BOUNDARY ASYMPTOTICS OF THE RELATIVE BERGMAN KERNEL METRIC FOR HYPERELLIPTIC CURVES AND JACOBIANS

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The Bergman kernel on each complex manifold is a canonical volume-form determined by the complex structure, and we study the variation (in particular its asymptotic behaviors) of the Bergman kernel at degeneration. For a holomorphic family of hyperelliptic curves and their Jacobians, we estimate asymptotic behaviors of the horizontal curvature forms of the relative Bergman kernel metrics near the degenerate boundaries with nodes or cusps. Specifically, the curvature form tends near a node to an incomplete metric on the parameter space, but tends near a cusp to 0. These results are different from the elliptic curve case where hyperbolic growth exists, and the type of singularities surely determines various boundary asymptotics. For the genus-two case particularly, asymptotic formulas with precise coefficients involving the complex structure information are written down explicitly.

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