On rapidly rotating geometry

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In collaboration with

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Masashi Kimura (YITP, Japan)
Mandar Patil (TIFR, India)

§ 1 Introduction

Kerr spacetime: vacuum, stationary, axi-symmetric, asymptotically flat

In Boyer-Lindquist coordinates

$$ds^2 = -\frac{\Sigma\Delta}{A}dt^2 + \frac{A}{\Sigma}\sin^2\theta\left(d\varphi - \frac{2aMr}{A}dt\right)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2$$

where $\Delta(r) = r^2 - 2Mr + a^2$

$$\Sigma(r,\theta) = r^2 + a^2\cos^2\theta \qquad A(r,\theta) = \left(r^2 + a^2\right)^2 - \Delta(r)a^2\sin^2\theta$$

$$-\infty < t < +\infty, \quad -\infty < r < +\infty, \quad 0 \le \theta \le \pi, \quad 0 \le \varphi < 2\pi$$

M : ADM mass $\qquad$ a : Kerr parameter (angular momentum $L=aM$)

$a^2 \le M^2$: larger root of $\Delta(r) = 0$ $\left(r = M + \sqrt{M^2 - a^2} \ge M\right)$ is the event horizon

$a^2 > M^2$: always $\Delta(r) > 0$ holds. No event horizon.

Ring singularity at $r = 0$, $\theta = \pi/2$ is naked.

The over-spinning Kerr geometry is very interesting.

It is very important but very difficult to predict
the observable signatures from the naked singularity.

Let's focus on the vicinity of the *singularity*.

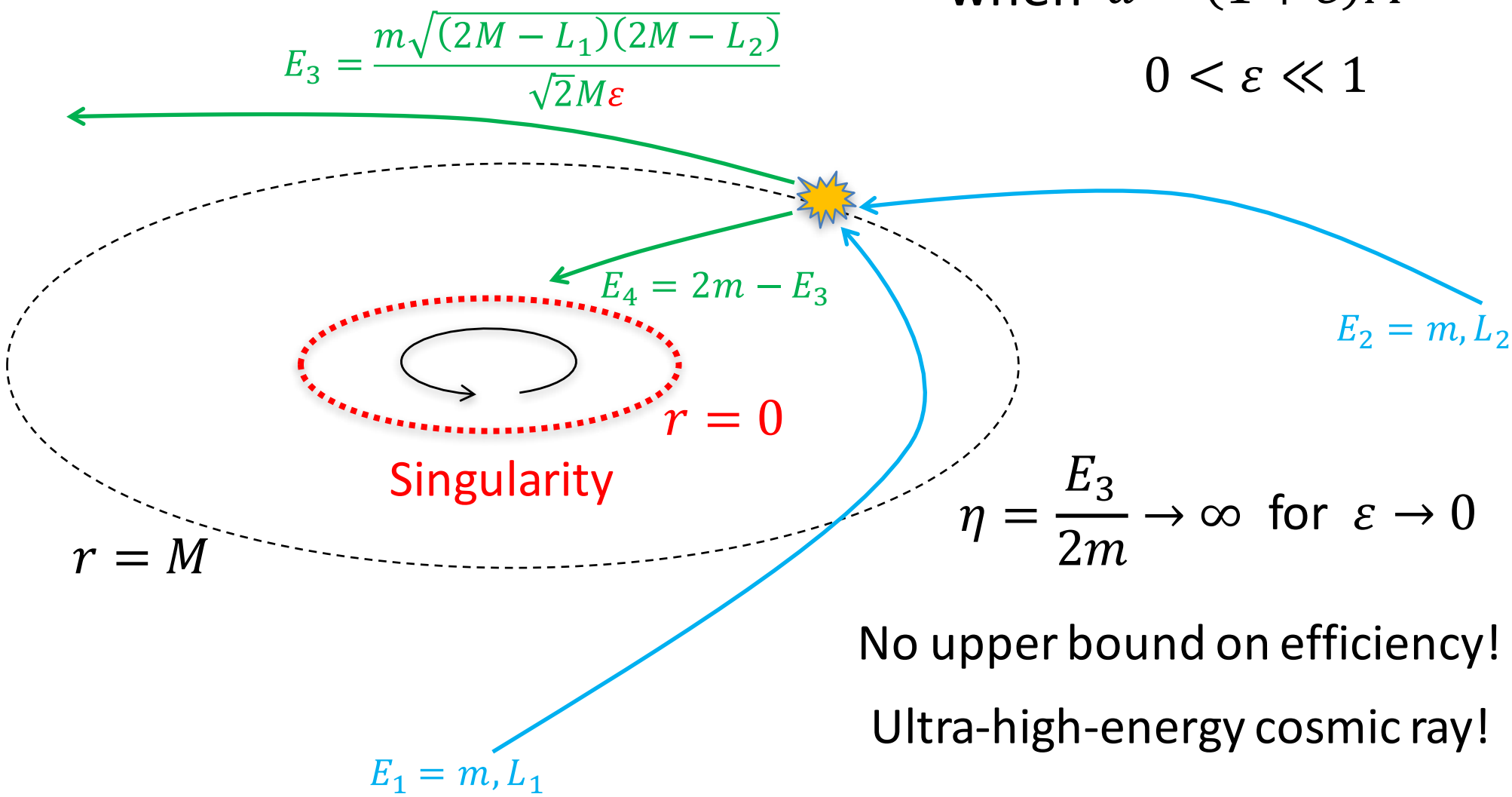
Sorry for being a bit out of the main topics of the workshop…

Collisional Penrose process

M. Patil, T. Harada, KN, P.S. Joshi and M. Kimura (2015)

when $a = (1+\varepsilon)M$

$0 < \varepsilon \ll 1$



$$\eta = \frac{E_3}{2m} \to \infty \quad \text{for} \quad \varepsilon \to 0$$

No upper bound on efficiency!

Ultra-high-energy cosmic ray!

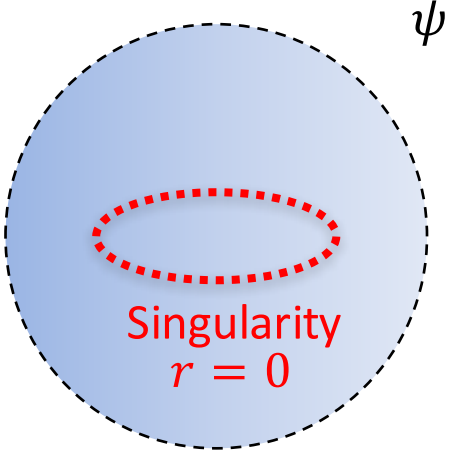
§ 2 Stability of over-spinning Kerr

Cardoso, Pani, Cadoni, Cavaglia (2008); Pani, Barausse, Berti and Cardoso (2010)

Quasi-normal modes of perturbations

Imaginary part of ω is positive →unstable

$$\psi = e^{-i\omega t + im\varphi} R(r) S(\theta)$$



Boundary condition
on $r = r_0$ =constant

- Reflecting BC
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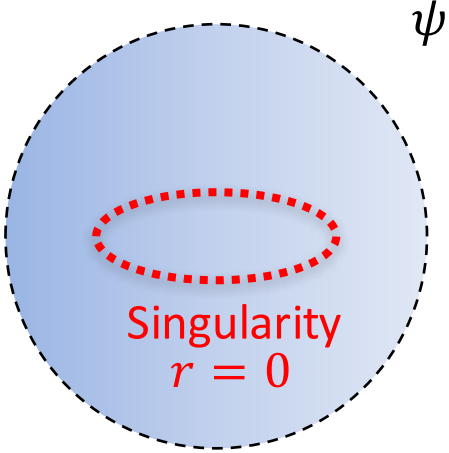

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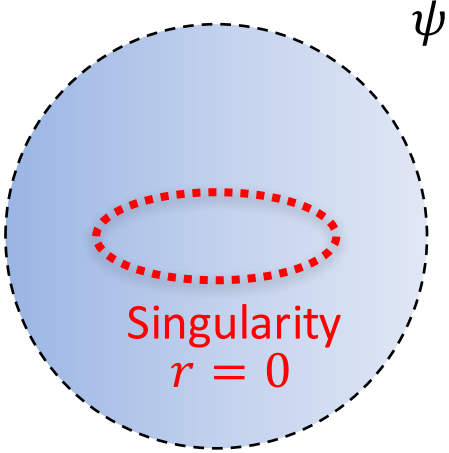

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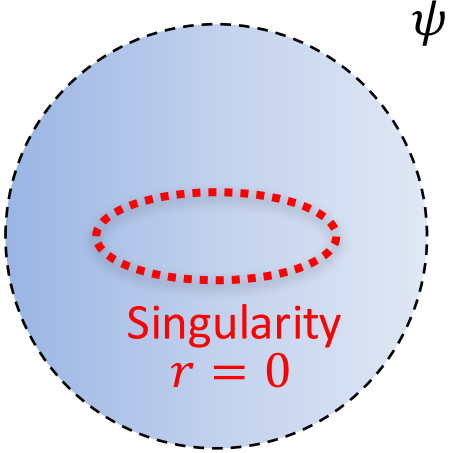

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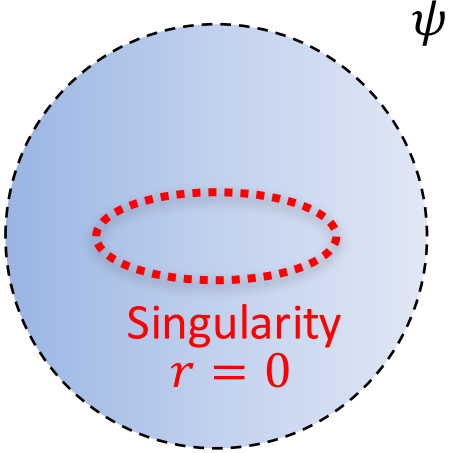

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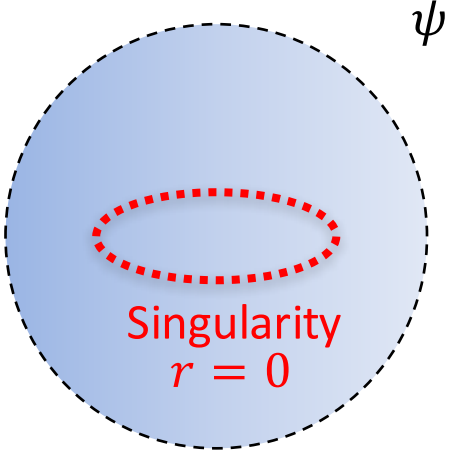

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Near extremal over-spinning Kerr has very long life time!

We are interested in

$$a = (1+\varepsilon)M \quad 0 < \varepsilon \ll 1$$

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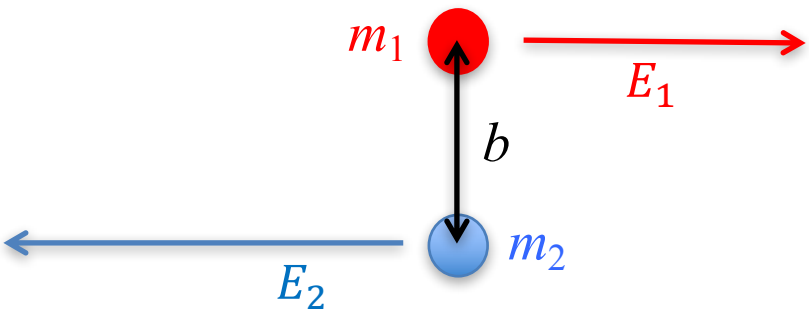
§ 3 How small is an over-spinning body?

Two relativistic particles without any interactions



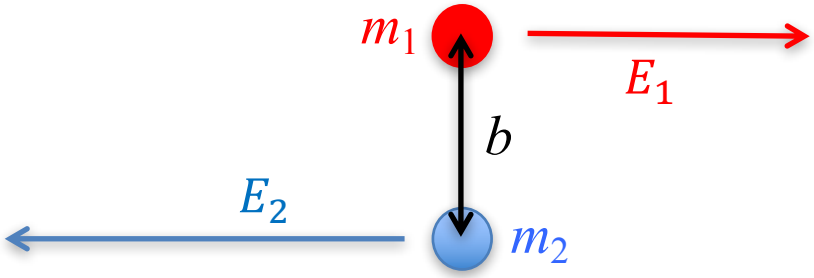
angular momentum $L = b\mu v$ in center of mass frame

$v = |\vec{v}_1 - \vec{v}_2|$: relative velocity

$$\mu = \frac{E_1 + E_2}{c^2(E_1 + E_2)}$$: relativistic reduced mass

Kerr parameter of this system $a = \dfrac{L}{Mc}$

$$M = \frac{E_1 + E_2}{c^2}$$: total mass of this system



Over-spinning condition: $a > \dfrac{GM}{c^2}$

$$\Rightarrow \quad b > \frac{(E_1 + E_2)^2}{E_1 E_2} \frac{c}{v} \frac{GM}{c^2} > \frac{2GM}{c^2}$$

This result suggests that no over-spinning very compact body forms.

In order that the system is smaller than the gravitational radius,

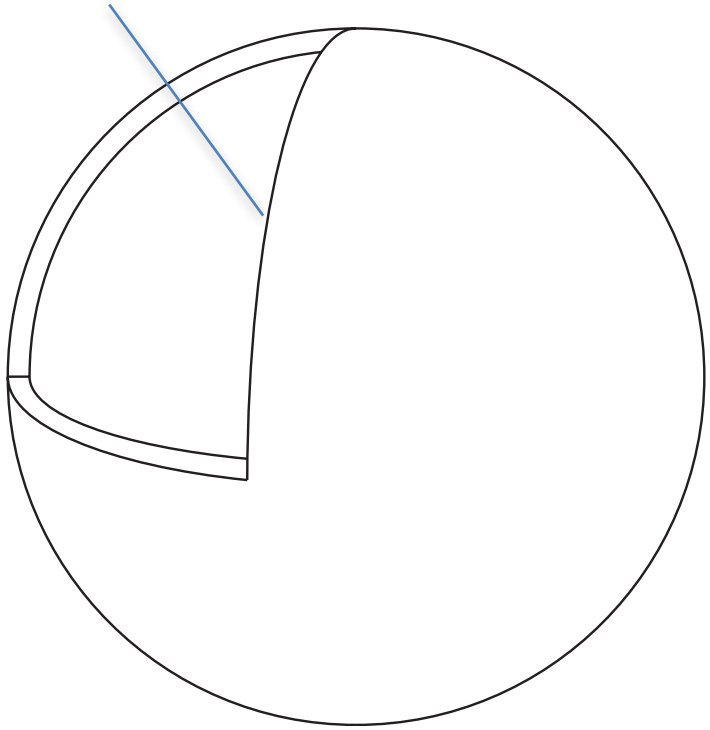
$v = |\vec{v}_1 - \vec{v}_2| > 2c$: Causality Violation

But does this inequality hold even when the gravity is taken into account?

The situation we consider: snapshot of an over-spinning thin shell

KN, M. Kimura, T. Harada, M. Patil, P.S. Joshi(2014) + some improvments

Inside ($r \leq R$): Regular curved space



Shell is located on $r = R = constant$.

3-dimensional metric: γ_{ij}

Continuous even on the shell

Extrinsic curvature: $K_{ij} = -\frac{1}{2}\mathcal{L}_n \gamma_{ij}$

Some components may be discontinuous

Outside ($r > R$): Over-spinning Kerr space

$a^2 > M^2$

Intrinsic Metric

$$dl^2 = \Phi^4(r,\theta)\left[\Lambda(r,\theta)dr^2 + \rho(r,\theta)d\theta^2 + P(r,\theta)\sin^2\theta d\varphi^2\right]$$

Outside the shell $r > R$: equivalent to the initial data of Kerr

$$\Phi(r,\theta) = 1$$

$$\Lambda(r,\theta) = \frac{\Sigma(r,\theta)}{\Delta(r)} \qquad \rho(r,\theta) = \Sigma(r,\theta) \qquad P(r,\theta) = \frac{A(r,\theta)}{\Sigma(r,\theta)}$$

where $\Delta(r) = r^2 - 2Mr + a^2$ $\qquad \Sigma(r,\theta) = r^2 + a^2\cos^2\theta$

$$A(r,\theta) = \left(r^2 + a^2\right)^2 - \Delta(r)a^2\sin^2\theta$$

$\Delta(r) = 0$: horizon

Over spinning: $a^2 > M^2$ $\Rightarrow$ $\Delta(r) > 0$ No horizon

Intrinsic Metric

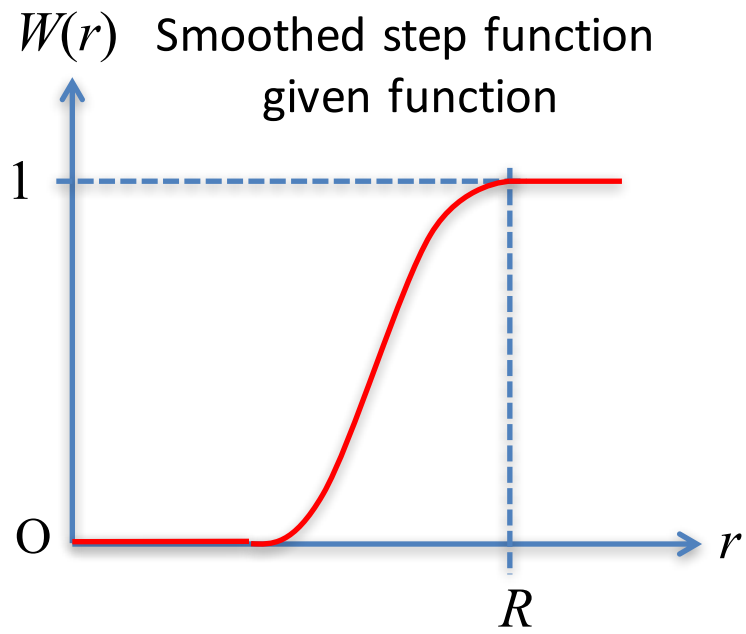
$$dl^2 = \Phi^4(r,\theta)\left[\Lambda(r,\theta)dr^2 + \rho(r,\theta)d\theta^2 + P(r,\theta)\sin^2\theta d\varphi^2\right]$$

Inside the shell $r \le R$

$\Phi(r,\theta) = 1$ on the shell $r = R$

$\boxed{\Phi(r,\theta) = ? \text{ inside the shell } r < R}$

Determined by the Hamiltonian constraint



$$\Lambda(r,\theta) = \frac{\Sigma(r,\theta)}{\Delta(r)}W(r) + \psi_C\left[1 - W(r)\right]$$

$$\rho(r,\theta) = \left(r^2 + a^2\cos^2\theta\right)W(r) + \psi_C r^2\left[1 - W(r)\right]$$

$$P(r,\theta) = \frac{A(r,\theta)}{\Sigma(r,\theta)}W(r) + \psi_C r^2\left[1 - W(r)\right]$$

ψ_C : Positive constant

Extrinsic Curvature

Outside the shell $r > R$: equivalent to the initial data of Kerr

$$K^{r\varphi} = -\sqrt{\frac{\Delta A}{\Sigma^3}}\frac{\partial}{\partial r}\left(\frac{aMr}{A}\right) \qquad K^{\theta\varphi} = -\sqrt{\frac{A}{\Delta\Sigma^3}}\frac{\partial}{\partial \theta}\left(\frac{aMr}{A}\right)$$

where

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$$A(r,\theta) = \left(r^2 + a^2\right)^2 - \Delta(r)a^2\sin^2\theta$$

Inside the shell $r < R$

$$K^{r\varphi} = -\frac{1}{\Phi^{10}\Lambda}\frac{\partial \boxed{X}}{\partial r} \qquad K^{\theta\varphi} = -\frac{1}{\Phi^{10}\Sigma}\frac{\partial \boxed{X}}{\partial \theta}$$

Determined by the Momentum Constraint

Numerically solve the constraint equations!

Momentum constraint: $D_a\left(K^{ab} - h^{ab} trK\right) = 0$

$$\frac{\partial}{\partial r}\left(\sqrt{\frac{\Sigma P^3}{\Lambda}}\frac{\partial X}{\partial r}\right) + \frac{1}{r^2 \sin^3\theta}\frac{\partial}{\partial\theta}\left(\sin^3\theta\sqrt{\frac{\Lambda P^3}{\Sigma}}\frac{\partial X}{\partial\theta}\right) = 0$$

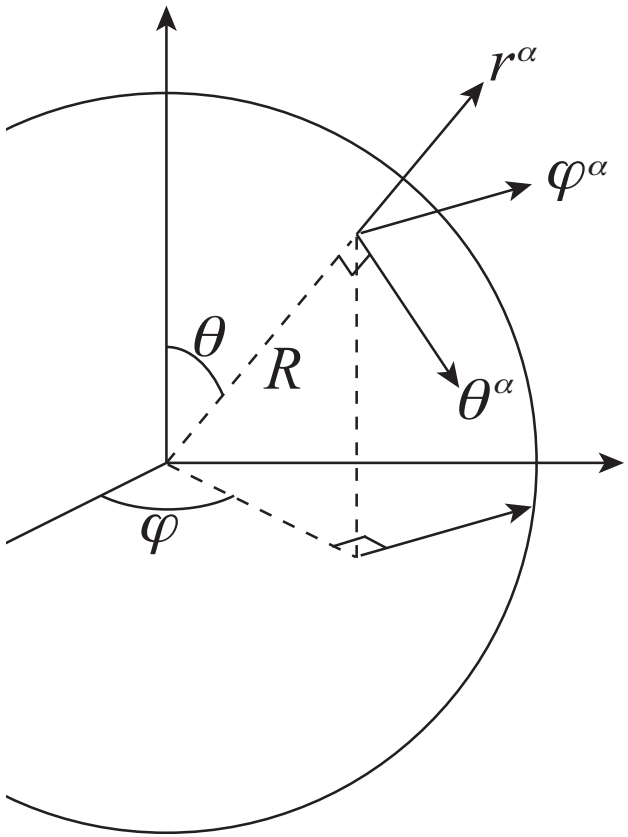
Hamiltonian constraint: ${}^3R - K^{ab}K_{ab} + trK^2 = 0$

$$\frac{1}{\sqrt{\Lambda\Sigma P}}\frac{\partial}{\partial r}\left(\sqrt{\frac{\Sigma P}{\Lambda}}\frac{\partial\Phi}{\partial r}\right) + \frac{1}{\sqrt{\Lambda\Sigma P}\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\sqrt{\frac{\Lambda P}{\Sigma}}\frac{\partial\Phi}{\partial\theta}\right)$$

$$= \frac{1}{8}{}^3\tilde{R}\Phi - \frac{1}{4}P\sin^2\theta\left[\frac{1}{\Lambda}\left(\frac{\partial X}{\partial r}\right)^2 + \frac{1}{\Sigma}\left(\frac{\partial X}{\partial\theta}\right)^2\right]\Phi^{-7}$$

From Israel's Junction Condition

$$\left[Q_{\mu\nu} - B_{\mu\nu} trQ\right]_{\pm} = \frac{8\pi G}{c^4} S_{\mu\nu}$$



$$S_{\mu\nu} = \sigma u_\alpha u_\beta + j\,(u_\alpha \varphi_\beta + \varphi_\alpha u_\beta) + p\,(\theta_\alpha \theta_\beta + \varphi_\alpha \varphi_\beta)$$

$$u_\alpha = \cosh\beta\, n_\alpha + \sinh\beta\, r_\alpha$$

$$\sigma = \frac{1}{8\pi}\sqrt{\frac{\Delta}{\Sigma}}\frac{\partial\Phi}{\partial r}\bigg|_{r=R-0} \cosh\beta$$: surface energy density

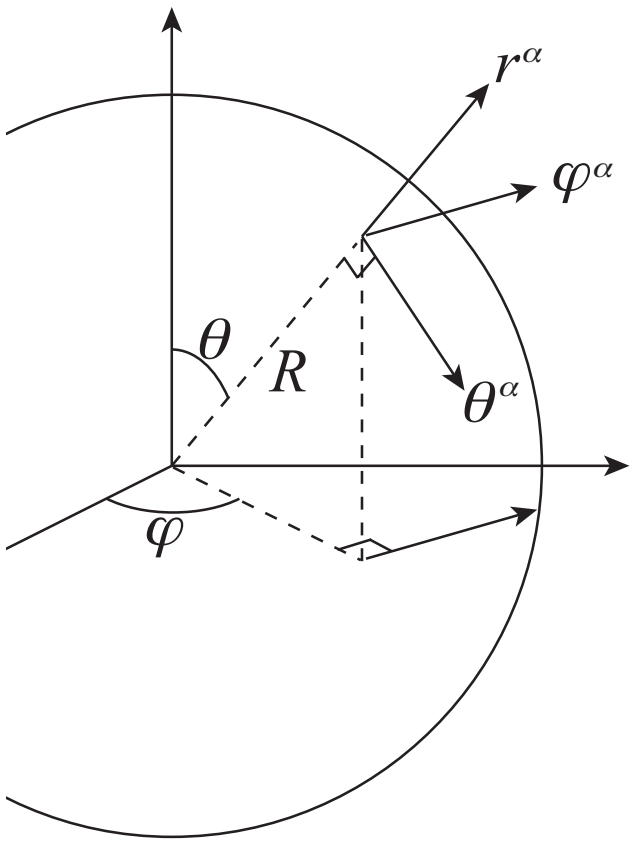
$$j = -\frac{1}{8\pi}\frac{\sqrt{A\Delta}}{\Sigma}\sin\theta\left[Ma\sqrt{\frac{A}{\Sigma\Delta}}\frac{\partial}{\partial r}\left(\frac{r}{A}\right) + \frac{\partial X}{\partial r}\right]\bigg|_{r=R}$$

: surface angular momentum density

p : stress on the shell specified by EOS

Stress-Energy tensor on the shell

From Israel's Junction Condition $$\boxed{\left[Q_{\mu\nu} - B_{\mu\nu} trQ\right]_{\pm} = \frac{8\pi G}{c^4} S_{\mu\nu}}$$



Shell is located on $r = R$.

Surface-stress-energy tensor on the shell

$$S_{\mu\nu} = \sigma\, u_\alpha u_\beta + j\,(u_\alpha \varphi_\beta + \varphi_\alpha u_\beta) + p\,(\theta_\alpha \theta_\beta + \varphi_\alpha \varphi_\beta)$$

$$= \rho\, v_\alpha v_\beta + p\, \theta_\alpha \theta_\beta + P\, \Phi_\alpha \Phi_\beta$$

Reasonable energy conditions

Weak Energy Condition: $\rho \geq 0,\ \ \rho + p \geq 0,\ \ \rho + P \geq 0$

Strong Energy Condition: $\rho + p \geq 0,\ \ \rho + P \geq 0,\ \ \rho + p + P \geq 0$

Dominant Energy Condition: $\rho + |p| \geq 0,\ \ \rho + |P| \geq 0$

Numerical Results and implications

Sufficient condition for the existence of EOS that does not conflict with reasonable energy conditions

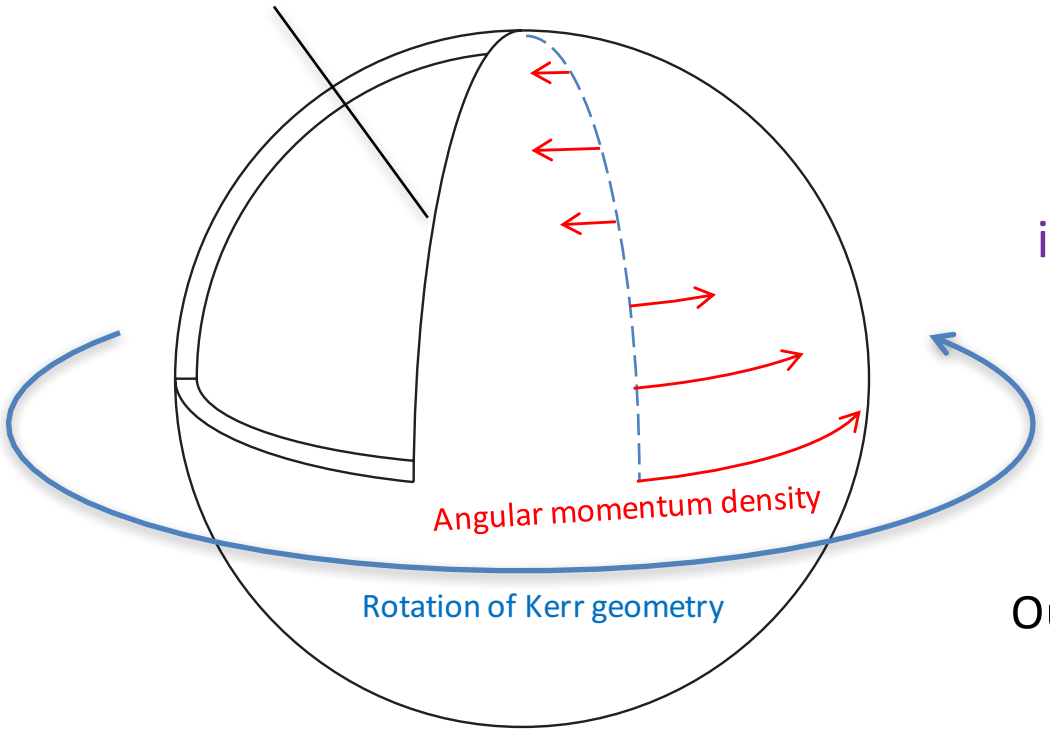
$$\sigma \geq \sqrt{2}|j|$$

Satisfied for any positive R?

Yes!

if it is radially moving with large velocity.

Inside ($r \leq R$): Regular curved space



Outside ($r > R$): Over-spinning Kerr space

$a^2 > M^2$

Shell is located on $r = R = constant$.

At least, very compact over-spinning object ($R < M$) can transiently exist.

High efficiency collisional Penrose process may occur around it.

We may find over-spinning geometry through ultra-high-energy cosmic ray.

At least, very compact over-spinning object ($R < M$) can transiently exist.

High efficiency collisional Penrose process may occur around it.

We may find over-spinning geometry through ultra-high-energy cosmic ray.

Maybe some implosion processes are necessary.

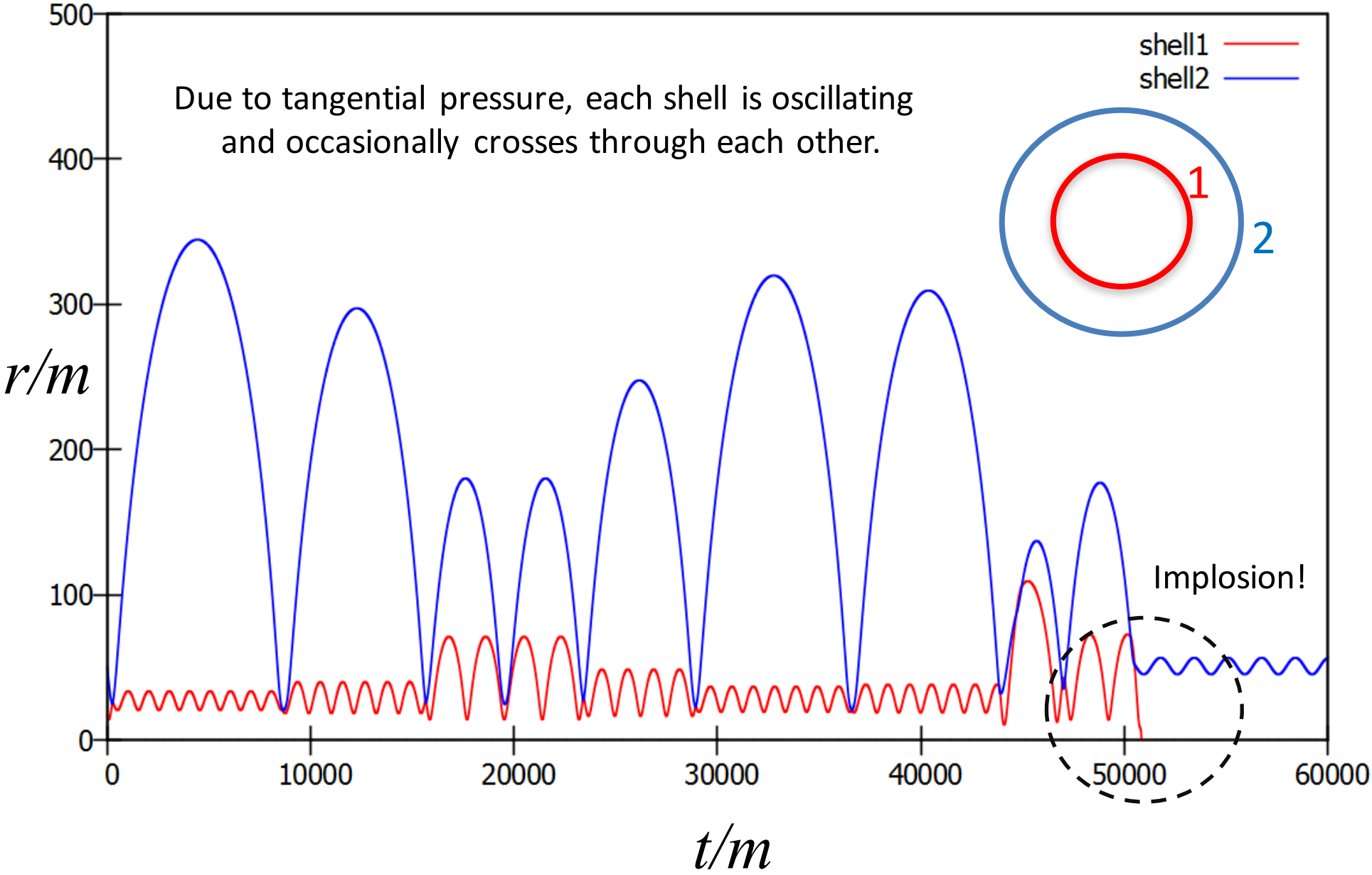
Not gravitational collapse,
but gravitationally unbound inward motion of very large kinetic energy.

Do implosions occur in astrophysical or cosmological situations?

Explosions are usual.

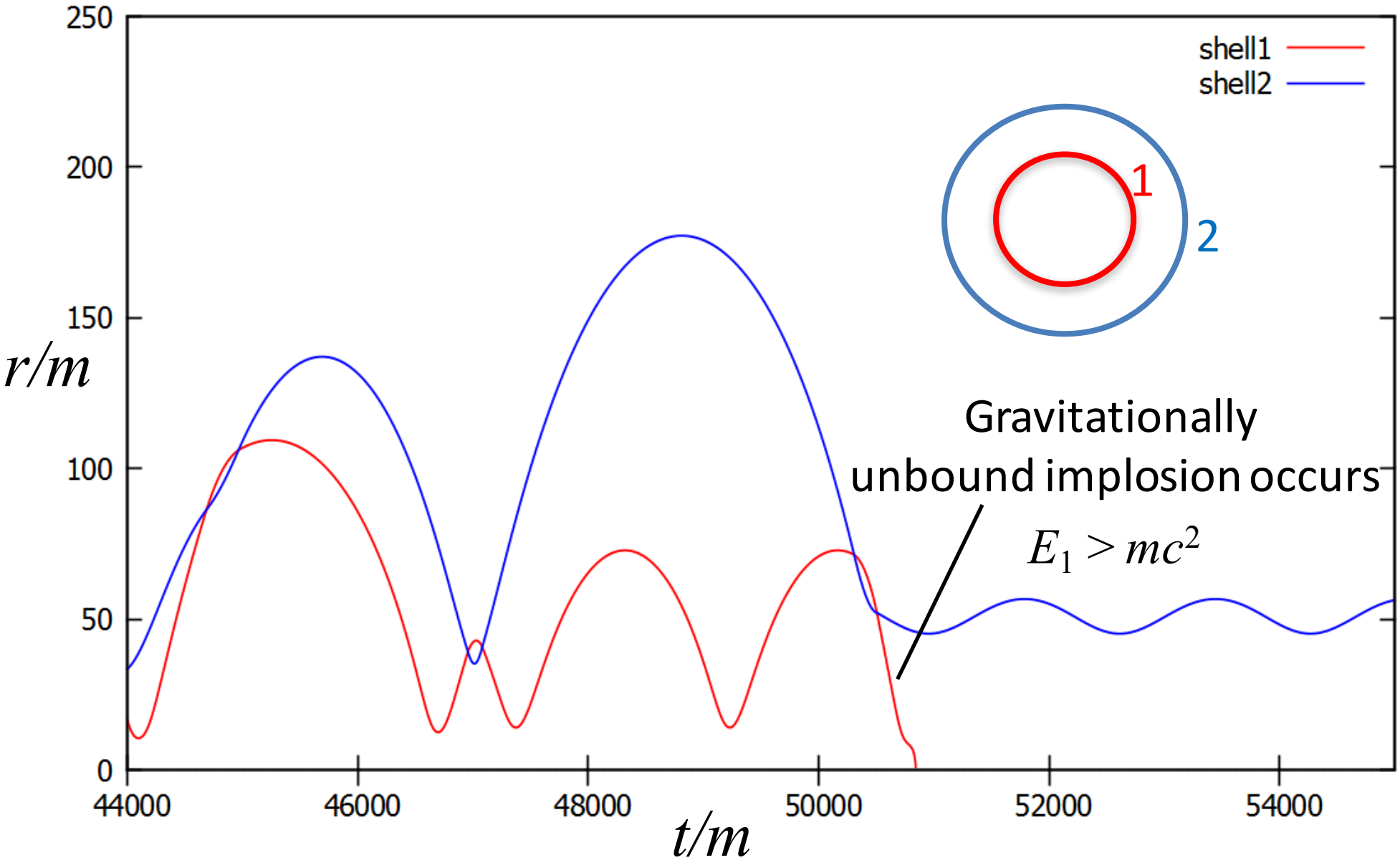
Two concentric spherical shells composed of collision-less particles

R. Yonekura and KN (2014)



V. Cardoso, J. V. Rocha (2016) from different point of view

CLOSEUP


shell1
shell2
1
2
Gravitationally
unbound implosion occurs
$E_1 > mc^2$
r/m
t/m
250
200
150
100
50
0
44000
46000
48000
50000
52000
54000

At least, very compact over-spinning object ($R < M$) can transiently exist.

High efficiency collisional Penrose process may occur around it.

We may find superspinars through ultra-high-enery cosmic ray.

Maybe some implosions process are necessary.

Not gravitational collapse,
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Implosions occur in astrophysical or cosmological situations?

Explosions are usual.

There is the possibility.