Shifting martingale measures
and the slow birth of a bubble

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We discuss some recent developments in the probabilistic analysis of asset price bubbles. Such a bubble is usually described as a local martingale appearing on top of the perceived fundamental value of a liquid asset, defined as the expected sum of future discounted dividends under a given equivalent martingale measure. In this view, however, either there is a bubble right at the beginning, or there is none at all. We then report on joint work with Francesca Biagini and Sorin Nedelcu where, in order to capture the slow birth of a perceived bubble, we study a flow in the space of equivalent martingale measures and the corresponding shifting perception of the fundamental value. This allows us to describe the birth of a bubble as an initial submartingale which then turns into a supermartingale before it falls back to its initial value zero.