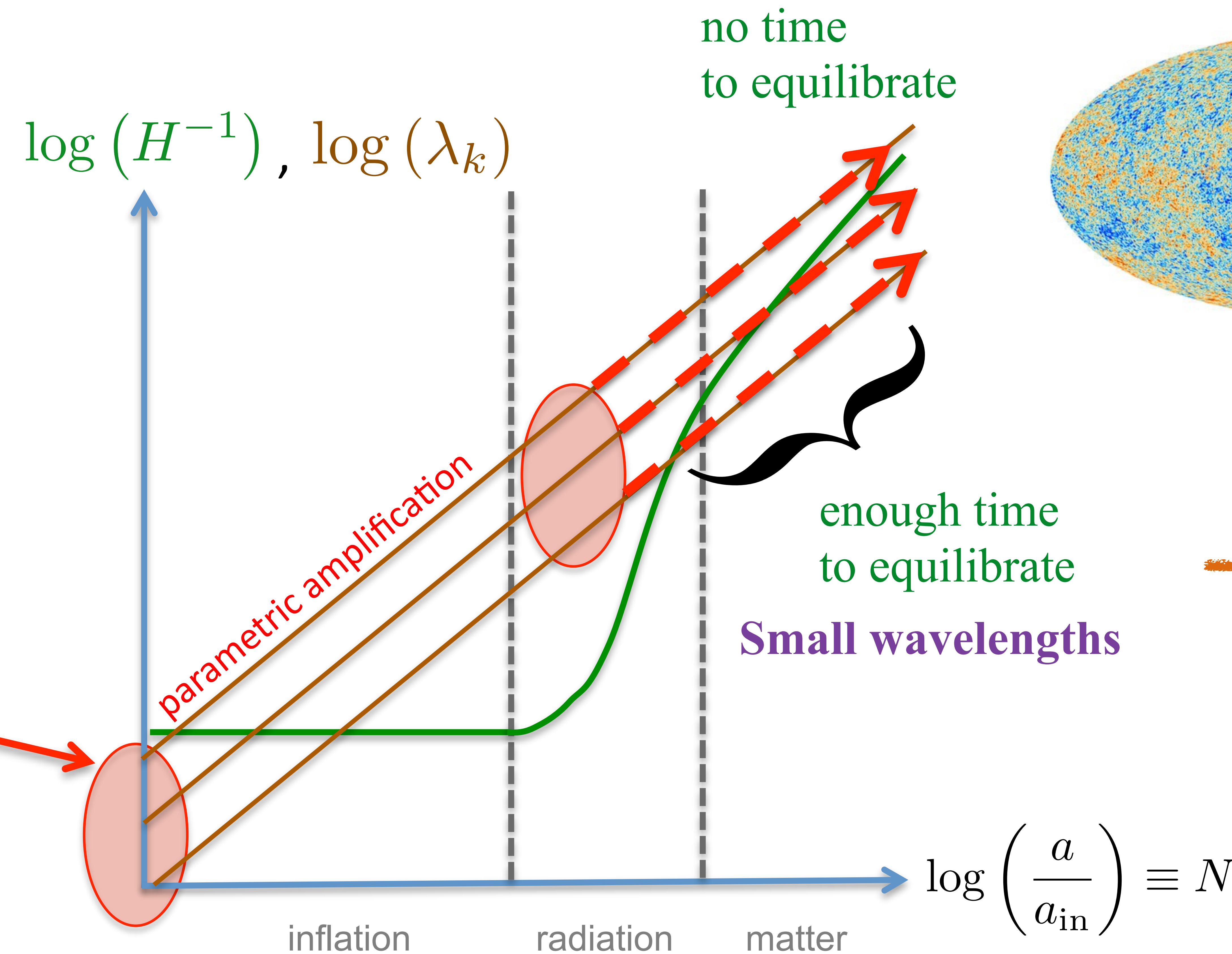


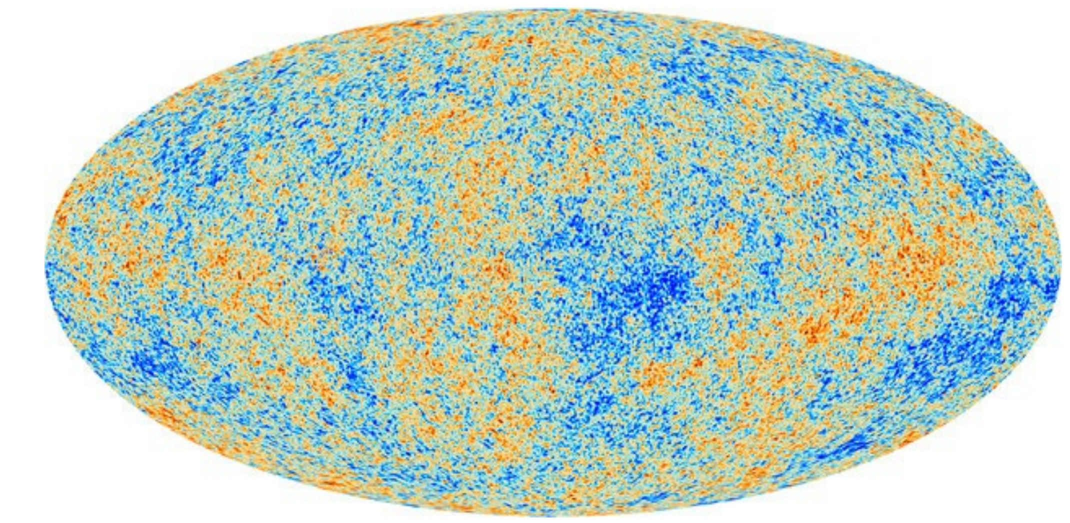


usual spectrum

Out-of-Equilibrium
initial density:
less quantum noise

less power ← **Very long wavelengths**





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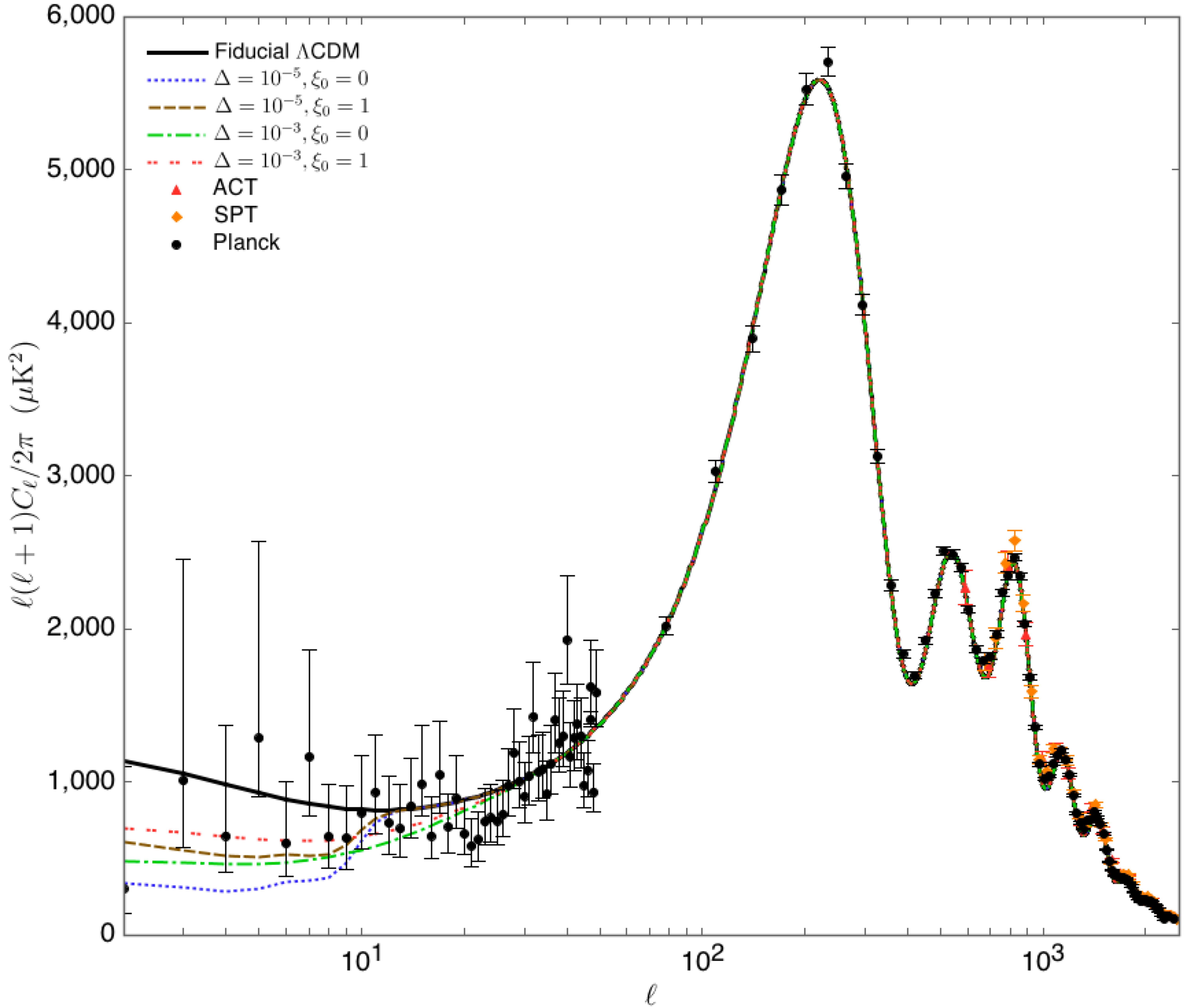
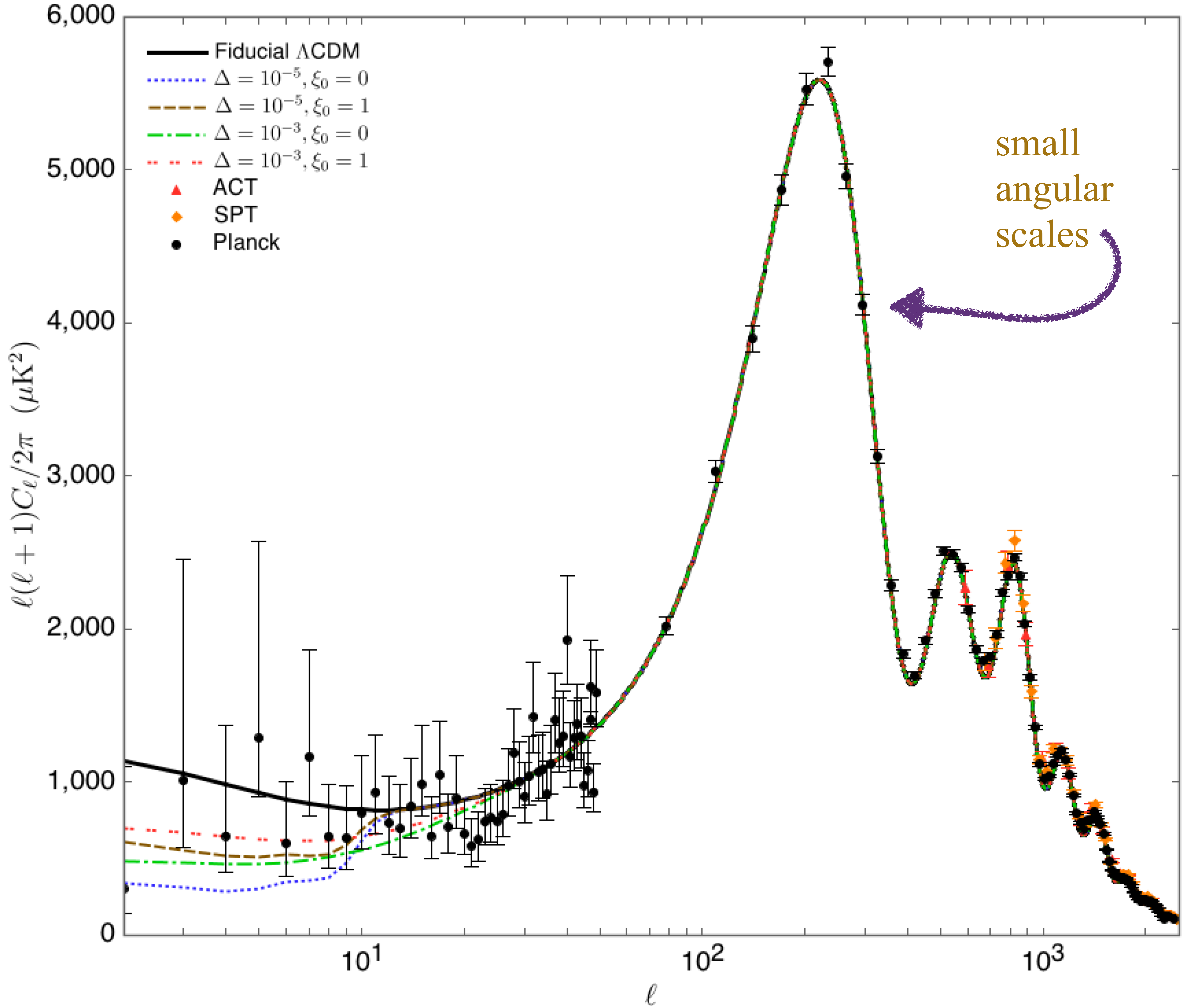


Fiducial ΛCDM
$\Delta = 10^{-5}, \xi_0 = 0$
$\Delta = 10^{-5}, \xi_0 = 1$
$\Delta = 10^{-3}, \xi_0 = 0$
$\Delta = 10^{-3}, \xi_0 = 1$
ACT
SPT
Planck
6,000
5,000
4,000
3,000
2,000
1,000
0
$\ell(\ell+1)C_\ell/2\pi \ (\mu K^2)$
10^1
10^2
10^3
ℓ



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SPT
Planck
small angular scales
$\ell(\ell+1)C_\ell/2\pi \ (\mu K^2)$
ℓ
6,000
5,000
4,000
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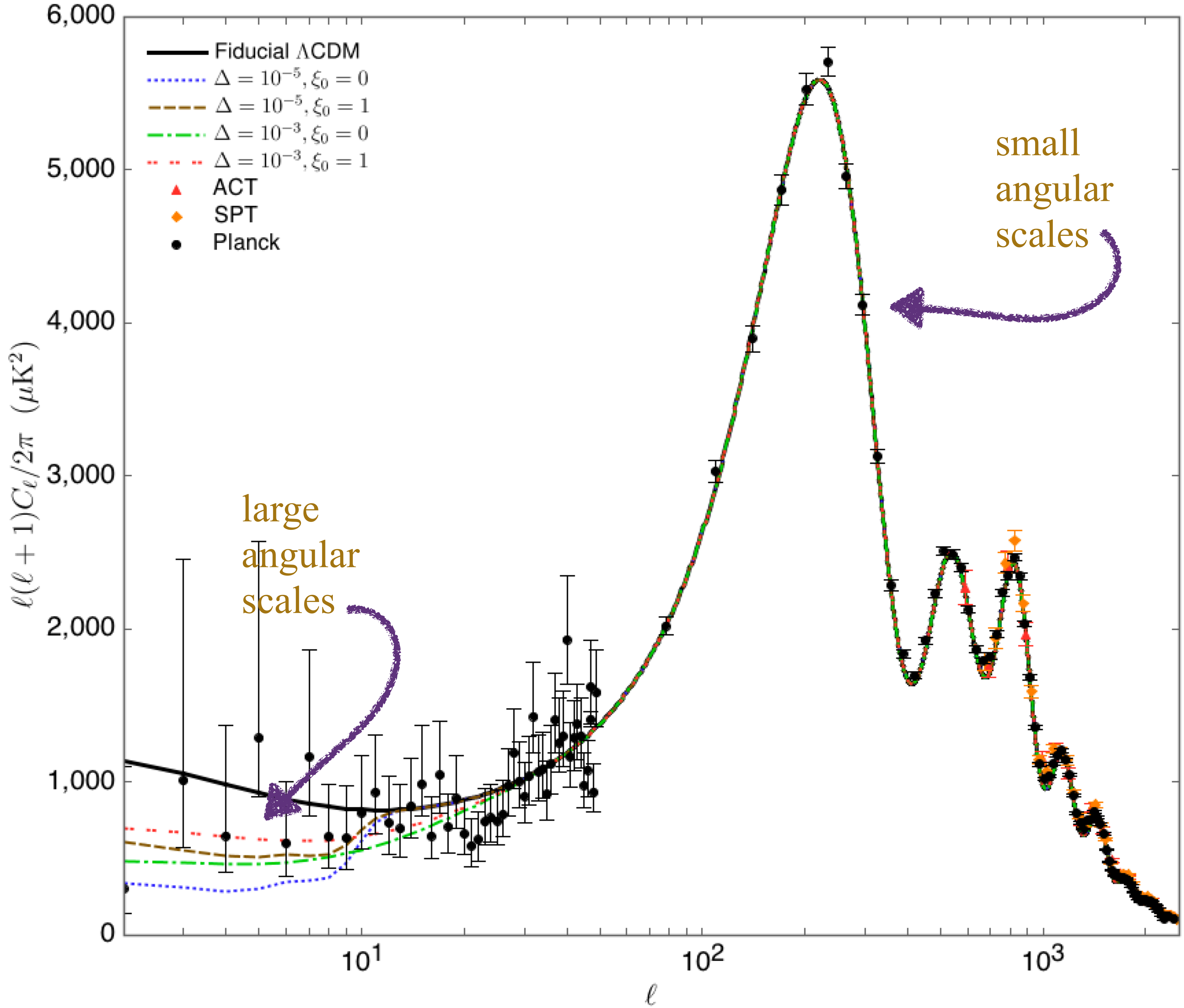

6,000
5,000
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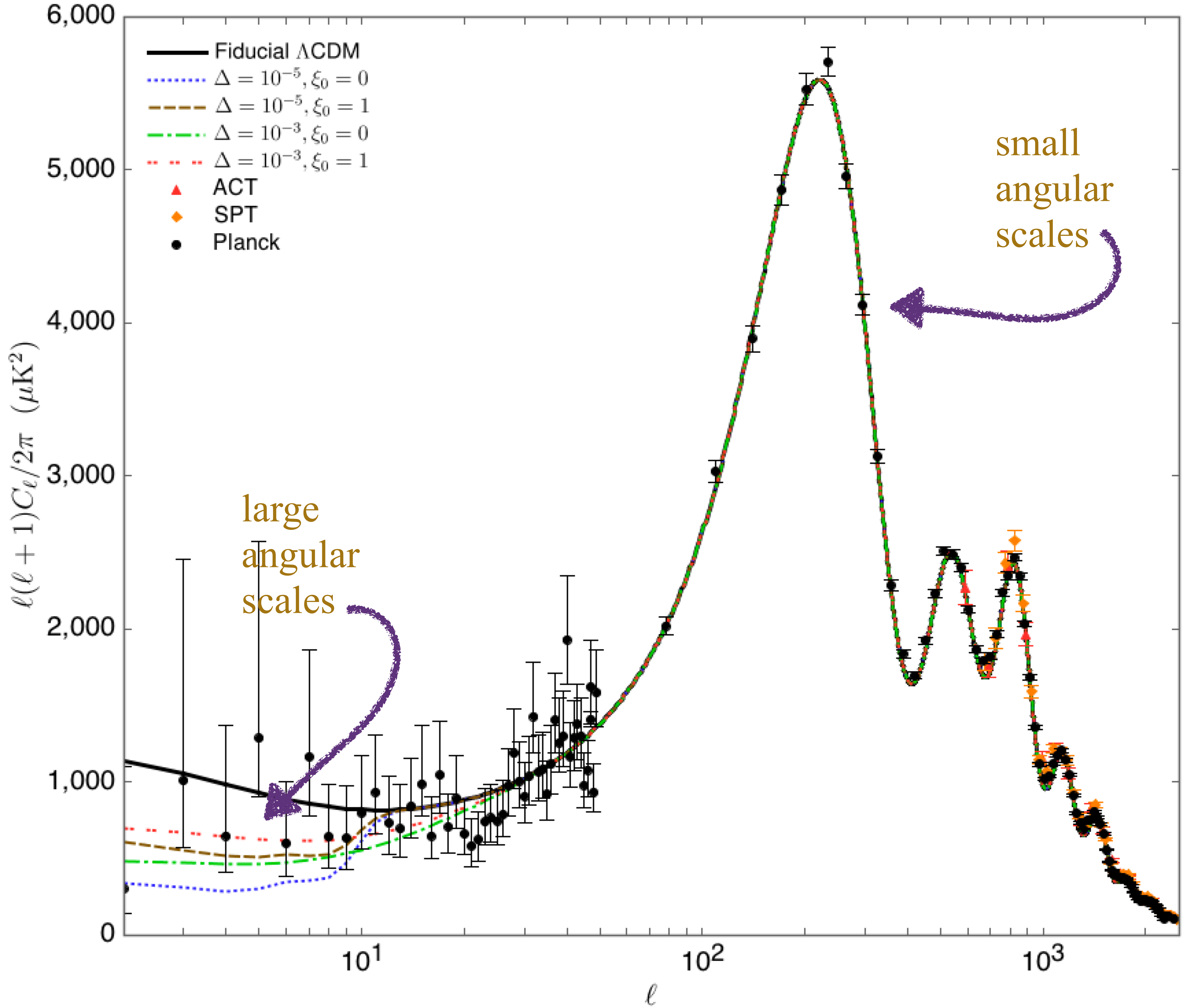
large angular scales



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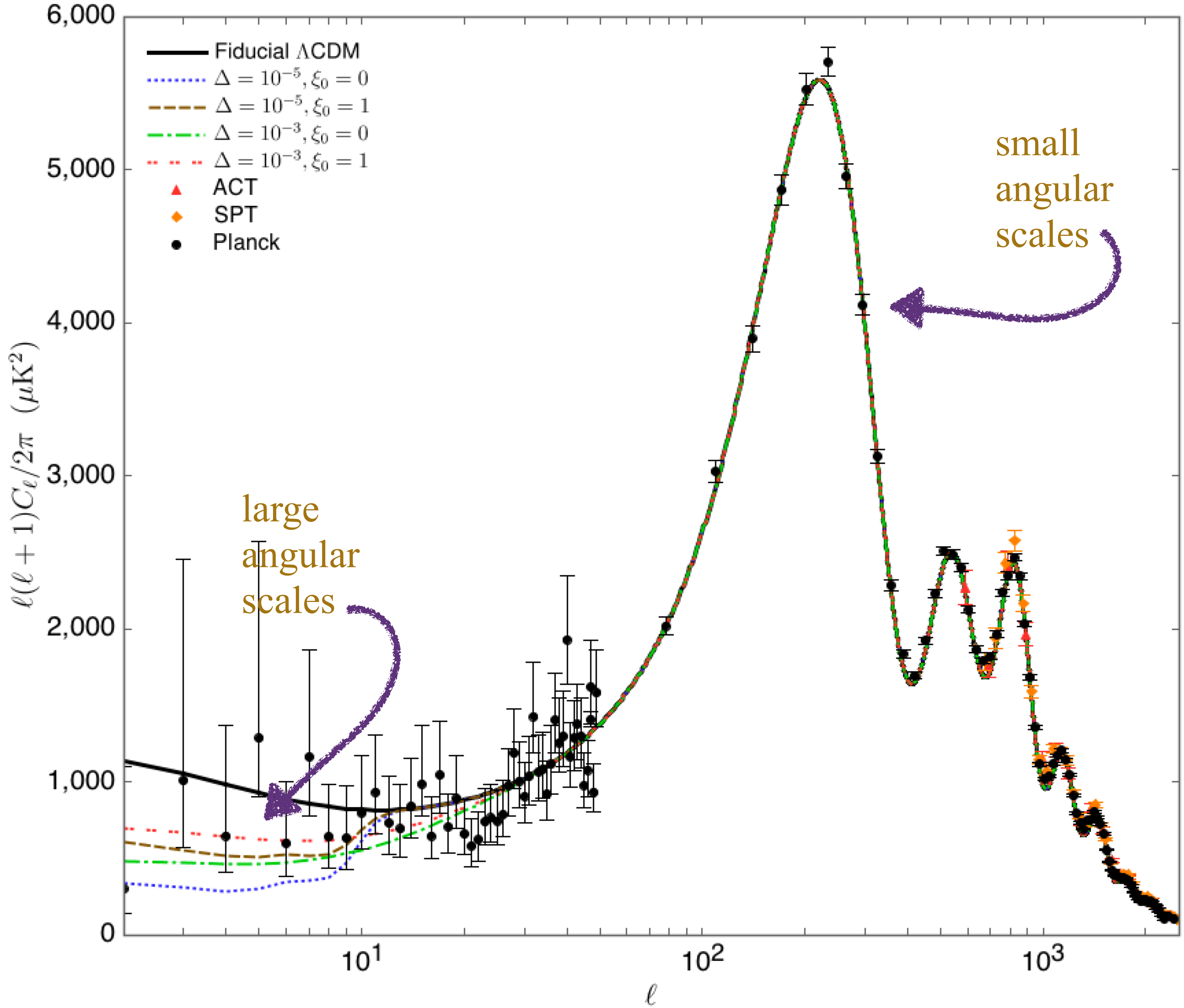
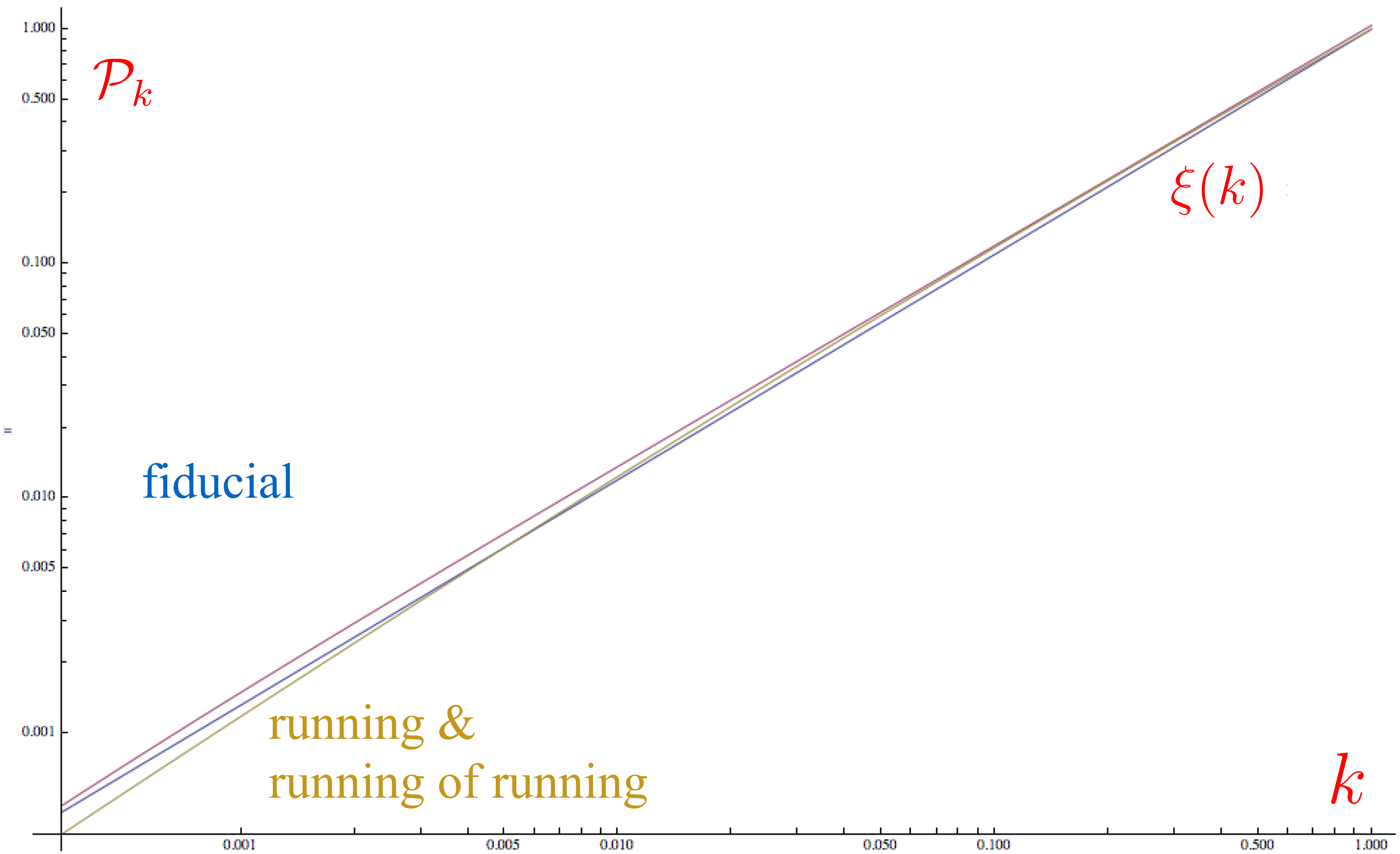
Better fit???



Better fit???

CosmoMC chains

with only one parameter added, others held fixed: $\xi(k) = \tan^{-1}\left[c_1\left(\frac{k}{k_\star}\right) + \underbrace{c_2}_{3.0}\right] + \underbrace{c_3}_{0.85} - \frac{\pi}{2}$



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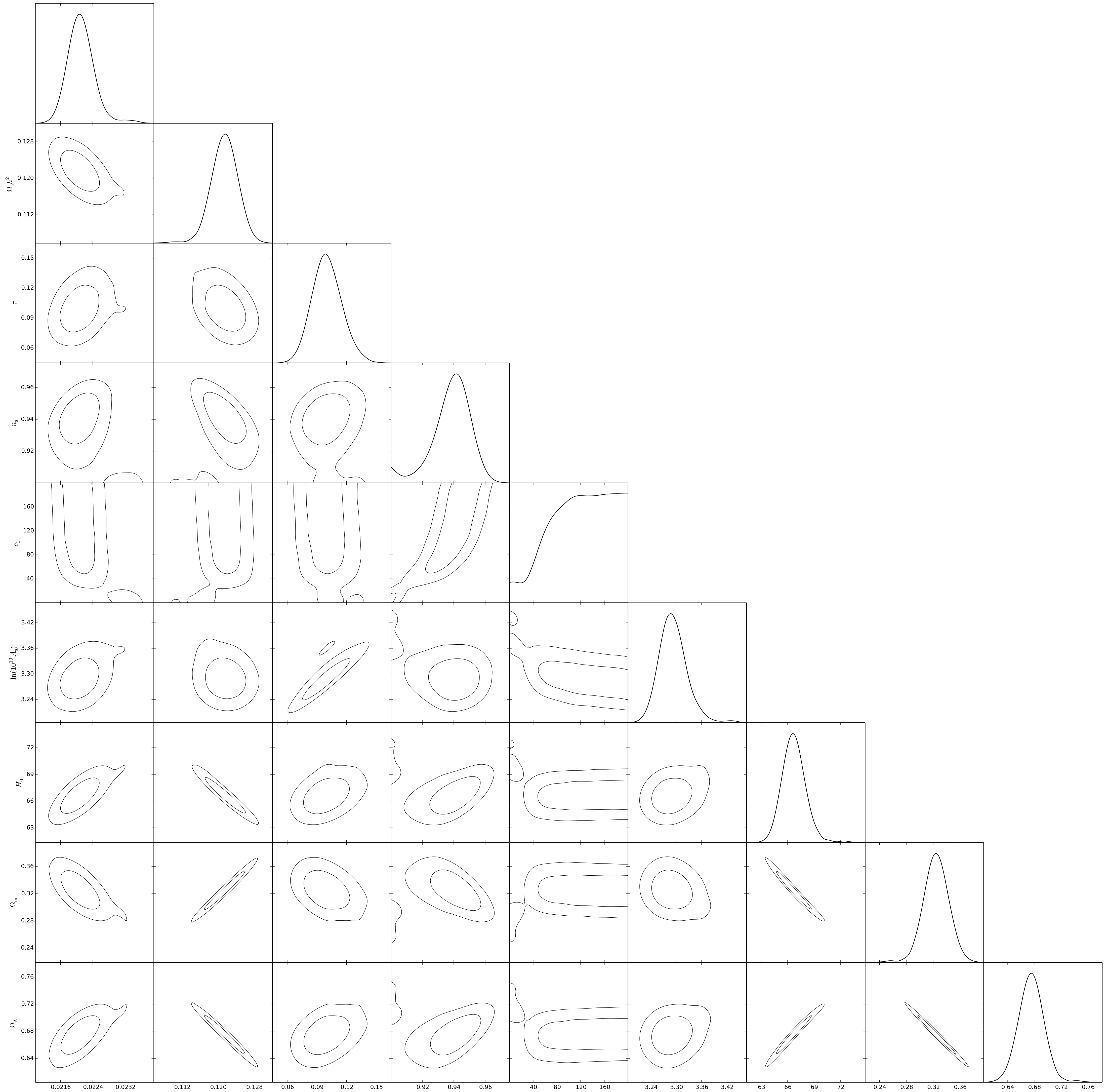

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3.0 0.85

with S. Vitenti & A. Valentini

Results…

work in progress!

Usual Planck best-fit

The GRW dynamical collapse model

Ghirardi - Rimini - Weber

Modified Schrödinger equation with collapse towards $\hat{C}$ eigenstates

$$\mathrm{d}|\Psi\rangle = -i\hat{H}|\Psi\rangle \mathrm{d}t + \sqrt{\gamma}\left(\hat{C} - \langle\hat{C}\rangle\right)\mathrm{d}W_t|\Psi\rangle - \underbrace{\frac{\gamma}{2}\left(\hat{C} - \langle\hat{C}\rangle\right)^2 \mathrm{d}t|\Psi\rangle}_{\text{normalization}}$$

Hamiltonian

non linear stochastic

$$\langle\hat{C}\rangle \equiv \langle\Psi|\hat{C}|\Psi\rangle$$

random outcomes

Born rule

$$\mathbb{E}\left(\mathrm{d}W_t\right) = 0$$

$$\mathbb{E}\left(\mathrm{d}W_t \mathrm{d}W_{t'}\right) = \mathrm{d}t\mathrm{d}t'\delta(t-t')$$

break superposition principle

Wiener process

BONUS: Amplification mechanism

Grown perturbations

Big objects are classical
small objects are quantum!

Primordial perturbations

Conclusions

(1) dBB = testable formulation of QM

(2) quantum non-equilibrium may produce new effects

(3) most systems did reach equilibrium

(4) primordial perturbations maybe not…

(5) specific shape for the primordial spectrum

(6) comparable with data!

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more work still needs be done
(other modifications of QM can be tested…)

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Dziękuję!

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