X Forum of Partial Differential Equations

Będlewo, 19–24 June 2016

Book of Abstracts

X Forum of Partial Differential Equations

Będlewo, 19–24 June 2016

Book of Abstracts

PARTICIPANTS LIST

Tomasz Adamowicz (Institute of Mathematics of the Polish Academy of Sciences)
Włodzimierz Bąk (Uniwersytet Opolski)
Bartosz Bieganowski (Nicolaus Copernicus University)
Piotr Michał Bies (Politechnika Warszawska)
Piotr Biler (Instytut Matematyczny Uniwersytetu Wrocławskiego)
Alexander Bobylev (Keldysh Institute of Applied Mathematics)
Mariusz Bodzioch (University of Warmia and Mazury in Olsztyn)
Bogusław Bożek (Akademia Górniczo-Hutnicza im. St. Staszica)
Zdzislaw Brzezniak (University of York)
Jan Burczak (IMPAN)
Rafał Celiński (Instytut Matematyczny UWr)
Krzysztof Chełmiński (MiNI, Politechnika Warszawska)
Jan Cholewa (Uniwersytet Śląski)
David Cruz-Uribe (University of Alabama)
Antoni Leon Dawidowicz (Jagiellonian University)
Łukasz Dawidowski (Uniwersytet Śląski w Katowicach)
Jacek Gałęski (University of Warsaw)
Michał Gaczkowski (Warsaw University of Technology)
David Gérard-Varet (University Paris Diderot)
Viktor Gerasimenko (Institute of mathematics of NAS of Ukraine)
Paweł Goldstein (University of Warsaw)
Jan Goncerzewicz (Wrocław University of Science and Technology)
Piotr Gwiazda (Instytut Matematyczny PAN/ MIMUW)
Martina Hofmanova (Technical University Berlin)
Tadeusz Iwaniec (Syracuse University)
Piotr Kalita (Jagiellonian University)
Agnieszka Kałamajska (University of Warsaw, Institute of Mathematics)
Grzegorz Karch (Uniwersytet Wrocławski)
Adrian Karpowicz (University of Gdańsk)
Antoni Kijowski (Politechnika Warszawska/Warsaw University of Technology)
Konrad Kisiel (Warsaw University of Technology)
Filip Klawe (Institute of Applied Mathematics, University of Heidelberg)
Piotr Knosalla (Uniwersytet Opolski)
Piotr Kokocki (Nicolaus Copernicus University)
Sławomir Kolasiński (Max Planck Institute for Gravitational Physics)
Marta Kornafel (Cracow University of Economics)
Igor Kossowski (Institute of Mathematics, Lodz University of Technology)
Tomasz Kostrzewa (Warsaw University of Technology)
Karolina Kropielnicka (University of Gdańsk)
Andrzej Krzywicki (Uniwersytet Wrocławski)
Mirosław Lachowicz (University of Warsaw)
Henryk Leszczyński (University of Gdańsk)

Michał Łasica (University of Warsaw)
Grzegorz Łukaszewicz (University of Warsaw)
Katarzyna Mazowiecka (University of Warsaw)
Jarosław Mederski (Nicolaus Copernicus University)
Piotr Minakowski (MIMUW / IWR Heidelberg University)
Piotr Miszczak (MIMUW)
Michał Miśkiewicz (Uniwersytet Warszawski)
Elżbieta Motyl (Uniwersytet Łódzki)
Piotr Mucha (Uniwersytet Warszawski)
Tadeusz Nadzieja (Uniwersytet Opolski)
Andrzej Nowakowski (University of Lodz)
Anna Ochal (Jagiellonian University in Krakow)
Aleksandra Orpel (Faculty of Mathematics and Computer Science,University of Lodz)
Sebastian Owczarek (Warsaw University of Technology)
Andrzej Palczewski (Department of Mathematics, University of Warsaw)
Martyna Patera (Nicolaus Copernicus University)
Zbigniew Peradzyński (University of Warsaw)
Szymon Peszat (UJ Kraków i IMPAN)
Jan Peszek (University of Warsaw)
Tomasz Piasecki (Institute of Mathematics of the Polish Academy of Sciences)
Dominika Pilarczyk (Uniwersytet Wrocławski, Instytut Matematyczny)
Antoni Pierzchalski (Faculty of Matematics and Computer Science, Lodz University)
Łukasz Płociniczak (Politechnika Wrocławska)
Bogdan Przeradzki (Instytut Matematyki Politechniki Łódzkiej)
Andrzej Raczyński (Instytut Matematyczny, Uniwersytet Wrocławski)
Andrzej Rozkosz (Uniwersytet Mikołaja Kopernika)
Ryszard Rudnicki (Institute of Matheamtics PAS)
Lucjan Sapa (AGH, Faculty of Applied Mathematics)
Jakub Siemianowski (Uniwersytet Mikołaja Kopernika w Toruniu)
Mikołaj Sierżęga (Uniwersytet Warszawski)
Iwona Skrzypczak (Uniwersytet Warszawski)
Robert Stańczy (Uniwersytet Wrocławski)
Paweł Strzelecki (Instytut Matematyki UW)
Katarzyna Szymańska-Dębowska (Instytut Matematyki Politechnika Łódz)
Agnieszka Świerczewska-Gwiazda (University of Warsaw)
Olli Toivanen (Institute of Mathematics of the Polish Academy of Sciences)
Krzysztof Topolski (University of Gdansk)
Giuseppe Toscani (University of Pavia)
Andrea Tosin (Politecnico di Torino)
Plamen Turkedjiev (King's College London)
Damian Wiśniewski (Uniwersytet Warmińsko - Mazurski w Olsztynie)
Aneta Wróblewska-Kamińska (Institute of Mathematics, Polish Academy of Sciences)
Dariusz Wrzosek (University of Warsaw)
Jerzy Zabczyk (Instytut Matematyczny Polskiej Akademii Nauk)

Wojciech Zajączkowski (Instytut Matematyczny Polskiej Akademii Nauk)
Ewelina Zatorska (Imperial College London)
Anna Zatorska-Goldstein (University of Warsaw)
Piotr Zgliczyński (Jagiellonian University)

PLAN OF THE CONFERENCE

Monday 20.06.2016

KINETIC THEORY (Room C)

9:00 - 9:10	OPENING OF THE CONFERENCE
9:10 - 10:00	Alexander V. Bobylev *Maxwellian bounds for solutions of the spatially homogeneous Boltzmann equation*
10:00 - 10:50	Giuseppe Toscani *Kinetic theory of wealth distribution*
10:50 - 11:20	Coffee break
11:20 - 12:10	David Gérard-Varet *The mean field Kuramoto model*
12:10 - 13:00	Piotr Gwiazda *Concentrated polymers*
13:00	Lunch
15:00 - 15:50	Grzegorz Karch *Eternal and infinite energy solutions of homogeneous Boltzmann equitation*
15:50 - 16:30	Agnieszka Świerczewska-Gwiazda *Trailer of the Simons Semester*
16:30 - 17:00	Coffee break
19:00	Banquet

Tuesday 21.06.2016

KINETIC THEORY (Room C)

9:00 - 9:50	Andrea Tosin *A Boltzmann-type kinetic approach to the modelling of vehicular traffic*
9:50 - 10:40	Mirosław Lachowicz *Multiscale descriptions of swarming phenomena*
10:40 - 11:10	Coffee break

MATHEMATICAL FLUID MECHANICS (Room C)

11:10 - 12:00	Piotr Mucha *Transport in fluid mechanics*
12:00 - 12:30	Piotr Zgliczyński *Symbolic dynamics (chaos) for Kuramoto-Sivashinsky PDE on the line - a computer assisted proof*
12:30 - 13:00	Grzegorz Łukaszewicz *Bounds on the vertical heat transfer for the Rayleigh–Bénard Convection in the micropolar fluid*
13:00	Lunch
14:30 - 15:00	Ewelina Zatorska *Incompressible congestions modelled by the compressible Navier-Stokes equations*
15:00 - 15:30	Piotr Minakowski *The Taylor-Galerkin method and its applications*
15:30 - 16:00	Jan Burczak *Existence and optimal regularity of solutions to a quasilinear Stokes system with a very weak forcing*
16:00 - 16:30	Coffee break
16:30 - 17:00	Aneta Wróblewska-Kamińska *Non-Newtonian flow over a rough surface*
17:00 - 17:30	Tomasz Piasecki *On the stationary flow of reactive gaseous mixture*
17:30 - 18:00	Piotr Kalita *Smooth attractors for weak solutions of the SQG equation with critical dissipation*

CHEMOTAXIS AND OTHER NONLOCAL PROBLEMS (Room A)

11:10 - 12:00	Piotr Biler *Radial solutions of chemotaxis systems with nonlocal diffusion*
12:00 - 12:30	Dariusz Wrzosek *Interspecies competition and chemorepulsion*
12:30 - 13:00	Ryszard Rudnicki *Phenotype-structured population models. Does assortative mating lead to speciation?*
13:00	Lunch
14:30 - 15:00	Karolina Kropielnicka *Structured population models in a space of measures; From analytical foundations to numerical results*
15:00 - 15:30	Lucjan Sapa *On local weak solutions to Nernst–Planck–Poisson system*
15:30 - 16:00	Rafał Celiński *Asymptotic profile to the chemotaxis model with chemoatractant consumption*
16:00 - 16:30	Coffee break
16:30 - 17:00	Krzysztof Topolski *Sobolev type equations: analytical and numerical approach*
17:00 - 17:30	Robert Stańczy *Evolution for diffusing particles in gravitation*
19:00	Dinner

Wednesday 22.06.2016

STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS (Room C)

9:00 - 9:50	Szymon Peszat *Stochastic Partial Differential Equations - a short overview*
9:50 - 10:20	Jerzy Zabczyk *On Musiela equation for the forward rates*
10:20 - 10:50	Elżbieta Motyl *Stochastic partial differential equations in hydrodynamics*
10:50 - 11:20	Coffee break
11:20 - 11:50	Zdzislaw Brzezniak *On the (deterministic and stochastic) Navier-Stokes equations with constrained L^2 energy of the solution*
11:50 - 12:20	Martina Hofmanova *Stochastic mean curvature flow*
12:20 - 12:50	Plamen Turkedjiev *Representation of and numerics for SPDEs using backward doubly stochastic differential equations*

GEOMETRIC ANALYSIS AND RELATED PROBLEMS (Room A)

9:00 - 9:50	Tadeusz Iwaniec *The Principle of Non-Interpenetration of Matter*
9:50 - 10:20	Michał Miśkiewicz *Weak compactness for systems of n-harmonic type*
10:20 - 10:50	Anna Zatorska-Goldstein *Elliptic problems with critical growth, the Hardy inequality and the existence for parabolic problems*
10:50 - 11:20	Coffee break
11:20 - 11:50	Agnieszka Kałamajska *Strongly nonlinear multiplicative inequalities and Boyed indices*
11:50 - 12:20	Aleksandra Orpel *On positive evanescent solutions of a class of singular elliptic problems*
12:20 - 12:50	Michał Łasica *The l1-anisotropic total variation flow in the plane*

13:00	Lunch
20:00	Bonfire

Thursday 23.06.2016

GEOMETRIC ANALYSIS AND RELATED PROBLEMS (Room C)

9:00 - 9:50	David Cruz-Uribe *H=W in matrix weighted spaces, with applications to mappings of finite distortion*
9:50 - 10:20	Paweł Goldstein *Continuity of finite distortion Sobolev mappings between manifolds*
10:20 - 10:50	Sławomir Kolasiński *New solutions to a generalized Plateau problem*
10:50 - 11:20	Coffee break
11:20 - 11:50	Antoni Pierzchalski *On the reciprocity formula for two and more moduli*
11:50 - 12:20	Jarosław Mederski *Ground states and bound states of semilinear Maxwell equations*
12:20 - 12:50	Iwona Skrzypczak *Liouville theorems for elliptic problems in variable exponent spaces*
13:00	Lunch
14:30 - 15:00	Olli Toivanen *Harnack's inequality for quasiminimizers with generalized Orlicz growth conditions*
15:00 - 15:30	Piotr M. Bies *Linear elliptic equations in variable Hölder spaces*
15:30 - 16:00	Michał Gaczkowski *Variable Sobolev spaces on complete Riemannian manifolds*
16:00 - 16:30	Coffee break
16:30 - 17:00	Tomasz Kostrzewa *Sobolev spaces on metric groups*

CONTRIBUTED TALKS (Room A)

11:20 - 11:50	Viktor Gerasimenko *On approaches to derivation of kinetic equations for hard sphere fluids*
11:50 -12:20	Jan Peszek *Dynamics of particles with nonlocal singular interactions*
12:20 - 12:50	Katarzyna Szymańska-Dębowska *Second order ordinary differential systems with nonlocal Neumann conditions at resonance*
13:00	Lunch
14:30 - 15:00	Andrzej Rozkosz *Renormalized solutions of semilinear elliptic equations involving measure data and Dirichlet operator*
15:00 - 15:30	Sebastian Owczarek *Renormalised solutions in thermo-visco-plasticity for a Norton-Hoff type model*
15:30 - 16:00	Konrad Kisiel *Dynamical poroplasticity model with gradient type nonlinearity*
16:00 - 16:30	Coffee break

17:00 - 18:30 POSTER SESSION (Room A)

Bartosz Bieganowski	*Ground-state solutions for the semilinear Schrodinger equation with sign-changing nonlinearities*
Antoni Kijowski	*Various types of harmonic functions on˜metric˜measure˜space*
Piotr Knosalla	*Aerotaxis equations*
Igor Kosowski	*Existence of solutions for boundary value problem with strong nonlinear nonlocal conditions*
Martyna Patera	*The boundary Harnack principle*
Lucjan Sapa	*Existence and uniqueness of global weak solutions to interdiffusion with Vegard rule*
Jakub Siemianowski	*Thermomicropolar fluid*

19:00	Dinner

Friday 24.06.2016

CONTRIBUTED TALKS (Room C)

9:00 - 9:30	Jan Cholewa *On a nonlinear evolution equation in scales*
9:30 - 10:00	Filip Klawe *Weak-mild solution to microscopic simplified Multiple Myeloma model*
10:00 - 10:30	Coffee break
10:30 - 11:00	Andrzej Nowakowski *Optimal blowup time, approximate blowup time, sufficient condition, dual dynamic programming*
11:00 - 11:30	Jan Goncerzewicz *Porous media equation in tubular domains: large time behaviour of solutions*

CONTRIBUTED TALKS (Room A)

9:00 - 9:30	Antoni Leon Dawidowicz *On the properties of multidimensional Lasota equation in Orlicz spaces*
9:30 - 10:00	Adrian Karpowicz *The Maximum Principle for Viscosity Solutions of Elliptic Differential Functional Equations*
10:00 - 10:30	Coffee break
10:30 - 11:00	Łukasz Dawidowski *Solvability of the quasilinear parabolic equation of Kirchhoff type*
11:00 - 11:30	Anna Ochal *Variational-hemivariational inequalities in mathematical modeling*

11:35 - 11:45	CLOSING OF THE CONFERENCE (Room C)
12:00	Lunch

Abstracts

Ground-state solutions for the semilinear Schrödinger equation with sign-changing nonlinearities

Bartosz Bieganowski

Nicolaus Copernicus University,
ul. Chopina 12/18, 87-100 Torun, Poland,
bartoszb@mat.umk.pl

We are concerned with the following Schrödinger equation

$$-\Delta u + V(x)u = f(x,u) - \Gamma(x)|u|^{q-2}u, \quad x \in \mathbb{R}^N$$

where f, Γ are periodic in $x \in \mathbb{R}^N$, $\Gamma(x) \geqslant 0$ and 0 lies below the spectrum of the Schrödinger operator $-\Delta + V(x)$, $V \in L^\infty(\mathbb{R}^N)$. Observe that the right side $f(x,u) - \Gamma(x)|u|^{q-2}u$ is sign-changing and does not satisfy the monotonicity condition. The problem appears in nonlinear optics, where gap solitons in photonic crystals are studied and potential V is of the form

$$V(x) = V_{per}(x) + V_{loc}(x), \quad x \in \mathbb{R}^N.$$

We assume that V_{per} periodic, but V_{loc} is a localized potential that vanishes at infinity and is responsible for the linear defect in a photonic crystal. We impose some general assumptions on f and we find a ground-state solution on the Nehari manifold $\mathcal{N}$ in the subcritical case.

References

[1] B. Bieganowski, J. Mederski: *Nonlinear Schrödinger equations with sum of periodic and vanishig potentials and sign-changning nonlinearities*, arXiv:1602.05078

Linear elliptic equations in variable Hölder spaces

Piotr Michał Bies

Department of Mathematics and Information Sciences,
Warsaw University of Technology,
Ul. Koszykowa 75, 00-662 Warsaw, Poland
biesp@mini.pw.edu.pl

We study partial differential equations of second order with the right side in variable Hölder space. We show that solutions of this problem are in variable Hölder space and that they are uniqueness. We are doing it by proving Schauder estimates in such spaces. Moreover, in the talk we want to say something about Cordes-Nirenberg theory for spaces with variable exponent. This theory is about regularity of weak solutions of elliptic equations. Morrey and Campanato Theorems for variable Hölder spaces are important in this researches. These Theorems characterize Hölder continuity by some integral conditions. We show that weak solutions of elliptic equations are Hölder spaces with variable exponent by these Theorems.
The talk is based on results obtained together with P. Górka.

Radial solutions of chemotaxis systems with nonlocal diffusion

Piotr Biler

Instytut Matematyczny, Uniwersytet Wrocławski,
pl. Grunwaldzki 2/4, 50-384 Wrocław, Poland
Piotr.Biler@math.uni.wroc.pl

We study radially symmetric solutions of the Keller-Segel model with the diffusion described by fractional powers of the Laplacian in d dimensions

$$u_t + (-\Delta)^{\alpha/2} u + \nabla \cdot (u \nabla v) = 0,$$

$$\Delta v + u = 0, \quad x \in \mathbb{R}^d, \; t > 0.$$

In particular, global in time solutions are constructed, and a finite time blowup is shown for solutions with “big” initial data. Criteria for this dichotomic behavior of solutions are expressed in terms of the norms of suitable Morrey spaces.

References

[1] P. Biler, T. Cieślak, G. Karch, J. Zienkiewicz, *Local criteria for blowup of solutions in two-dimensional chemotaxis models*, Disc. Cont. Dynam. Syst. A, to appear.
[2] P. Biler, G. Karch, J. Zienkiewicz, *Optimal criteria for blowup of radial and N-symmetric solutions of chemotaxis systems*, Nonlinearity **28** (2015), 4369–4387.
[3] P. Biler, G. Karch, J. Zienkiewicz, in preparation.

Maxwellian bounds for solutions of the spatially homogeneous Boltzmann equation

Alexander V. Bobylev

Keldysh Institute of Applied Mathematics, Russian Academy of Sciences, Moscow,
alexander.bobylev@kau.se

The talk is based on a joint paper with Irene Gamba. We consider the spatially homogeneous Boltzmann equation and assume that the initial distribution function is bounded by a Maxwellian. A natural conjecture is that the corresponding solution is also bounded uniformly in time by another Maxwellian with constant parameters. The conjecture was considered earlier by several authors and finally it was proved for hard spheres and hard potentials with cut-off. The proof, however, does not work for pseudo-Maxwell molecules. We discuss related questions in the talk and present another way of proof, which can be applied to the Maxwell case. Various aspects of the so-called "comparison principle" for the Boltzmann equations are also explained in the talk.

On the (deterministic and stochastic) Navier-Stokes equations with constrained L^2 energy of the solution

Zdzislaw Brzezniak

Department of Mathematics, University of York,
Heslington, York YO10 5DD, United Kingdom,
zdzislaw.brzezniak@york.ac.uk

We study deterministic and stochastic Navier-Stokes equations with a constraint on L^2 energy of the solution. We prove the existence and uniqueness of local strong solutions and the existence of a global solutions for the constrained 2D Navier-Stokes equations on the torus on the whole Euclidean space. This is based on joint works with Gaurav Dhariwal (York) and Mauro Mariani (Roma I).

Existence and optimal regularity of solutions to a quasilinear Stokes system with a very weak forcing

Jan Burczak

Institute of Mathematics, Polish Academy of Sciences,
Warsaw, Poland,
jb@impan.pl

Miroslav Bulíček, Sebastian Schwarzacher

Charles University,
Prague, Czech Republic

In contrast to linear systems, providing a full-range L^q Calderón-Zygmund-type theory in quasilinear case is difficult for two main reasons: For 'high' q's, due to lack of smoothing property of a homogenous problem in a general case (i.e. lack of a Uhlenbeck-type structure). For 'low' q's — due to lack of existence theory below the duality exponent. Focusing on the latter case, this problem has been recently resolved by Bulíček, Diening & Schwarzacher for a quasilinear, quadratic system that can be seen as an intermediary step between the Laplacian and the p-Laplacian. In my talk, I will present an analogous theory for the following stationary quasilinear Stokes system (that does not posses the Uhlenbeck structure)

$$\begin{aligned} -\mathrm{div}(\mathcal{A}(x,\varepsilon(u))) + \nabla p = -\mathrm{div} f \qquad & \text{in } \Omega, \\ \mathrm{div} u = 0 \qquad & \text{on } \partial\Omega \\ u = 0 \qquad & \text{on } \partial\Omega. \end{aligned}$$

Our assumptions include, for instance, a Carreau fluid. This is a joint work with M. Bulíček and S. Schwarzacher.

Asymptotic profile to the chemotaxis model with chemoatractant consumption

Rafał Celiński

Mathematical Insitute, University of Wrocław,
pl. Grunwaldzki 2/4, 50-384 Wrocław,
Rafal.Celinski@math.uni.wroc.pl

We consider the following initial-boundary value problem

$$n_t = \Delta n - \nabla \cdot (n \nabla c) \quad \text{with} \quad x \in \mathbb{R}^2,\ t > 0, \tag{1}$$

$$c_t = \Delta c - nc, \tag{2}$$

supplemented with nonnegative initial conditions

$$n(x,0) = n_0(x),\ c(x,0) = c_0(x). \tag{3}$$

It was recently proved by Zhang and Zheng [1] that system (1)–(3) has a global-in-time solutions. The aim of my talk is to deliver the asymptotic profile as $t \to \infty$ of those solutions without any smallness assumption on initial data. Moreover, I will try to point out the main difficulty in extending this result to the more general initial value problem called chemotaxis-Navier-Stokes model.

References

[1] Q. Zhang, X. Zheng, *Global well-posedness for the two-dimensional incompressible chemotaxis-Navier-Stokes equations*, SIAM J. Math. Anal. **46** (2014), pp. 3078–3105.

On a nonlinear evolution equation in scales

Jan Cholewa

Institute of Mathematics
Silesian University
40-007 Katowice
jan.cholewa@us.edu.pl

Following [1] an abstract integral equation is analyzed in a scale of Banach spaces and a range of spaces is exhibited in which a problem can be locally be well posed and in which the solution smooths. If the solution ceases to exist in a finite time, an estimate of a blow up rate is also obtained.
The approach is applicable, e.g., to parabolic equations in Lebesgue's spaces [1,2], strongly damped wave equations [1,3], fractional Navier-Stokes equations [4].

References

[1] J. W. Cholewa, C. Quesada, A. Rodriguez-Bernal, Nonlinear evolution equations in scales of Banach spaces and applications to PDEs, preprint.

[2] H. Brezis, T. Cazenave, A nonlinear heat equation with singular initial data, J. Anal. Math., 68, 277-304, 1996.

[3] A. N. Carvalho, J. W. Cholewa, Local well posedness for strongly damped wave equations with critical nonlinearities, Bull. Austral. Math. Soc., 66, 443-463, 2002.

[4] J. W. Cholewa, Tomasz Dlotko, Fractional Navier-Stokes Equations, preprint.

$H = W$ in matrix weighted spaces, with applications to mappings of finite distortion

David Cruz-Uribe, OFS

University of Alabama,
Tuscaloosa, AL, 35401, USA
dcruzuribe@ua.edu

We consider degenerate Sobolev spaces where the degeneracy is controlled by a matrix A_p weight, a weight class introduced by Nazarov, Treil and Volberg. We prove that the classical Meyers-Serrin theorem $H = W$ holds in this setting. The proof requires extending the classical machinery of scalar A_p weights to matrix weights; even in the scalar case our approach has found other applications. We apply our results to study partial regularity of degenerate p-Laplacian equations. As an application we can prove partial regularity results for mappings of finite distortion such that $f \in W^{1,p}$, $n - 1 \leqslant p < n$. We construct an example to show that our results are the best possible.

On the properties of multidimensional Lasota equation in Orlicz spaces

Antoni Leon Dawidowicz

Jagiellonian University, Faculty of Mathematics and Computer Science,
ul. Łojasiewicza 6, 30-348 Kraków, Poland
Antoni.Leon.Dawidowicz@im.uj.edu.pl

Anna Poskrobko

Bialystok University of Technology, Faculty of Computer Science,
ul. Wiejska 45A, 15-351 Białystok, Poland,
a.poskrobko@pb.edu.pl

In the papers of Brzeźniak, Haribash and the authors the properties of asymptotic behavior of the equation

$$\frac{\partial u}{\partial t} + x\frac{\partial u}{\partial x} = \lambda u \tag{4}$$

are considered. The dynamical system given by (4) can be chaotic in the sense of Devaney or asymptotically stable. The properties depend on functional space on which the dynamical system is considered. We generalize the obtained results studying multidimensional version of equation (4), i.e.

$$\frac{\partial u}{\partial t} + \sum_{i=1}^{n} c_i(x)\frac{\partial u}{\partial x_i} = \lambda u. \tag{5}$$

We consider its asymptotic properties in Orlicz space L^φ generated by so-called φ-function $\varphi : \mathbb{R} \to \mathbb{R}$. For φ-function, we can define the lower and upper Matuszewska-Orlicz indices $\underline{p}$ and $\overline{q}$ by the formulas

$$\underline{p} = \sup\{p : \text{ for some } C > 0 \text{ we have } \varphi(at) \geqslant Ca^p\varphi(t) \text{ for } 0 \leqslant t < \infty \text{ and } a > 1\},$$

$$\overline{q} = \inf\{q : \text{ for some } C < \infty \text{ we have } \varphi(at) \leqslant Ca^q\varphi(t) \text{ for } 0 \leqslant t < \infty \text{ and } a > 1\}.$$

We prove the asymptotic behavior of (4) and (5) in the space L^φ with dependence on Matuszewska - Orlicz indices.

References

[1] Antoni Leon Dawidowicz, Anna Poskrobko, On chaotic and stable behaviour of the von Foerster-Lasota equation in some Orlicz spaces, Proc. Estonian Acad. Sci. Phys. Math. (2008), 61–69.

[2] Antoni Leon Dawidowicz, Anna Poskrobko, Asymptotic properties of the von Foerster-Lasota equation and indices of Orlicz space (submitted to Differential and Integral Equations).

Solvability of the quasilinear parabolic equation of Kirchhoff type

Łukasz Dawidowski

University of Silesia,
ul. Bankowa 14, 40-007 Katowice,
lukasz.dawidowski@us.edu.pl

Consider the Dirichlet problem for quasilinear generalized degenerate Kirchhoff equation

$$u_t - (1 + \|\nabla u\|^2_{L^2(\Omega)})\Delta u + g(u,x) = 0 \tag{6}$$

with initial condition

$$u(0,x) = u_0(x), x \in \Omega,$$

and boundary condition of the Dirichlet type

$$u|_{\partial\Omega} = 0,$$

We will assume that $u_0 \in H^2(\Omega)$ and $\Omega \subseteq \mathbb{R}^N$ is a domain of the class C^2.
The existence of solution of problem (6) under some assumptions will be studied using the Leray – Schauder principle.

References

[1] Ł. Dawidowski, *The quasilinear parabolic Kirchhoff equation*, submitted.

[2] M. Gobbino, *Quasilinear Degenerate Parabolic Equations of Kirchhoff Type*, Math. Meth. Appl. Sci., 22, 375-388 (1999).

[3] M. Nakao, *An attractor for a nonlinear dissipative wave equation of Kirchhoff type*, J. Math. Anal. Appl. 353 (2009) 652-659.

Variable Sobolev spaces on complete Riemannian manifolds

Michał Gaczkowski

Warsaw University of Technology,
ul. Koszykowa 75, 00-662 Warsaw, Poland,
M.Gaczkowski@mini.pw.edu.pl

In this talk we are going to introduce variable Sobolev space on Riemannian manifolds. Continuous and compact embedding will be discussed in the case of complete manifold. For non compact manifolds, compact embedding will require a space of functions invariant under the action of some group. As an application we will study the PDE problem

$$-\Delta_{q(x)}u(x) + |u(x)|^{q(x)-2}u(x) = f(x, u(x)).$$

The talk is based on results obtained together with P.Górka and Daniel Pons.

References

[1] M. Gaczkowski, P. Górka, *Variable Hajłasz-Sobolev spaces on compact metric spaces*, Math. Slov., to appear.

[2] M. Gaczkowski, P. Górka, D. Pons, *Sobolev spaces with variable exponents on complete manifolds*, J. Func. Anal., **270** (2016), 1379-1415.

The mean field Kuramoto model

David Gerard-Varet

Université Paris Diderot,
Département de Mathématiques, 5 rue Thomas Mann, 75013 Paris
david.gerard-varet@imj-prg.fr

The Kuramoto equation is a popular mean-field model that describes a large population of coupled oscillators. Its popularity is due to the fact that it exhibits synchronization behaviour. Mathematically, it corresponds to the convergence of the phase distribution of the oscillator towards a Dirac mass, as time goes to infinity. Although identified for long, this convergence has remained widely unjustified from the mathematical point of view. We shall report on recent progress on this question, discussing similarities and differences with Landau damping for Vlasov equations. This is joint work with H. Dietert and B. Fernandez.

On approaches to derivation of kinetic equations for hard sphere fluids

V.I. Gerasimenko

Institute of Mathematics of NAS of Ukraine,
3, Tereshchenkivs'ka Str., Kyiv, Ukraine,
gerasym@imath.kiev.ua

In the talk we consider a new approach to the problem of the rigorous description of kinetic evolution of large hard sphere systems.

For this purpose we establish the Boltzmann–Grad asymptotic behavior of a solution of the Cauchy problem of the dual BBGKY hierarchy for marginal observables of hard spheres. The constructed scaling asymptotics is governed by the set of recurrence evolution equations, namely, by the dual Boltzmann hierarchy with hard sphere collisions. For initial states specified in terms of a one-particle distribution function we prove that the mean value functional for the constructed limit of additive-type marginal observables is equivalent to the mean value functional determined by a one-particle distribution function governed by the Boltzmann kinetic equation and the evolution of nonadditive-type marginal observables is equivalent to the property of the propagation of initial chaos for states.

One of the advantages of this approach to the derivation of kinetic equations from underlying hard sphere dynamics consists in an opportunity to construct the Boltzmann-like kinetic equation with initial correlations and it gives to describe the propagation of initial correlations in the Boltzmann–Grad scaling limit.

Moreover, using suggested approach, we also derive the non-Markovian generalization of the Enskog kinetic equation and construct the marginal functionals of states, describing the creation of all possible correlations of particles with hard sphere collisions in terms of a one-particle distribution function. The Boltzmann–Grad asymptotic behavior of a non-perturbative solution of the stated Enskog equation and the marginal functionals of states are established.

The obtained results we extend on systems of hard spheres with inelastic collisions. In particular, we established that in a one-dimensional space the kinetic evolution of a system of hard rods with inelastic collisions is governed by the certain generalization of the known Boltzmann equation for a one-dimensional granular gases.

References

[1] V.I. Gerasimenko, *On the approaches to the derivation of the Boltzmann equation with hard sphere collisions.* Proc. Inst. Math. NASU, v.10, No.2, pp. 71–95, 2013.

Continuity of finite distortion Sobolev mappings between manifolds.

Paweł Goldstein

We prove that if a $W^{1,n}$ mapping between two closed, n-dimensional manifolds has finite distortion (e.g. if its Jacobian determinant is positive a.e.), then it is continuous. This result is well known in the Euclidean setting, and in that case carries over through essentially the same arguments to Orlicz-Sobolev mappings sufficiently close to $W^{1,n}$. However, in the case of mappings between manifolds, the argumentation is different – and the result is not true for Orlicz-Sobolev mappings, which we illustrate on a counterexample. This study has been motivated by and has applications to problems in regularity of Sobolev isometric embeddings.
This is joint work with Piotr Hajłasz and M. Reza Pakzad.

Porous media equation in tubular domains: large time behaviour of solutions

Jan Goncerzewicz

Faculty of Pure and Applied Mathematics, Wrocław University of Technology
Wybrzeże Wyspiańskiego 27, 50 - 370 Wrocław, Poland
jan.goncerzewicz@pwr.edu.pl

This is a report of our developments on large-time behaviour of solutions of the porous media equation $\partial_t u = \Delta u^m$, $m > 1$, posed in infinite tubular domains, certain subdomains of these domains, and, their higher dimensional analogues. For the homogeneous Cauchy–Dirichlet problem with initial data that have one-sidedly bounded support it is shown that there is a universal pattern of convergence to a self-similar solution. Moreover, the large-time behaviour of the free boundary in every solution mimics that of the self-similar one. The results complete and improve earlier results of [?] and [?]. This is a joint work with B.H. Gilding [?].

References

[1] Gilding, B. H., Goncerzewicz, J. *The porous media equation in an infinite cylinder, between two infinite parallel plates, and like spatial domains.*, Interfaces Free Bound. 18 (2016), 47-75.
[2] Vázquez, J. L. *Porous medium flow in a tube. Traveling waves and KPP behavior.* Comm. Contemporary Math. 9 (2007), 731–751.
[3] Vázquez, J. L. *The Porous Medium Equation*, Clarendon Press, Oxford 2007.

Concentrated polymers

Piotr Gwiazda

Institute of Mathematics, Polish Academy of Sciences, Poland,
ul. Śniadeckich 8, 00-656 Warszawa,
pgwiazda@mimuw.edu.pl

We will concentrate on a class of mathematical models for polymeric fluids, which involves the coupling of the Navier–Stokes equations for a viscous, incompressible, constant-density fluid with a parabolic-hyperbolic integro-differential equation describing the evolution of the polymer distribution function in the solvent, and a parabolic integro-differential equation for the evolution of the monomer density function in the solvent. The viscosity coefficient, appearing in the balance of linear momentum equation in the Navier–Stokes system, includes dependence on the shear-rate as well as on the weight-averaged polymer chain length. The system of partial differential equations under consideration captures the impact of polymerization and depolymerization effects on the viscosity of the fluid. We discuss the existence of global-in-time, large-data weak solutions under fairly general hypotheses. The talk is based on the common result with M. Bulicek, P. Gwiazda and E. Süli [1] and with Camillo De Lellis [2].

References

[1]M. Bulicek, P. Gwiazda, E. Süli, A. Świerczewska-Gwiazda *Analysis of a viscosity model for concentrated polymers*, arXiv:1501.05766, to appear in Math. Models Meth. Appl. Sci.

[2] C. De Lellis, P. Gwiazda, A. Świerczewska-Gwiazda, *Transport equation with integral terms*, arXiv:1602.03193.

Stochastic mean curvature flow

Martina Hofmanová

Technical University Berlin
Strasse des 17. Juni 136, 10623 Berlin
hofmanov@math.tu-berlin.de

Motion by mean curvature of embedded hypersurfaces in $\mathbb{R}^{N+1}$ is an important prototype of a geometric evolution law and has been intensively studied in the past decades. Mean curvature flow is characterized as a steepest descent evolution for the surface area energy and constitutes a fundamental relaxation dynamics for many problems where the interface size contributes to the systems energy. One of the main difficulties of the mean curvature flow is the appearance of topological changes and singularities in finite time. Further issues then arise in the mathematical treatment of the stochastic mean curvature flow, which was introduced as a refined model incorporating the influence of thermal noise.
We study a stochastically perturbed mean curvature flow for graphs in $\mathbb{R}^3$ over the two-dimensional unit-cube subject to periodic boundary conditions. In particular, we establish the existence of a weak martingale solution. The proof is based on energy methods and therefore presents an alternative to the stochastic viscosity solution approach. To overcome difficulties induced by the degeneracy of the mean curvature operator and the multiplicative gradient noise present in the model we employ a three step approximation scheme together with refined stochastic compactness and martingale identification methods. The talk is based on a joint work with Matthias Röger and Max von Renesse [1].

References

[1] M. Hofmanová, M. Röger, M. von Renesse: Weak solutions for a stochastic mean curvature flow of two-dimensional graphs, arXiv:1412.5863, to appear in Probab. Theory Related Fields.

The Principle of Non-Interpenetration of Matter

Tadeusz Iwaniec

Syracuse University (USA)
tiwaniec@syr.edu

It is axiomatic in the theory of elasticity that the energy-minimal displacement should be a homeomorphism. However, from the mathematical point of view, this is highly oversimplified precondition. One quickly runs into a serious difficulty when passing to the limit of an energy-minimizing sequence of homeomorphisms; injectivity is lost. In search for mathematical models of hyper-elasticity , we must accept and explore the weak limits of the energy- minimizing sequences of Sobolev homeomorphisms. For 2D theory of plates and thin films (surfaces), these are none other than monotone mappings. It is characteristic to a monotone map to squeeze some parts of the elastic body (to a points or an arc) but not to fold the body.

Smooth attractors for weak solutions of the SQG equation with critical dissipation

Piotr Kalita

Jagiellonian University,
Kraków, Poland
piotr.kalita@ii.uj.edu.pl

Michele Coti Zelati

University of Maryland,
College Park, USA,
micotize@umd.edu

We study the two dimensional forced and critically damped surface quasi-geostrophic equation on the torus

$$\begin{aligned} \partial_t\theta + \boldsymbol{u}\cdot\nabla\theta + (-\Delta)^{1/2}\theta = f, \\ \boldsymbol{u} = \mathcal{R}^\perp\theta = \nabla^\perp(-\Delta)^{-1/2}\theta, \\ \theta(0) = \theta_0, \quad \int_{\mathbb{T}^2}\theta_0(x) = 0. \end{aligned}$$

The equation is used in modeling of geophysical flows. If $\theta_0 \in L^2$ and $f \in L^p$ for some $p > 2$ we consider a class of vanishing viscosity weak solutions which are nonunique, cf. [2, 3]. The evolutionary system defined by those solutions has a global attractor in L^2 [3]. On the other hand if $f \in L^\infty \cap H^1$ and $\theta_0 \in H^1$, there exists the unique strong solution. The semiflow governed by the strong solutions has a global attractor in H^1 of finite fractal dimension [4]. We present the results of [1] where we show, using appropriate bootstrapping techniques, that if forcing term is smooth enough, the global attractor in the sense of the multivalued theory coincides with the global attractor for the strong solutions.

References

[1] M. Coti Zelati and P. Kalita, Smooth attractors for weak solutions of the SQG equation with critical dissipation, DCDS-B, to appear.

[2] L. A. Caffarelli and A. Vasseur, Drift diffusion equations with fractional diffusion and the quasi-geostrophic equation, Ann. of Math. (2) 171 (2010), no. 3, 1903–1930.

[3] A. Cheskidov, M. Dai, The existence of a global attractor for the forced critical surface quasi-geostrophic equation in L^2, arXiv:1402:2801, 2014.

[4] P. Constantin, M. Coti Zelati, and V. Vicol, Uniformly attracting limit sets for the critically dissipative SQG equation. Nonlinearity 29 (2016), no. 2, 298–318.

Eternal and infinite energy solutions of homogeneous Boltzmann equitation

Grzegorz Karch

Uniwersytet Wrocławski,
Instytut Matematyczny, pl. Grunwaldzki 2/4, 50-384 Wrocław, Poland,
grzegorz.karch@uwr.edu.pl

I will announce our recent results on the existence of eternal solutions to the homogeneous Boltzmann equation for Maxwellian molecules. Such solutions are obtained in a space of probability measures of infinite energy (*i.e.* infinite second moment). They describe the large time behavior of other infinite energy solutions and appear as well as intermediate time asymptotic states of finite, but arbitrary high, energy solutions.

References

[1] M. Cannone, G. Karch: Infinite energy solutions to the homogeneous Boltzmann equation, *Communications on Pure and Applied Mathematics* **63** (2010), 747–778.

[2] M. Cannone, G. Karch: On self-similar solutions to the homogeneous Boltzmann equation. *Kinetic and Related Models* **6** (2013), no. 4, 801–808.

[3] M. Cannone, G. Karch, E. Ntovoris: Eternal solutions to the homogeneous Boltzmann equation *Work in progress* (2016).

The Maximum Principle for Viscosity Solutions of Elliptic Differential Functional Equations

Adrian Karpowicz

University of Gdańsk,
Wit Stwosz St. 57, 80-952 Gdańsk, Poland,
Adrian.Karpowicz@mat.ug.edu.pl

Let Ω be a bounded and open subset of $\mathbb{R}^n$. For $x \in \overline{\Omega}$, we define

$$A[x] = \left\{ y \in \mathbb{R}^n : x + y \in \overline{\Omega} \right\}.$$

Let $u : \Omega \to \mathbb{R}$ and $x \in \overline{\Omega}$. We define $u_x : A[x] \to \mathbb{R}$ by the formula

$$u_x(y) = z(x+y) \text{ for } y \in A[x].$$

We shall discuss the Maximum Principle for viscosity solutions of the following functional differential elliptic problem:

$$\begin{cases} F\left(x, u(x), u_x, Du(x), D^2u(x)\right) = 0 & \text{in } \Omega \\ u = \phi & \text{on } \partial\Omega. \end{cases}$$

We suppose that function $F : \overline{\Omega} \times \mathbb{R} \times C(E, \mathbb{R}) \times \mathbb{R}^n \times S_{n \times n} \to \mathbb{R}$ of the variables (x, r, q, p, X) is nondecreasing in r and nonincreasing in X.
We prove that if $u \in C(\bar{\Omega}, \mathbb{R})$ (respectively, $v \in C(\bar{\Omega}, \mathbb{R})$) is a subsolution (respectively, supersolution) of $F = 0$ in Ω and $u \leqslant v$ on $\partial\Omega$ then $u \leqslant v$ in Ω.

References

[1] A. Karpowicz, *The Maximum Principle for Viscosity Solutions of Elliptic Differential Functional Equations*, Opuscula Math. 33, no. 1 (2013), 99–105.
[2] A. Karpowicz, *Viscosity Solutions of Elliptic Differential Functional Equations*, to appear.

Strongly nonlinear multiplicative inequalities and Boyed indices

Agnieszka Kałamajska

Warsaw University, Department of Mathematics, Informatics and Computer Science, ul Banacha 2, 02-097 Warsaw, Poland

kalamajs@mimuw.edu.pl

We are interested in the inequality:

$$\int_{(a,b)} |f^{'}(x)|^q h(f(x))dx \leqslant C \int_{(a,b)} \left(\sqrt[p]{|f''(x)\mathcal{T}_{h,p}(f(x))|} \right)^q h(f(x))dx,$$

and its Orlicz variants, where $\mathcal{T}_{h,p}(\cdot)$ is certain transformation of function f with the property $\mathcal{T}_{h\equiv 1,2}(f) = f$. In some restricted variants inequality was earlier obtained by Oppiel and Mazya in the 60ties and 70ties of the last century. I would like to overview the variants of this inequality, applications to regularity for singular elliptic pde's, perspectives for its future applications, as well as its recent development toward inequalities involving nonlocal operators, achieved by exploiting certain invariances of the inequality. The talk will be based on chain of works obtained together with Katarzyna Pietruska-Pałuba, Jan Peszek, Katarzyna Mazowiecka, Tomasz Choczewski, Ignacy Lipka, Alberto Fiorenza and Claudia Capogne.

Various types of harmonic functions on metric measure space

Antoni Kijowski

Warsaw University of Technology,
Ul. Koszykowa 75, 00-662 Warsaw, Poland,
kijowskia@student.mini.pw.edu.pl

We will consider a setting of metric measure space. There are many different approaches to defining p-harmonic functions. One of them is introduced by Gaczkowski and Górka using mean value property. The other way is done by using space of Newtonian functions and the notion of p-weak upper gradient. In this case harmonic functions are minimas of Dirichlet energy. I will compare those two definitions and present their propierties.

Dynamical poroplasticity model with gradient type nonlinearity

Konrad Kisiel

Warsaw University of Technology,
Koszykowa 75, 00–662 Warsaw,
K.Kisiel@mini.pw.edu.pl

We will discuss the existence theory to a model for the poroplastic behaviour of soil. It is given by the following system of equations

$$\begin{aligned} \rho u_{tt}(x,t) - \mathrm{d}iv_x T(x,t) + \alpha \nabla_x p(x,t) &= F(x,t), \\ c_0 p_t(x,t) - c\Delta_x p(x,t) + \alpha\, \mathrm{d}iv_x u_t(x,t) &= f(x,t), \\ \varepsilon\left(u(x,t)\right) &= \frac{1}{2}\left(\nabla_x u(x,t) + \nabla_x^T u(x,t)\right), \\ T(x,t) &= \mathcal{D}\left(\varepsilon(u(x,t)) - \varepsilon^p(x,t)\right), \\ \varepsilon_t^p(x,t) &= A(T(x,t)). \end{aligned} \tag{7}$$

System (7) is equipped with nonhomogeneous initial–boundary conditions.
We are interested in finding the following functions

- the displacement field $u : \Omega \times [0, T_e] \to \mathbb{R}^3$,
- the pore pressure of the fluid $p : \Omega \times [0, T_e] \to \mathbb{R}$,
- the inelastic deformation tensor $\varepsilon^p : \Omega \times [0, T_e] \to \mathcal{S}(3) = \mathbb{R}^{3\times3}_{sym}$,
- the Cauchy stress tensor $T : \Omega \times [0, T_e] \to \mathcal{S}(3)$,

We assume that the constitutive function (right hand side of $(7)_5$) is deviatoric and it is a sum of two maps, where one is the gradient of convex function and second is globally Lipschitz i.e.

$$A(T) = \nabla_T M(T) + l(T) =: G(T) + l(T),$$

where $l : \mathcal{S}(3) \to \mathcal{S}(3)$ is a globally Lipschitz and $M : \mathcal{S}(3) \to \mathbb{R}$ is a differentiable convex function. Symbol ∇_T denotes the gradient operator with respect to $T \in \mathcal{S}(3)$.
Without any additional growth conditions for function $G : \mathcal{S}(3) \to \mathcal{S}(3)$ we were able to prove the existence of a solution such that equations in (7) are satisfied almost everywhere. It is worth mentioning that in a lot of cases authors are able only to show the existence of weak solutions (constitutive equation is satisfied only in the measure valued sense).

References

[1] K. Kisiel, K. Kosiba, *D*ynamical poroplasticity model with mixed boundary conditions – theory for $\mathcal{LM}$–type nonlinearity, J. Math. Anal. Appl., 2016, accepted.
[2] S. Owczarek, *C*onvergence of coercive approximations for a model of gradient type in poroplasticity, Math. Methods Appl. Sci., 32 (12):1541-–1563, 2009.

Weak-mild solution to microscopic simplified Multiple Myeloma model

Filip Z. Klawe

Heidelberg University,
Im Neuenheimer Feld 205, 69120 Heidelberg, Germany

Institute of Applied Mathematics and Mechanics, University of Warsaw,
Banacha 2,02-097 Warsaw, Poland

fzklawe@mimuw.edu.pl

Our research is directed to describe evolution of cancer cells in domain Ω and, furthermore, to describe the changes in shape of this domain. We consider a case of circular geometry, i.e. let $\Omega(0)$ be a circle and changes of its geometry are uniform in all directions. $\Gamma(t)$ is a boundary of $\Omega(t)$ and its evolution is defined by ODE on radius of $\Omega(t)$. Combination of these issues and additional equation which describes concentration of surface component leads to the following system of equations

$$\begin{cases} \frac{\partial u}{\partial t} - \Delta u &= F_u(u) & x \in \Omega(t),\ t \in [0,T], \\ \frac{\partial u}{\partial \vec{n}} + Vu &= F_\gamma(u,B) & x \in \Gamma(t),\ t \in [0,T], \\ \frac{\partial B}{\partial t} + BVH - \Delta_\Gamma B &= F_B(u,B) & x \in \Gamma(t),\ t \in [0,T], \\ \frac{dR(t)}{dt} = V(t) &= g(\overline{B})(R_{\max} - R(t))(R(t) - R_{\min}), & \end{cases} \tag{8}$$

where V is normal velocity of the boundary and H stays for mean curvature of $\Gamma(t)$, that is $H(t) = \frac{1}{R(t)}$. Additionally, we assume that the velocity of boundary depends on radius $R(t)$ and mean value of surface's concentration, i.e. $\overline{B} = \frac{1}{|\Gamma(t)|}\int_{\Gamma(t)} B\,\mathrm{d}S$.
Similar problem was considered in [2]. However, in [2] there was no surface component and surface moves only in one direction. Some additional tips to surface equation may be found in [1].
We present the existence of weak-mild solution to (8). Moreover, we prove the uniqueness of this solution.

References

[1] G. Dziuk and C.M. Elliott. Finite element methods for surface pdes. *Acta Numer.*, 22:289–396, 2013.

[2] A. Fasano and A. Mikelić. The 3d flow of a liquid through a porous medium with absorbing and swelling granules. *Interfaces and Free Boundaries*, 4:329 – 261, 2002.

Aerotaxis equations

Piotr Knosalla

Opole University,
ul. Oleska 48, Opole, Poland
pknosalla@math.uni.opole.pl

A colony of bacteria lives in some bounded region filled with oxygen dissolved in water. The metabolism of the bacteria depends on the concentration of oxygen, which plays the role of both attractant (at moderate concentrations) and repellent (at high and low concentrations). Aerotaxis is the movement of bacteria toward the optimal concentration of oxygen for their growth. Some model describing this biological phenomena will be analyzed.

New solutions to a generalized Plateau problem

Sławomir Kolasiński

Max Planck Institute for Gravitational Physics (Albert Einstein Institute),
Am Mühlenberg 1 D-14476 Potsdam-Golm, Germany,
skola@mimuw.edu.pl

Plateau problem is about finding a surface that spans a given boundary and has the minimal area. Precise formulations depend on the meaning of the words *surface*, *spans*, *boundary*, and *area*. I shall briefly describe the classical formulation of the problem given by Reifenberg and Almgren and also the more recent approaches suggested by David and by Harrison and Pugh. After that, I shall present a modified Almgren's construction which gives a general existence result for an abstract Plateau problem encompassing many different formulations.

This is a joint work with Yangqin Fang (AEI Potsdam-Golm), Xiangyu Liang (Université Claude Bernard Lyon 1), and Ulrich Menne (AEI Potsdam-Golm).

References

[1] F. J. Almgren, Jr. Existence and regularity almost everywhere of solutions to elliptic variational problems among surfaces of varying topological type and singularity structure. *Ann. of Math. (2)*, 87:321–391, 1968.

[2] Guy David. Should we solve Plateau's problem again? In *Advances in analysis: the legacy of Elias M. Stein*, volume 50 of *Princeton Math. Ser.*, pages 108–145. Princeton Univ. Press, Princeton, NJ, 2014.

[3] C. De Lellis, F. Ghiraldin, F. Maggi. A direct approach to Plateau's problem. J. Eur. Math. Soc. (JEMS) (2015).

[4] G. De Philippis, A. De Rosa, and F. Ghiraldin. A direct approach to Plateau's problem in any codimension. Adv. in Math., 288:59–80, (2015).

[5] J. Harrison. Soap film solutions to Plateau's problem. *J. Geom. Anal.*, 24(1):271–297, 2014.

Existence of solutions for boundary value problem with strong nonlinear nonlocal conditions

Igor Kossowski

Łódź University of Technology,
90-924 Łódź, ul. Wólczańska 215, Poland,
kossowski.igor@gmail.com

Let $f\colon [0,1] \times \mathbb{R}^k \to \mathbb{R}^k$ be a given continuous function, $g = (g_1, \ldots, g_k)$, $g_i\colon [0,1] \to \mathbb{R}$, be a given function of bounded variation and $h\colon \mathbb{R}^k \to \mathbb{R}^k$ also be a given continuous function.
We consider the following boundary value problem

$$x' = f(t,x), \quad \int_0^1 h(x(s))\, dg(s) = 0, \tag{9}$$

where

$$\int_0^1 h(x(s))\, dg(s) = \left(\int_0^1 h_1(x(s))\, dg_i(s), \ldots, \int_0^1 h_k(x(s))\, dg_k(s) \right),$$

and the integrals $\int_0^1 h_i(x(s)) dg_i(s)$ are meant in the sense of Riemann-Stieltjes, $i = 1, \ldots, k$.
We prove an existence of solutions for (9). Our method is based on applying Leray-Schauder topological degree.

References

[1] D. Franco, J. J. Nieto, D. O'Regan, *Existence of solutions for first order ordinary differential equations with nonlinear boundary conditions*, Applied Mathematics and Computation 153 (2004), 793-802.

[2] B. Yan, D. O'Regan, R. P. Agarwal, *Multiplicity and uniqueness results for the singular nonlocal boundary value problem involving nonlinear integral conditions*, Boundary Value Problems a Springer Open Journal, 2014:148.

Sobolev spaces on metric groups

Tomasz Kostrzewa

Warsaw University of Technology,
Ul. Koszykowa 75, 00-662 Warsaw, Poland,
kostrzewat@mini.pw.edu.pl

Sobolev space were first introduced on subsets of $\mathbb{R}^n$ and later generalized to more complicated structures e.g. Riemannian manifolds and metric measure spaces. In my talk I will introduce Sobolev spaces on locally compact abelian groups. Those space share many properties with the classical ones. For instance, the Sobolev embeddings and Rellich-Kondrachov compact embedding.
In my talk I will concentrate on properties of Sobolev spaces on metric groups. In this case we can prove some stronger embedding results and also find good dense subsets in our spaces. Furthermore, the analogue of the Trace Theorem can be proven. I will also discuss which results can be applied to the p-adic groups case. This talk is based on joint work with P. Górka.

References

[1] P. Górka, Pego theorem on locally compact abelian groups. *J. Algebra Appl.* Vol. 13, No. 4 (2014), 135-143.

[2] P. Górka, T. Kostrzewa, Pego everywhere. J. Algebra Appl. Vol. 15 No. 4 (2016) 1650074

[3] P. Górka, T. Kostrzewa, Sobolev spaces on metrizable groups. Ann. Acad. Sci. Fenn. Math. 40 (2015), 837-849.

[4] P. Górka, T. Kostrzewa, E.G. Reyes, The Rellich lemma on compact abelian groups and equations of infinite order. *Int. J. Geoem. Meth. Mod. Phys.*, Vol.10, No.2, 2013.

[5] P. Górka, T. Kostrzewa, E.G. Reyes, Sobolev spaces on locally compact abelian groups: compact embeddings and local spaces. *Journal of Function Spaces and Applications.*

[6] P. Górka and E.G. Reyes, Sobolev spaces on locally compact abelian groups and the bosonic string equation. *J. Aust. Math. Soc.* **98** (2015), 39 - 53.

Structured population models in a space of measures; From analytical foundations to numerical results

Karolina Kropielnicka

University of Gdańsk,
Poland,
karolina.kropielnicka@mat.ug.edu.pl

The aim of this talk is to present recent results obtained in the numerical approach to Fredrickson–Hoppensteadt model, which describes the evaluation of the age-structured, two-sex populations. To be more specific, we will present the Escalator Boxcar Train (EBT) method derived for this model in [1], and present the recently proved theorem on its convergence. Due to the fact that all the problem and its analysis are embedded in a space of nonnegative Radon measures equipped with flat metric (also known as bounded Lipschitz or Fourtet–Mourier distance), see [2] and [3], large part of the talk will be devoted to the justification of the choice of the proper metric space.

References

[1] Piotr Gwiazda, Karolina Kropielnicka, Anna Marciniak-Czochra, *The escalator boxcar train method for a system of age–structured equations*, Netw. Heterog. Media, **11** 2016, no. 1, 123-143, arXiv:1506.00016v2

[2] Agnieszka Ulikowska, *An age-structured two-sex model in the space of Radon measures: well posedness*, J Kinet. Relat. Models, **5** 2012, no. 4, 873-900

[3] Piotr Gwiazda, Anna Marciniak-Czochra, *Structured population equations in metric spaces*, J. Hyperbolic Differ. Equ., **7** 2010, no. 4, 733-773

Multiscale descriptions of swarming phenomena

Mirosław Lachowicz

University of Warsaw,
Institute of Applied Mathematics and Mechanics,
ul. Banacha 2, 02–097 Warsaw
lachowic@mimuw.edu.pl

Henryk Leszczyński

University of Gdańsk,
ul. Wita Stwosza 57, 80–952 Gdańsk

A general class of mathematical structures (integro–differential equations) that models swarming behavior at the mesoscopic level is proposed. These structures lead to interesting mathematical problems of blow–up versus global existence ([2], [3]). Macroscopic ("*hydrodynamic*") limits are discussed ([4]). The corresponding individually–based (microscopic) model is proposed (cf. [1]).

References

[1] J. Banasiak, M. Lachowicz, *Methods of small parameter in mathematical biology*, Birkhäuser, Boston 2014.
[2] M. Lachowicz, H. Leszczyński, M. Parisot, *A simple kinetic equation of swarm formation: blow–up and global existence*, Appl. Math. Letters, 57, 2016, 104-107.
[3] M. Lachowicz, H. Leszczyński, M. Parisot, *Blow–up and global existence for a kinetic equation of swarm formation*, to appear.
[4] M. Parisot, M. Lachowicz, *A kinetic model for the formation of swarms with nonlinear interactions*, Kinetic Related Models, 9, 1, 131–164, 2016.

The l^1-anisotropic total variation flow in the plane

Michał Łasica

University of Warsaw,
ul. Banacha 2, 02-097 Warszawa, Poland,
lasica@mimuw.edu.pl

Salvador Moll

Universitat de València,
C/Dr. Moliner, 50, Burjassot, Spain,
j.salvador.moll@uv.es

Piotr B. Mucha

University of Warsaw,
ul. Banacha 2, 02-097 Warszawa, Poland,
mucha@hydra.mimuw.edu.pl

We consider the l^1-anisotropic total variation flow in the plane, corresponding formally to the equation

$$u_t = \operatorname{div}(\operatorname{sgn} u_{x_1}, \operatorname{sgn} u_{x_2}).$$

This is a member of the family of anisotropic total variation flows $u_t = \operatorname{div}(\partial\varphi(Du))$ with $\varphi = \|\cdot\|_1$, whose discretised versions have been particularly often applied to image denoising and decomposition as an alternative to the isotropic case ($\varphi = \|\cdot\|_2$).
We prove that the flow preserves the class of functions piecewise constant on rectangles, in fact we provide explicit description of evolution in this class. It is crucial to understand what exactly happens at time instances when several regions where the function is constant merge and non-local phenomenon of breaking may be exhibited, leading to expansion of the jump set, which is not observed in the isotropic case.
Nevertheless, approximating continuous functions with piecewise constant functions, we are able to show that continuous data stay continuous. In fact, a large class of moduli of continuity is preserved by the flow.
An essential ingredient is a lemma, where a class of Cheeger problems for l^1-anisotropic perimeter is solved.

Bounds on the vertical heat transfer for the Rayleigh–Bénard Convection in the micropolar fluid

Grzegorz Łukaszewicz

University of Warsaw,
glukasz@mimuw.edu.pl

Piotr Kalita

Jagiellonian University,
piotr.kalita@ii.uj.edu.pl

José A. Langa

Universidad de Sevilla,
langa@us.es

We consider the Rayleigh–Bénard setting of a horizontal layer of fluids confined by two parallel planes a distance h apart. The fluid is heated at the bottom plane at temperature T_0 and cooled at the top plane at temperature T_1 ($T_0 > T_1$). The dynamic model consists of the advection-diffusion equation for the temperature coupled with the incompressible micropolar fluid equations via a buoyancy force proportional to the temperature.
We establish connections between the heat flux and the energy dissipation for given Prandtl and Rayleigh numbers. Moreover, we obtain physically relevant bounds on the Nusselt number in terms of the Rayleigh number and the nondimensional micropolar parameters and compare them with that for the classical Boussinesq model, namely,

$$\mathrm{Nu} \leqslant \frac{1}{4}\sqrt{\mathrm{Ra}} - 1.$$

(For the overview of the classical problem cf. references [1–3]).
It occurs that the presence of the microrotation field in the fluid stabilizes the fluid flow (due to internal friction) and even may stop the convective heat transport.

References

[1] P. Constantin, Ch. Doering, *Heat transfer in convective turbulence*, Nonlinearity, 9 (1996) 1049–1060.
[2] Ch. Doering, P. Constantin, *Variational bounds on energy dissipation in incompressible flows. III. Convection*, Physical Review E, 53 (1996), 5957–5981.
[3] X.Wang, *Bound on vertical heat transport at large Prandtl number*, Physica D, 237 (2008) 854–858.

Higher order Pizzetti's formulas and polyharmonic functions

Grzegorz Łysik

Jan Kochanowski University in Kielce
lysik@impan.pl

We introduce integral mean value functions which are averages of integral means over spheres/balls and over their images under the action of a discrete group of complex rotations. In the case of real analytic functions we derive higher order Pizzetti's formulas. As applications we obtain a maximum principle for polyharmonic functions and a characterization of convergent solutions to higher order heat type equations. Finally we state a Dirichlet type problem for polyharmonic functions and give its solution in the case of the unit ball in $\mathbb{R}^n$.

References

[1] G. Łysik, *Higher order Pizzetti's formulas*, Rend. Lincei Mat. Appl. **27** (2016), 105–115.

Ground states and bound states of semilinear Maxwell equations

Jarosław Mederski

Nicolaus Copernicus University,
ul. Chopina 12/18, 87-100 Toruń,
jmederski@mat.umk.pl

We look for solutions $E : \Omega \to \mathbb{R}^3$ of the problem

$$\begin{cases} \nabla \times (\nabla \times E) + \lambda E = |E|^{p-2}E & \text{in } \Omega \\ \nu \times E = 0 & \text{on } \partial\Omega \end{cases}$$

on a bounded Lipschitz domain $\Omega \subset \mathbb{R}^3$, where $\nabla\times$ denotes the curl operator in $\mathbb{R}^3$. The equation describes the propagation of the time-harmonic electric field $\Re\{E(x)e^{i\omega t}\}$ in a nonlinear isotropic material Ω with $\lambda = -\mu\epsilon\omega^2 \leqslant 0$, where μ and ϵ stand for the permeability and the linear part of the permittivity of the material. The nonlinear term $|E|^{p-2}E$ with $p > 2$ is responsible for the nonlinear polarisation of Ω and the boundary conditions are those for Ω surrounded by a perfect conductor. The problem has a variational structure and we deal with the subcritical values $p < 6 = 2^*$ as well as with the critical one $p = 6$, where $6 = 2^*$ is the Sobolev critical exponent. We show that there is a ground state solution and at least finite number of bound states depending on parameter $\lambda \leqslant 0$.

References

[1] T. Bartsch, J. Mederski: *Ground and bound state solutions of semilinear time-harmonic Maxwell equations in a bounded domain*, Arch. Rational Mech. Anal., **215** (1), (2015), 283–306.

[2] J. Mederski: *The Brezis-Nirenberg problem for the curl-curl operator*, in preparation.

The Taylor–Galerkin method and its applications

Piotr Minakowski

Interdisciplinary Center for Scientific Computing (IWR),
University of Heidelberg,
Im Neuenheimer Feld 205, Heidelberg, Germany
p.minakowski@iwr.uni-heidelberg.de

I will present numerical studies on the time-accurate finite element methods. The Taylor-Galerkin approach is to incorporate the structure of the model into the numerical scheme and obtain stabilization-like contribution in the most direct and natural way. We develop the Taylor-Galerkin discretization schemes [1], for transport problem with source, visco-elastic materials [2], and the Euler system with congestion constraint [3].

References

[1] Donea, J. (1984), A Taylor–Galerkin method for convective transport problems. Int. J. Numer. Meth. Engng.

[2] Carew, E. O., Townsend, P. and Webster, M. F. (1994), Taylor-Galerkin algorithms for viscoelastic flow: Application to a model problem. Numer. Methods Partial Differential Eq., 10: 171-190.

[3] Pierre Degond, Jiale Hua, Laurent Navoret (2011), Numerical simulations of the Euler system with congestion constraint, Journal of Computational Physics, Volume 230, Issue 22

Weak compactness for systems of n-harmonic type

Michał Miśkiewicz

University of Warsaw,
Faculty of Mathematics, Informatics and Mechanics,
Banacha 2, 02-097 Warszawa
m.miskiewicz@mimuw.edu.pl

Weak compactness of n-harmonic maps into arbitrary manifolds was shown by Changyou Wang [1] in 2005. In this talk I discuss the structural properties of the n-harmonic system that are crucial for the proof. This leads to a generalization [2] to a wider class of critical elliptic systems

$$-\operatorname{div}(|\nabla u|^{n-2}|\nabla u|) = |\nabla u|^{n-2}\Omega \cdot \nabla u$$

on a bounded domain in $\mathbb{R}^n$, where $u \in W^{1,n}$ and $\Omega \in L^n$. The matrix Ω (which is allowed to depend on u) satisfies some additional structural assumptions. I show that if a sequence of weak solutions (u_k, Ω_k) is weakly convergent (in respective spaces) to (u, Ω), then the limit pair also satisfies the equation.

References

[1] C. Wang, *A compactness theorem of n-harmonic maps*, Ann. Inst. H. Poincaré Anal. Non Linéaire, 22 (2005), pp. 509–519.

[2] M. Miśkiewicz, *A weak compactness result for critical elliptic systems of n-harmonic type*, J. Math. Anal. Appl., vol. 439, 1, pp. 370–384.

Stochastic partial differential equations in hydrodynamics

Elżbieta Motyl

University of Łódź, Poland

The talk will be concerned mainly with the martingale solutions of the stochastic Navier-Stokes equations. The problem of the existence and uniqueness of the solutions will be considered. This approach can be also applied to other equations, e.g. Boussinesq equations and magneto-hydrodynamic equations (MHD).

Transport in fluid mechanics

Piotr Bogusław Mucha

University of Warsaw,
Banacha 2, Warszawa, Poland,
p.mucha@mimuw.edu.pl

I would like to concentrate my attention of the transport equations in models arising from fluid mechanics. Two basic places where we touch the issue are the Lagrangian coordinates and the continuity equation for compressible models.
Within the talk, I will present classical results which still are useful tools in nowadays problems of mathematical fluid mechanics. The issue of integrability and regularity will be discussed in various functional frameworks. The state of the art for this subject will be presented too.

Optimality and approximate optimality Conditions for the Blowup Time of diffusion equations

Andrzej Nowakowski

Faculty of Math & Computer Sciences, University of Lodz
Banacha 22, 90-238 Lodz, Poland
annowako@math.uni.lodz.pl

We study the blowup problems for semilinear parabolic differential equations with control function. Sufficient optimality conditions for controlled blowup time are derived in terms of dula dynamic programming methodology. We define ε-optimal value function and we construct sufficient ε-optimaity conditions for that function again in terms of dual dynamic programming inequality.

Variational-hemivariational inequalities in mathematical modeling

Anna Ochal

Jagiellonian University in Krakow,
ul. Stanisława Łojasiewicza 6, 30-348 Kraków, Poland
ochal@ii.uj.edu.pl

We study a class of elliptic variational-hemivariational inequalities in reflexive Banach spaces. An inequality in the class involves a nonlinear operator, a convex set of constraints and two nondifferentiable functionals, among which at least one is convex.
Let X be a reflexive Banach space. Given a set $K \subset X$, an operator $A\colon X \to X^*$ and functions $\varphi\colon K \times K \to \mathbb{R}$, $j\colon X \to \mathbb{R}$, we consider the following problem:

Find an element $u \in K$ such that

$$\langle Au, v-u\rangle + \varphi(u,v) - \varphi(u,u) + j^0(u; v-u) \geqslant \langle f, v-u\rangle \quad \text{for all } v \in K.$$

The motivation to study the problem comes from the fact that it contains, as particular cases, various problems considered in the literature.
We deliver a result on existence and uniqueness of a solution to the inequality. Next, we consider a mathematical model which describes the equilibrium of an elastic body in unilateral contact with a foundation. The model leads to a variational-hemivariational inequality for the displacement field, that we analyse by using our abstract results.

References

[1] S. Migórski, A. Ochal, M. Sofonea, A Class of Variational-Hemivariational Inequalities in Reflexive Banach Spaces, submitted to *Journal of Elasticity* (2016).

On positive evanescent solutions of a class of singular elliptic problems

Aleksandra Orpel

Faculty of Mathematics and Computer Science,University of Łódź,
S.Banacha 22, 90-238 Łódź, Poland
orpela@math.uni.lodz.pl

We investigate the existence and properties of solutions of the following class of elliptic differential equations

$$div(a(||x||)\nabla u(x)) + f(x,u(x)) - (u(x))^{-\alpha}||\nabla u(x)||^{\beta} + g(||x||)x \cdot \nabla u(x) = 0,$$

for $x \in R^n$, $||x|| > R$, with the condition $\lim\limits_{||x|| \to \infty} u(x) = 0$. We present the approach based on the subsolution and supersolution method for bounded subdomains and a certain convergence procedure. Our results cover both sublinear and superlinear cases of f. The speed of decaying of solutions will be also characterized more precisely.

References

[1] D.P.Covei, Existence and asymptotic behavior of solution to a singular elliptic problem, Surveys in Mathematics and its Application, 6 (2011), 127-136

[2] S.Cui, Existence and nonexistence of positive solutions of singular semilinear elliptic boundary value problems, Nonlinear Analysis, 41, (2000), 149-176

[3] A. Orpel, Increasing sequences of positive evanescent solutions of nonlinear elliptic equations, J. Differential Equations 259 (2015) 1743–1756

[4] A. Orpel, Positive evanescent solutions of singular elliptic problems in exterior domains, Elec. J. Qualit. Theor. Diff. Eqts., publication in preparation

Renormalised solutions in thermo-visco-plasticity for a Norton-Hoff type model

Sebastian Owczarek

Faculty of Mathematics and Information Science, Warsaw University of Technology, Koszykowa 75, 00-662 Warsaw, Poland,
s.owczarek@mini.pw.edu.pl

Krzysztof Chełmiński

Faculty of Mathematics and Information Science, Warsaw University of Technology, Koszykowa 75, 00-662 Warsaw, Poland,
kchelmin@mini.pw.edu.pl

The presentation deals with the existence of a renormalized solution to a nonlinear system which concerns a thermo-visco-plasticity Norton-Hoff type model. The main difficulties are the strong coupling between the equations, the nonlinearities and the fact that the right-hand side of the energy balance equation lies in L^1. The proof bases on the approximate technique, Boccardo and Gallout's approach and Minty's monotonicity trick. First the temperature and the dissipation term are estimated by truncation for which the existence is known. Then, by using the definition of a strong solution, one shows that the almost pointwise convergence of temperature's approximate sequence to a measurable function θ. However, this is not enough to pass to a limit. The key step is to use the monotone character of the flow rule and Minty trick to identify weak limits. The final step is to validate that the limit is a renormalised solution.

References

[1] Chełmiński K. and Owczarek S., *Renormalized solutions in thermo-visco-plasticity for a Norton-Hoff type model. Part I: The truncated case*, Nonlinear Anal. Real World Appl. 28 (2016), 140-152.

[2] Chełmiński K. and Owczarek S., *Renormalised solutions in thermo-visco-plasticity for a Norton-Hoff type model. Part II: The limit case*, Nonlinear Anal. Real World Appl. 31 (2016), 643-660.

The boundary Harnack principle

Martyna Patera

Nicolaus Copernicus University,
Toruń,
martynapatera@gmail.com

The boundary Harnack principle is a property saying that two positive harmonic functions in a domain vanishing on a portion of the boundary decay at the same rate toward a smaller portion of the boundary.
First, we will look at this property in a Euclidean space. Next we will define a harmonic function on a metric space using the mean value property and investigate a Carleson type estimate for such a function in a John domain.

References

[1] T. Adamowicz, M. Gaczkowski, P. Górka: Harmonic functions on metric measure spaces. arXiv:1601.03919.
[2] H. Aikawa, T. Kilpeläinen, N. Shanmugalingam, X. Zhong: Boundary Harnack principle for p-harmonic functions in smooth Euclidean domains. Potential Analysis (2007) 26:281-301.
[3] H. Aikawa and N. Shanmugalingam: Carleson-type estimates for p-harmonic functions and the conformal Martin boundary of John domains in metric measure spaces. Michigan Math. J. 53 (2005), no. 1, 165–188.

Stochastic Partial Differential Equations - a short overview

Szymon Peszat

Institute of Mathematics,
Jagiellonian University and Institute of Mathematics,
Polish Academy of Sciences

At the beginning I would like to derive some Stochastic Partial Differential Equations (SPDEs in short) from interacting particle models. Then I would like to talk about several SPDEs having natural interpretation. In particular Zakai equation of filtering, the famous KPZ equation, and the so-called Parabolic Anderson models with be discussed. Finally I going to discuss applications of SPDEs to the construction of Markov processes.

Dynamics of particles with nonlocal singular interactions

Jan Peszek

University of Warsaw,
Banacha 2, 02-097 Warsaw, Poland,
j.peszek@mimuw.edu.pl

I will present some recent developments in models of dynamics of particles with nonlocal interactions generated by a singular potential (e.g. [1], [2], [3]). Singularity of the potential enables sticking of the trajectories of the particles, which can be viewed as an extreme example of compressibility (see [4]). Such phenomenon causes a number of problems from the point of view of both qualitative and quantitative analysis (see [5]).

References

[1] J. A. Carrillo, Y.-P. Choi, and M. Hauray. Local well-posedness of the generalized Cucker-Smale model with singular kernels. *MMCS* vol. 47 of *ESAIM Proc. Surveys,* 2014,
[2] S.-Y. Ha and J.-G. Liu, A simple proof of the Cucker-Smale flocking dynamics and mean-field limit. *Commun. Math Sci.*, 2009,
[3] M. Hauray and P.-E. Jabin. Particle approximation of Vlasov equations with singular forces: propagation of chaos, *Ann Sci. Ec.*, 2015,
[4] J. Peszek. Existence of piecewise weak solutions of a discrete Cucker-Smale flocking model with a singular communication weight. *J. Differential Equations*, 2014,
[5] P. Mucha and J. Peszek, The Cucker-Smale equation: singular communication weight, measure solutions and weak-atomic uniqueness, *preptint: arXiv:1509.07673v2*, 2016.

On the stationary flow of reactive gaseous mixture

Tomasz Piasecki

Polish Academy of Sciences, Institute of Mathematics
Warsaw
tpiasecki@mimuw.edu.pl

We are interested in a system of equations describing stationary flow of a mixture of gases undergoing reversible chemical reactions which reads

$$\begin{gathered}\operatorname{div}(\rho\mathbf{u}) = 0,\\ \operatorname{div}(\rho\mathbf{u}\otimes\mathbf{u}) - \operatorname{div}\mathbf{S} + \nabla\pi = \rho\mathbf{f},\\ \operatorname{div}(\rho E\mathbf{u}) + \operatorname{div}(\pi\mathbf{u}) + \operatorname{div}\mathbf{Q} + \operatorname{div}(\mathbf{S}\mathbf{u}) = \rho\mathbf{f}\cdot\mathbf{u},\\ \operatorname{div}(\rho Y_k\mathbf{u}) + \operatorname{div}\mathbf{f}_k = m_k\mathbf{w}_k, \quad k\in\{1,\ldots,n\},\end{gathered} \tag{10}$$

where $\mathbf{u}$ is the velocity of the fluid, ρ is the density of the mixture which is a sum of species densities ρ_k and $Y_k = \frac{\rho_k}{\rho}$ are the species mass fractions. Furthermore, $\mathbf{S}$ denotes the viscous stress tensor, π the internal pressure of the fluid, $\mathbf{f}$ the external force, E the specific total energy and $\mathbf{Q}$ the heat flux. The first three equations form the well known stationary compressible Navier-Stokes system and the equations $(10)_4$ describe the balance of masses of n species. The precise form of all quantities and constitutive relations will be explained in the talk.
I will start with presentation on known results concerning existence of weak solutions to the system (10). Then I will present some ideas of how to strengthen these results, which is a work in progress with M. Pokorný and E. Zatorska. It turn out that new pressure estimates developed in [3] for compressible Naver-Stokes system can be applied also to the mixture model (10) and enable to generalize the results from [1] on wider range of γ in the pressure law $\pi(\rho) = \rho^\gamma + \rho\theta$. We also introduce a slightly more general definition of solutions, so called *variational entropy solutions*. This type of solutions is considered for compressible Navier-Stokes equations in papers [2],[3] and allows for further relaxation of the range of γ. On the other hand, for sufficiently high γ variational entropy solutions are weak solutions.

References

[1] V. Giovangigli, M. Pokorný, E. Zatorska *On the steady flow of reactive gaseous mixture*, Analysis (Berlin) 35 (2015), no. 4, 319–341

[2] A. Novotný, M. Pokorný *Steady compressible Navier-Stokes-Fourier system for monoatomic gas and its generalizations*, J. Differential Equations 251 (2011), no. 2, 270–315

[3] A. Novotný, M. Pokorný *Weak and variational solutions to steady equations for compressible heat conducting fluids*, SIAM J. Math. Anal. 43 (2011), no. 3, 1158–1188

On the reciprocity formula for two and more moduli

A. Pierzchalski

Faculty of Mathematics and Computer Science, University of Lodz,
antoni@math.uni.lodz.pl

The following subjects will be discussed:
- the modulus of a family of curves and the capacity of a condenser
- the reciprocity formula for two moduli or capacities
- conjugacy of the extremal functions
- foliations of a Riemannian manifold and their moduli
- conjugate submersions
- the reciprocity formula for two transversal foliations (global version)
- partition along or/and across the leaves
- the reciprocity formula (local version)
- generalizations for more than two moduli
- examples

References

[1] D. Blachowska, *A modulus and the extremal form of a foliation*, Demonstratio Math. 37 (2004), no. 4, 939–954.
[2] M. Ciska, *T*he modulus of pairs of transversal foliations (in Polish), PhD thesis, University of Lodz, Poland, 2012.
[3] M. Ciska, A. Pierzchalski On the modulus of level sets of conjugate submersions Differential Geometry and its Applications, 36 (2014), 90–97.
[4] J. Kalina, A. Pierzchalski, *A* variation of the modulus of submanifold families, Analytic functions, Kozubnik 1979 (Proc. Seventh Conf., Kozubnik, 1979), Lecture Notes in Math., 798, Springer, Berlin, 1980, 250–257.
[5] A. Kaźmierczak, A. Pierzchalski *A* reciprocity type formula for systems of orthogonal foliations, to appear.
[6] A. Pierzchalski, *T*he k-module of level sets of differential mappings, Scientific Communications of the Czechoslovakian-GDR-Polish School on Differential Geomerty at Boszkowo 1978, Math. Inst. Polish Acad. Sci.,Warsaw, 1979, 180–185.
[7] A. Romanov, *C*apacity relations in a planar quadrilateral, Siberian Mathematical Journal, Vol. 53 (2008) 886–897.

Renormalized solutions of semilinear elliptic equations involving measure data and Dirichlet operator

Andrzej Rozkosz

Nicholaus Copernicus University
Toruń, Poland
rozkosz@mat.umk.pl

Let E be a locally compact separable metric space, $(\mathcal{E}, D(\mathcal{E}))$ be a regular Dirichlet form on $L^2(E;m)$, μ be a bounded smooth measure (with respect to the capacity associated with $(\mathcal{E}, D(\mathcal{E}))$ and $f : R \times \mathbb{R} \to \mathbb{R}$ be a measurable function. We consider semilinear equation of the form

$$(*) \qquad -Lu = f(\cdot, u) + \mu \quad \text{in} \quad E,$$

where L is the operator associated with $(\mathcal{E}, D(\mathcal{E}))$. The class of such operators is quite large. It includes local operators (the model example is the Laplacian Δ or uniformly elliptic divergence form operator) as well as nonlocal operators (the model example is the fractional Laplacian $\Delta^{\alpha/2}$).

We first give a definition of a renormalized solution to $(*)$ recently introduced in [3] and show that u is a renormalized solution of $(*)$ if and only if u is a probabilistic solution of $(*)$, i.e satisfies some equation, which may be viewed as a nonlinear Feynman-Kac formula associated with $(*)$. This result enables studying $(*)$ by probabilistic methods. By way of illustration what can be proved by these methods, we show some results on existence, uniqueness and regularity of solutions of $(*)$ proved in [2]. These results generalize to the case of Dirichlet operators the corresponding results from [1] proved in case $L = \Delta$.

Talk is based on joint work with Tomasz Klimsiak.

References

[1] P. Bènilan, L. Boccardo, T. Gallouët, R. Gariepy, M. Pierre and J.L. Vazquez, An L^1-theory of existence and uniqueness of solutions of nonlinear elliptic equations, *Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4)* **22** (1995) 241–273.

[2] T. Klimsiak and A. Rozkosz, Dirichlet forms and semilinear elliptic equations with measure data, *J. Funct. Anal.* **265** (2013) 890–925.

[3] T. Klimsiak and A. Rozkosz, Renormalized solutions of semilinear equations involving measure data and operator corresponding to Dirichlet form, *NoDEA Nonlinear Differential Equations Appl.* **22** (2015) 1911–1934.

Phenotype-structured population models. Does assortative mating lead to speciation?

Ryszard Rudnicki

Institute of Mathematics,
Polish Academy of Sciences, Bankowa 14, 40-007 Katowice, Poland.

We consider two models of phenotypic evolution in hermaphroditic populations which two types of mating of individuals: random and assortative, i.e., the individuals with similar traits mate more often than they would choose a partner randomly. In the case of random mating the existence of an one-dimensional attractor is proved. In the case of assortative mating we show show that it converges to a combination of Dirac's delta functions (or more generally to a multimodal distribution). This result means that assortative mating can lead to a polymorphic population and adaptive speciation.
The talk is based on the following papers:
1. R. Rudnicki, P. Zwoleński, Model of phenotypic evolution in hermaphroditic populations, J. Math. Biol. 2015.
2. R. Rudnicki, R. Wieczorek, Does assortative mating lead to a polymorphic population? A toy model justification. (in preparation).

On local weak solutions to Nernst–Planck–Poisson system

Lucjan Sapa

AGH University of Science and Technology, Faculty of Applied Mathematics, Poland

sapa@agh.edu.pl

Piotr Kalita

Jagiellonian University, Faculty of Mathematics and Computer Science, Poland

piotr.kalita@ii.uj.edu.pl

Robert Filipek, Krzysztof Szyszkiewicz

AGH University of Science and Technology, Faculty of Material Science and Ceramics, Poland

rof@agh.edu.pl szyszkin@agh.edu.pl

We study the following initial and boundary value problem for one-dimensional Nernst–Planck–Poisson (NPP) system with nonlinear boundary conditions

$$\begin{cases} u_t = \alpha_1 u_{xx} - \alpha_2 (u\varphi_x)_x, \\ v_t = \beta_1 v_{xx} + \beta_2 (v\varphi_x)_x, \\ \varphi_{xx} = \lambda(u - v), \end{cases}$$

$$u(0,x) = u_0(x) \quad \text{and} \quad v(0,x) = v_0(x),$$

$$\begin{cases} \alpha_1 u_x(t,0) - \alpha_2 u(t,0)\varphi_x(t,0) = f_1(t, u(t,0)), \\ \alpha_1 u_x(t,1) - \alpha_2 u(t,1)\varphi_x(t,1) = f_2(t, u(t,1)), \\ \beta_1 v_x(t,0) + \beta_2 v(t,0)\varphi_x(t,0) = g_1(t, v(t,0)), \\ \beta_1 v_x(t,1) + \beta_2 v(t,1)\varphi_x(t,1) = g_2(t, v(t,1)), \\ \varphi(t,0) = h_1(t), \\ \varphi(t,1) = h_2(t). \end{cases}$$

The functions $u_0, v_0 : \Omega \to \mathbb{R}$, $f_i, g_i : [0,T] \times \mathbb{R} \to \mathbb{R}$, $h_i : [0,T] \to \mathbb{R}$ and constants $\alpha_i, \beta_i, \lambda > 0$ for $i = 1, 2$ are given, where $\Omega = (0,1)$, $T > 0$. The boundary conditions cover the special case of the full Chang-Jaffé (CJ) conditions. The system describes many important physical and biological processes, for example ionic diffusion in porous media, electrochemical and biological membranes as well as electrons and holes transport in semiconductors. With the considered boundary conditions the physical system need not be closed, it can also be open. We will present theorems on existence, uniqueness, and nonnegativity of weak solutions and some numerical simulations.

References

[1] Filipek R., Kalita P., Sapa L., Szyszkiewicz K., *On local weak solutions to Nernst–Planck–Poisson system*, submitted.

Existence and uniqueness of global weak solutions to interdiffusion with Vegard rule

Lucjan Sapa

AGH University of Science and Technology, Faculty of Applied Mathematics, Poland

sapa@agh.edu.pl

Marek Danielewski

AGH University of Science and Technology, Faculty of Material Science and Ceramics, Poland

daniel@agh.edu.pl

Bogusław Bożek

AGH University of Science and Technology, Faculty of Applied Mathematics, Poland

bozek@agh.edu.pl

We study the problem of the one-dimensional interdiffusion in the r-component solid solution. Denote by $M_i = \text{const} > 0$, $\Omega_i = \text{const} > 0$ and $\Theta_i : [0, \frac{M_1}{\Omega_1}] \times ... \times [0, \frac{M_r}{\Omega_r}] \to \mathbb{R}_+$, $i = 1, ..., r$, the molecular mass, molar volume and diffusion coefficient of the ith component of the mixture, respectively. Let v^D be the Darken drift velocity. The local mass conservation low for the Darken flux of the ith component of the mixture

$$J_i = -\theta(\varrho_1, ..., \varrho_r)\partial_x \varrho_i + \varrho_i v^D,$$

$i = 1, ..., r$, and the Vegard rule on the concentrations ϱ_i lead to the strongly coupled (i.e. by the second derivatives) differential system

$$\partial_t \varrho_i + \partial_x \left(-\Theta_i(\varrho_1, ..., \varrho_r)\partial_x \varrho_i + \varrho_i \sum_{j=1}^{r} \frac{\Omega_j \Theta_j(\varrho_1, ..., \varrho_r)}{M_j} \partial_x \varrho_j \right) + K(t)\partial_x \varrho_i = 0,$$

$i = 1, ..., r$, with the initial condition and the coupled nonlinear boundary conditions

$$\varrho_i(0, x) = \varrho_i^0(x),$$

$$\begin{cases} \left(-\Theta_i(\varrho_1, ..., \varrho_r)\partial_x \varrho_i + \varrho_i \left(K(t) + \sum_{j=1}^{r} \frac{\Omega_j \Theta_j(\varrho_1, ..., \varrho_r)}{M_j} \partial_x \varrho_j \right) \right)(t, -\Lambda) = j_{i,L}(t), \\ \left(-\Theta_i(\varrho_1, ..., \varrho_r)\partial_x \varrho_i + \varrho_i \left(K(t) + \sum_{j=1}^{r} \frac{\Omega_j \Theta_j(\varrho_1, ..., \varrho_r)}{M_j} \partial_x \varrho_j \right) \right)(t, \Lambda) = j_{i,R}(t). \end{cases}$$

The functions ϱ_i^0, $j_{i,L}$, $j_{i,R}$ are given and the function $K(t)$ is found. We will present theorems on existence and uniqueness of global weak solutions and numerical simulations.

References

[1] B. Bożek, M. Danielewski, L. Sapa, *Existence and uniqueness of global weak solutions to interdiffusion with Vegard rule*, in preparation.

Thermomicropolar fluid

Jakub Siemianowski

Nicolaus Copernicus University,
Faculty of Mathematics and Computer Sciences,
Chopina 12/18, 87-100 Toruń, Poland,
jsiem@mat.umk.pl

Grzegorz Łukaszewicz

University of Warsaw,
Institute of Applied Mathematics and Mechanics,
Banacha 2, 02-097 Warsaw, Poland
glukasz@mimuw.edu.pl

The thermomicropolar fluid motion is induced by differential heating of a layer of such fluid bounded by two horizontal one-dimensional plates a distance 1 apart. We consider a system of equations describing 2D thermomicropolar fluid in the dimensionless form

$$\begin{cases} \frac{1}{\mathrm{Pr}}(u_t + (u\cdot\nabla)u) + \nabla p = \Delta u + 2N^2 \mathrm{rot}\omega + e_2 \mathrm{Ra} T, \\ \mathrm{div} u = 0, \\ \frac{1}{\mathrm{Pr}}(\omega_t + u\cdot\nabla\omega) + 4N^2\omega = \frac{1}{L^2}\Delta\omega + 2N^2\mathrm{rot} u, \\ T_t + u\cdot\nabla T = \Delta T + D\mathrm{rot}\omega\cdot\nabla T, \end{cases} \tag{11}$$

where $u = (u_1, u_2)$ is the velocity field, p is the pressure, ω is the microrotation, T is the temperature and e_2 is the unit upward vector. The constant Ra is the Rayleigh number, Pr is the Prandtl number and N^2, L^2, $D > 0$ are constants related to viscosity coefficients. The fluids occupy the region $\Omega = (-\infty, \infty) \times [0, 1]$. We include boundary and intial conditions

$$u|_{y=0,1} = 0, \quad \omega|_{y=0,1} = 0,$$
$$T|_{y=0} = 1, \quad T|_{y=1} = 0,$$
$$u|_{t=0} = u_0, \quad \omega|_{t=0} = \omega_0, \quad T|_{t=0} = T_0$$

with periodicity in the horizontal direction assumed. Since the thermomicropolar fluids has not yet been studied by mathematicians, we show that there are solutions to the problem (11) in a certain sense.
The thermomicropolar fluid equations were introduced by A. C. Eringen in 1972, see [1].

References

[1] Straughan B., *The Energy Method, Stability, and Nonlinear Convection*, Applied Mathematical Sciences, 91. Springer-Verlag, New York, (1992).

Liouville theorems for elliptic problems in variable exponent spaces

by Sylwia Dudek and Iwona Skrzypczak.

Iwona Skrzypczak
Institute of Applied Mathematics and Mechanics,
Warsaw University
Polska (Poland)

iskrzypczak@mimuw.edu.pl

We investigate nonexistence of nonnegative solutions to a partial differential inequality involving $p(x)$–Laplacian of the form

$$-\Delta_{p(x)} u \geqslant \Phi(x, u(x), \nabla u(x))$$

in $\mathbb{R}^n$, as well as in outer domain $\Omega \subseteq \mathbb{R}^n$, where $\Phi(x, u, \nabla u)$ is a locally integrable Carathéodory's function. We assume that $\Phi(x, u, \nabla u) \geqslant 0$ or compatible with p and u. Growth conditions on u and p lead to Liouville–type results for u.

Evolution for diffusing particles in gravitation

Robert Stańczy

Uniwersytet Wrocławski,
stanczr@math.uni.wroc.pl

Jean Dolbeault

Université Paris Dauphine,
dolbeaul@ceremade.dauphine.fr

We consider the generalized model of gravitating and diffusing particles with density $n = n(x,t)$ modeled by the evolution equation in $\Omega \subset \mathbb{R}^d$ of the form

$$n_t = \nabla \cdot (\nabla p + n\nabla(-\Delta)^{-1}n)$$

where the pressure function dependence on n and the temperature θ is in self-similar

$$p(n,\theta) = \theta^{d/2} P(n\theta^{-d/2})$$

Depending on the statistics governing pressure function P and global parameters of the system: mass, the energy or the entropy different phenomena can be observed: the gravothermal catastrophe or the convergence towards the stable steady states.

References

[0] J. Dolbeault, R. Stańczy, *Evolution for Fermi-Dirac statistics in gravitation,* in prep.
[1] R. Stańczy, *On stationary and radially symmetric solutions to some drift-diffusion equations with nonlocal term,* Applicable Analysis 95 (2016), 97-104.
[2] J. Dolbeault, R. Stańczy, *Bifurcation diagrams and multiplicity for nonlocal elliptic equations modeling gravitating systems based on Fermi-Dirac statistics,* Discrete and Continuous Dynamical Systems A 35 (2015), 139-154.
[3] P.H. Chavanis, M. Lemou, F. Mehats, *Models of dark matter halos based on statistical mechanics: I. The classical King model,* Physical Review D 91 (2015), 06353.
[4] E. Feireisl, Mathematics of complete fluid systems, Erwin Schroedinger International Institute for Mathematical Physics, Vienna, 2012.
[5] R. Stańczy, *On an evolution system describing self-gravitating particles in microcanonical setting,* Monatshefte für Mathematik 162 (2011), 197-224.
[6] J. Dolbeault, R. Stańczy, *Non-existence and uniqueness results for supercritical semilinear elliptic equations,* Annales Henri Poincaré 10 (2010), 1311-1333.
[7] R. Stańczy, *The existence of equilibria of many-particle systems,* Proceedings of the Royal Society of Edinburgh A 139 (2009), 623-631.
[8] R. K. Niven, M. Grendar, *Generalized Maxwell-Boltzmann, Bose-Einstein, Fermi-Dirac and Acharya-Swamy Statistics and the Pólya Urn Model,* ArXiv 0808.2102 (2008).
[9] P. Biler, R. Stańczy, *Parabolic-elliptic systems with general density-pressure relations,* Surikaisekikenkyusho Kokyuroku 1405 (2004), 31-53.

Second order ordinary differential systems with nonlocal Neumann conditions at resonance

Katarzyna Szymańska-Dębowska
joint work with Jean Mawhin

Institute of Mathematics, Łódź University of Technology,
90-924 Lódź, ul. Wólczańska 215, Poland,
katarzyna.szymanska-debowska@p.lodz.pl

Let $f : [0,1] \times \mathbb{R}^k \times \mathbb{R}^k \to \mathbb{R}^k$ be continuous and bounded, $g = \mathrm{diag}(g_1, \ldots, g_k)$ with $g_j : [0,1] \to \mathbb{R}$ having bounded variation, and let us consider the boundary value problem on $[0,1]$

$$x'' = f(t,x,x'), \;\; x'(0) = 0, \;\; x'(1) = \int_0^1 x'(s)\, dg(s). \tag{12}$$

It was recently considered in [1] in the special case with $f = f(t,x)$ such that the limit $h(t,\xi) := \lim_{r\to\infty} f(t,r\xi)$ exists uniformly in $\xi \in S^{k-1}$, with S^{k-1} the unit sphere in $\mathbb{R}^k$. We show that the result of [1] follows in a straightforward way from the simplest form of Leray-Schauder continuation theorem. This approach provides a generalization of the existence theorem of [1] to some f which may depend upon x', need not to belong to the Landesman-Lazer-Nirenberg class, but satisfy a condition of the type introduced in 1966 by Villari.

The use of more sophisticated techniques from coincidence degree theory provides other existence conditions for problem (12) in terms of the non-vanishing of the Brouwer degree of some mapping in $\mathbb{R}^k$ depending upon f and g. From this result follow also existence conditions for problem from [1] with Landesman-Lazer-Nirenberg nonlinearities, in terms of the non-vanishing of the Brouwer degree of some mapping in $\mathbb{R}^k$ depending upon h and g.

References

[1] K. Szymańska-Dębowska, k-dimensional nonlocal boundary-value problems at resonance, Electron. J. Diff. Equ., Vol. 2015 (2015), No. 148, pp. 1-8

[2] J. Mawhin and K. Szymańska-Dębowska, Second-order ordinary differential systems with nonlocal Neumann conditions at resonance, Ann. Mat. Pura ed Appl., to appear.

Harnack's inequality for quasiminimizers with generalized Orlicz growth conditions

Olli Toivanen

Institute of Mathematics, Polish Academy of Sciences
ul. Śniadeckich 8
00-656 Warszawa
otoivanen@impan.pl

Joint work with Petteri Harjulehto (Turku) and Peter Hästö (Turku, Oulu)

In a manuscript, we prove Harnack's inequality for local (quasi)minimizers in generalized Orlicz spaces without growth or coercivity conditions. As a consequence, we obtain the local Hölder continuity of local (quasi)minimizers. The results include as special cases standard growth assumptions, variable exponent growth and the double phase case.
The generalized Orlicz or Musielak-Orlicz space $L^{\varphi(\cdot)}(\Omega)$ is defined as the set of those functions f with $\lim_{\lambda\to 0}\int_\Omega \varphi(x,|\lambda f(x)|)\,dx = 0$, equipped with the Luxemburg norm. Our assumptions on φ (cf. the assumptions (A0)–(A2) in [1]) are natural generalizations of regularity assumptions in the variable exponent and double phase cases, and as they agree with the known optimal assumptions of two very disparate cases, we suggest they form a reasonable basis for a general theory.

References

[1] P. Harjulehto, P. Hästö, *The Riesz potential in generalized Orlicz spaces*, Forum Math., to appear

Sobolev type equations: analytical and numerical approach

Krzysztof A. Topolski

University of Gdańsk,
Wit Stwosz 57
80-952 Gdańsk, Poland,
matkt@mat.ug.edu.pl

Danuta Jaruszewska-Walczak

University of Gdańsk,
Wit Stwosz 57
80-952 Gdańsk, Poland,
dana@mat.ug.edu.pl

We consider initial-boundary-value problems for a class of Sobolev type equations. We prove a local-in-time existence results and nonexistence of global-in-time solutions. Such results are very important from the physical and mathematical point of view. For example, a disruption of a semiconductor can be described as a blow-up of a solution for a suitable differential problem.
We apply the method of lines to solve Sobolev type equations numerically and to estimate a blow-up time.

Kinetic theory of wealth distribution

Giuseppe Toscani

Department of Mathematics
University of Pavia ,
Via Ferrata 1, 27100 Pavia (Italy),
giuseppe.toscani@unipv.it

In the last decade, kinetic theory has proved a very effective tool in solving problems in social sciences and economics [1]. In particular, the distribution of wealth in a multi-agent society has been investigated by resorting to classical methods of kinetic theory of rarefied gases. In analogy with the Boltzmann equation, the change of wealth in these models is due to microscopic binary trades among agents. In this lecture, we present and discuss some recent results in the field. Among others, we consider the possible role and influence of knowledge in the evolution of wealth in a system of agents which interact through binary trades [2]. The trades, which include both saving propensity and the risks of the market, are here modified in the risk and saving parameters, which now are assumed to depend on the personal degree of knowledge. The numerical simulations show that the presence of knowledge has the potential to produce a class of wealthy agents and to account for a larger proportion of wealth inequality.

References

[1] L. Pareschi, G. Toscani, *Interacting Multiagent Systems: Kinetic Equations and Monte Carlo Methods*, Oxford University Press, Oxford (2014)
[2] L. Pareschi, G. Toscani, Wealth distribution and collective knowledge. A Boltzmann approach, *Phil. Trans. R. Soc. A* **372**, 20130396, 6 October (2014)

A Boltzmann-type kinetic approach to the modelling of vehicular traffic

Andrea Tosin

Department of Mathematical Sciences "G. L. Lagrange",
Politecnico di Torino,
Corso Duca degli Abruzzi 24, 10129 Turin, Italy,
andrea.tosin@polito.it

In this talk I will present a kinetic approach to the modelling of vehicular traffic. Sticking to the idea that the macroscopic characteristics of the flow of vehicles are ultimately due to microscopic interactions among single cars, the approach consists in implementing a probabilistic description of speed changes in a Boltzmann-type collisional operator. In particular, I will discuss how this approach allows one to study the fundamental diagrams of traffic, possibly also considering a heterogeneous composition of the flow of vehicles, up to some hydrodynamic/Fokker-Planck limits.

This is a joint work with: L. Fermo, P. Freguglia, M. Herty, G. Puppo, M. Semplice, G. Visconti.

References

1. L. Fermo and A. Tosin. Fundamental diagrams for kinetic equations of traffic flow. *Discrete Contin. Dyn. Syst. Ser. S*, 7(3):449–462, 2014.

2. P. Freguglia and A. Tosin. Proposal of a risk model for vehicular traffic: A Boltzmann-type kinetic approach. Preprint: arXiv:1506.05422, 2015.

3. G. Puppo, M. Semplice, A. Tosin, and G. Visconti. Analysis of a heterogeneous kinetic model for traffic flow. Preprint: arXiv:1511.06395, 2015.

4. G. Puppo, M. Semplice, A. Tosin, and G. Visconti. Fundamental diagrams in traffic flow: the case of heterogeneous kinetic models. *Commun. Math. Sci.*, 14(3):643–669, 2016.

5. G. Puppo, M. Semplice, A. Tosin, and G. Visconti. Kinetic models for traffic flow resulting in a reduced space of microscopic velocities. Preprint: arXiv:1507.08961, 2016.

Representation of and numerics for SPDEs using backward doubly stochastic differential equations

Plamen Turkedjiev
King's College London

Since the 90's, it has been known that a certain class of SPDEs admits a solution in the form of the conditional expectation of a functional of a standard Markov process. This Feynmann-Kac representation for SPDEs is based on so-called Backward-Doubly SDEs (BDSDEs). An important class of SPDEs that can be treated this way are the Zakai equations. These equations are significant for the nonlinear filtering problem, which arises in engineering and financial applications. Moreover, the BDSDE approach can be used to give existence and uniqueness results in Sobolev space for SPDEs whose coefficients are not smooth (hence admit no classical solution). In this talk, we start with an overview of the BDSDE approach to SPDEs, using the Zakai equation as a background example. We then turn our attention to a more recent use of the BDSDE to provide novel numerical approaches for SPDEs. As a numerical example, we present the so-called BDSDE filter for the nonlinear filtering problem.

Interspecies interactions and chemotaxis

Dariusz Wrzosek

University of Warsaw,
Banacha 2,Warszawa
darekw@mimuw.edu.pl

We consider a model which describes interspecies interactions in which chemical signaling plays a crucial rôle. The model belongs to the class of parabolic or degenerate parabolic systems with upper-triangular main part. The classical Lotka-Volterra model of competition was extended to account for the random dispersal of individuals and for their capability to avoid encounters with competitors by means of a chemo-sensory reaction to the smell of rivals (chemorepulsion). We consider the case of diffusing and non-diffusing repellent and study existence of non-constant steady states and long time-behavior.

Non-Newtonian flow over a rough surface

Aneta Wróblewska-Kamińska

Institute of Mathematics, Polish Academy of Sciences ,
ul. Śniadeckich 8, 00-656 Warszawa, Poland,
awrob@impan.pl

We analyse how small irregularities of the solid surface effects the steady flow of a general viscous fluid at larger scales. In particular we consider generalised Stokes system for incompressible non-Newtonian fluids of power-law type with zero Dirichlet boundary conditions when the surface of boundary is rough. Namely, it contains microscopic surface irregularities and an amplitude, and a wavelength of oscillations is described by a small parameter which converges to zero. Our aim is to derive effective boundary conditions - a wall law - on a smoothed boundary which gives a small approximation error. To this end we study corresponding boundary layer problem. This is a result of a joint research with David Gérad-Varet.

References

[1] Boundary Layer for a Non-Newtonian Flow over a Rough Surface. David Gérard-Varet, A. Wróblewska-Kamińska. arXiv:1511.01252

On Musiela equation for the forward rates

Jerzy Zabczyk

Instytut Matematyczny PAN,
Śniadeckich 8, Warszawa

Let $P(t,T)$ be the price, at moment $t \geqslant 0$, of a bond which pays 1 at (the maturity) time $T \geqslant t$. The forward rate process $r(t,x),\ t,x \geqslant 0$ is related to P by the formula:

$$P(t,T) = e^{-\int_0^{T-t} r(t,x)dx},\ \ T \geqslant t \geqslant 0.$$

In the paper [1] Musiela proposed a linear stochastic PDE as a model of the forward rates movements. In the talk we discuss a nonlinear version of the Musiela equation:

$$dr(t,x) = [\frac{\partial}{\partial x} r(t,x) + \frac{d}{dx} J(\int_0^x g(u,r(t,u))du)]dt + g(x,r(t-,x))dL(t)$$

where L is a stochastic process (Levy process), and J and g are functions. We concentrate on existence questions.

The presentation is based on a joint paper [2] with M. Barski.

References

[1]M. Musiela *Stochastic PDEs and term structure model.* 1993, Journees International de Finance, IGR-AFFI, La Baule

[2]M. Barski and J. Zabczyk, *Heath-Jarrow-Morton-Musiela equation with Levy perturbation.* JDE 253(2012), no.9, 2657-2697.

Incompressible congestions modelled by the compressible Navier-Stokes equations

Ewelina Zatorska

Imperial College London,
180 Queen's Gate, London SW7 2AZ, UK,
e.zatorska@imperial.ac.uk

We approximate a two–phase model used to describe traffic congestions by the compressible Navier-Stokes equations with a singular pressure term. The weak solutions of the latter approximate weak/measure-valued solutions to the compressible-incompressible two phase system: compressible for the density smaller than some barrier value and incompressible for the barrier value of the density. I will present a result in the multi-dimensional case with heterogeneous barrier [2] and a stronger results for the one-dimensional system [1].

References

[1] D. Bresch, C. Perrin, E. Zatorska: Singular limit of a Navier–Stokes system leading to a free/congested zones two-phase model.*C. R. Math. Acad. Sci. Paris* 352, no. 9, 685–690 (2014).

[2] C. Perrin, E. Zatorska: Free/Congested Two-Phase Model from Weak Solutions to Multi-Dimensional Compressible Navier–Stokes Equations. *Commun. PDEs*, 40: 1558–1589, (2015).

Elliptic problems with critical growth, the Hardy inequality and the existence for parabolic problems

Anna Zatorska-Goldstein

University of Warsaw
ul. Banacha 2, Warszawa
azator@mimuw.edu.pl

We consider a general two-weight Hardy-type inequality

$$K \int_{\Omega} |\xi|^p \omega_1(x) dx \leqslant \int_{\Omega} |\nabla \xi|^p \omega_2(x) dx,$$

discuss its relation with nonlinear elliptic problems with critical growth and connection to the existence problem for parabolic equations.

Symbolic dynamics (chaos) for Kuramoto-Sivashinsky PDE on the line - a computer assisted proof

Piotr Zgliczyński

For the Kuramoto-Sivashinsky PDE

$$u_t = -\nu u_{xxxx} - u_{xx} + (u^2)_x, \qquad \nu > 0 \tag{13}$$

on the line with odd and periodic boundary conditions with $\nu = 0.1212$ we give a computer assisted proof the existence of symbolic dynamics and countable infinity of periodic orbits.

Our approach is a mixture of rigorous numerics and topological methods and does not make use of any special features of Kuramoto-Sivashinsky PDE, or any global existence results nor spectral gap etc and therefore should be applicable to other systems of dissipative PDEs. The topological part exploits an apparent existence of transversal heteroclinic connections of two periodic orbits in both directions. The two approximate heteroclinic orbits connecting the periodic orbits are then used to obtain the topological horseshoe for some higher iterate of the Poincaré map.

This is a joint work with Daniel Wilczak.