Expected log-utility maximization under incomplete information and with Cox-process observations

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Preliminary remarks

- We consider the classical problem of **maximization of expected terminal log-utility**.
- The main novelty is the market model:

1. The coefficients in the asset price dynamics depend on an unobservable finite-state Markovian factor process θ_t *(regime-switching model)*.
2. the asset prices (their log-values) are observed, and consequently the portfolio is re-balanced, only at doubly stochastic random times, for which the associated counting process forms a *Cox process* having an intensity that depends on the same unobservable factor process.

Preliminary remarks

Financial relevance of the model

- Regime switching *(widely used)* may account for various stylized facts, such as *volatility clustering*.
- Random time observations are more realistic in comparison with diffusion-type models, especially on small time scales: prices do not vary continuously but by tick-size at random times in reaction to *arrival of significant new information*.

Preliminary remarks

Financial relevance of the model (continued)

- Restricting observations and trading to random times corresponds to the fact that portfolios cannot be re-balanced continuously: think of transaction costs and/or liquidity restrictions.
- The partial information setup allows for continuous updating of the underlying model.

Outline

- Description of the model and the objective.
- *Remarks on the problem setup and on the approach.*
- Filtering and an ensuing contraction operator.
- *Preliminary results:*
  - i) *Auxiliary results in view of determining the optimal strategy;*
  - ii) *an auxiliary value function.*
- Main result.

The model

The model

- θ_t : the hidden finite-state Markovian factor process

  $$d\theta_t = Q^* \theta_t dt + dM_t, \quad \theta_t \in E := \{e_1, \cdots, e_N\}, \quad \theta_0 = \xi \in E$$

  Q : transition intensity matrix; M_t : jump martingale on $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$.

The model

- With $p_t := (p_t^1, \cdots, p_t^N)$ the state-probability vector, i.e. $p_t^i = P\{\theta_t = e_i\}$, we consider on

$$\mathcal{S}_N := \left\{ p \in \mathbb{R}^N \mid \sum_{i=1}^{N} p^i = 1 >;\ 0 \leq p^i\ ,\ i = 1, \cdots, N \right\}$$

the Hilbert metric

$$d_H(p, \bar{p}) := \log \left(\sup_{\bar{p}(A)>0, A\subset E} \frac{p(A)}{\bar{p}(A)} \sup_{p(A)>0, A\subset E} \frac{\bar{p}(A)}{p(A)} \right)$$

The model

- Given are m risky assets with prices S_t^i satisfying
$$dS_t^i = S_t^i\{r^i(\theta_t)dt + \sum_j \sigma_j^i(\theta_t)dB_t^j\}$$
and let $X_t^i := \log S_t^i$.
- Given is also a non-risky asset with price S_t^0 satisfying
$$dS_t^0 = r_0 S_t^0 dt$$
and let $\tilde{S}_t^i := \frac{S_t^i}{S_t^0}$, with $\tilde{X}_t^i := \log \tilde{S}_t^i$ so that
$$d\tilde{X}_t^i = \{r^i(\theta_t) - r_0 - d(\sigma\sigma^*(\theta_t))^i\}dt + \sum_{j=1}^m \sigma_j^i(\theta_t)dB_t^j$$
with $d(\sigma\sigma^*(\theta)) = (\frac{1}{2}(\sigma\sigma^*)^{11}(\theta), \ldots, \frac{1}{2}(\sigma\sigma^*)^{mm}(\theta))$ *(column vector)*.

The model

- Prices *(and thus also the logarithms of their discounted values)* are only observed at the random times $\tau_0, \tau_1, \tau_2, \cdots$ so that, putting $\tilde{X}^i_k := \tilde{X}^i_{\tau_k}$, the observations $(\tau_k, \tilde{X}_k)$ form a multivariate marked point process with counting measure

$$\mu(dt, dx) = \sum_k \mathbf{1}_{\{\tau_k < \infty\}} \delta_{\{\tau_k, \tilde{X}_k\}}(t, x) dtdx$$

- The corresponding counting process

$$\Lambda_t := \int_0^t \int_{\mathbb{R}^m} \mu(dt, dx)$$

is supposed to be a Cox process with intensity $n(\theta_t)$, i.e.

$$\Lambda_t - \int_0^t n(\theta_s) ds \quad \text{is an} \quad (\mathcal{F}_t, P) - \text{martingale.}$$

The model

Consider the $\mathcal{F}-$subfiltrations

$$\mathcal{G}_t := \mathcal{F}_0 \vee \sigma\{\mu((0,s] \times B) : s \leq t, B \in \mathcal{B}(\mathbb{R}^m)\},$$

$$\mathcal{G}_k := \mathcal{F}_0 \vee \sigma\{\tau_0, \tilde{X}_0, \tau_1, \tilde{X}_1, \tau_2, \tilde{X}_2, \ldots, \tau_k, \tilde{X}_k\}.$$

- Below we shall need the the conditional (on $\mathcal{F}^\theta$) mean and variance of $\tilde{X}_t - \tilde{X}_k$, for which we put

$$\begin{aligned} m_k^\theta(t) &= \int_{\tau_k}^t [r(\theta_s) - r_0\mathbf{1} - d(\sigma\sigma^*(\theta_s))]ds, \\ \sigma_k^\theta(t) &= \int_{\tau_k}^t \sigma\sigma^*(\theta_s)ds \end{aligned}$$

and we let, for $z \in \mathbb{R}^m$,

$$\rho^\theta_{\tau_k,t}(z) \sim N(z; m_k^\theta(t), \sigma_k^\theta(t))$$

.

Investment strategies, portfolios

- N_t^i : number of assets of type i in the portfolio at time t :

$$N_t^i = \sum_k \mathbf{1}_{[\tau_k, \tau_{k+1})}(t) N_{\tau_k}^i$$

The wealth process at time t is then $V_t := \sum_{i=0}^m N_t^i S_t^i$. and the investment ratios

$$h_t^i := \frac{N_t^i S_t^i}{V_t}, \qquad (h_k^i := h_{\tau_k}^i)$$

are defined on

$$\bar{H}_m := \{(h^1, \ldots, h^m); h^1 + h^2 + \ldots + h^m \leq 1, 0 \leq h^i, i = 1, 2, \ldots, m\}$$

$\longrightarrow$ *No shortselling is allowed and $\bar{H}_m$ is closed and bounded.*

Investment strategies, portfolios

- The dynamics of a self-financing portfolio are ($h_t \in \bar{H}_m$)

$$dV_t = V_t \{[r_0 + h_t^* \{r(\theta_t) - r_0 \mathbf{1}\}]dt + h_t^* \sigma(\theta_t) dB_t\}$$

Defining $\gamma : \mathbb{R}^m \times \bar{H}_m \to \bar{H}_m$ by

$$\gamma^i(z,h) := \frac{h^i \exp(z^i)}{1 + \sum_{i=1}^{m} h^i(\exp(z^i) - 1)} , \qquad i = 1, , \ldots , m$$

one has that, for $t \in [\tau_k, \tau_{k+1})$,

$$h_t^i = \gamma^i(\tilde{X}_t - \tilde{X}_k, h_k)$$

$\longrightarrow$ h_t is thus determined by $h_k, \tilde{X}_k, \tilde{X}_t$ where $\tilde{X}_t$ is unobserved for $t \in (\tau_k, \tau_{k+1})$.

Investment strategies, portfolios

- The set $\mathcal{A}$ of admissible strategies is

$$\mathcal{A} := \{\{h_k\}_{k=0}^{\infty} | h_k \in \bar{H}_m,\ \mathcal{G}_k \text{ measurable and self-financing}\}$$

- For $n > 0$ let

$$\mathcal{A}^n := \{h \in \mathcal{A} | h_{n+i} = h_{\tau_{n+i}-} \text{ for all } i \geq 1\}$$

→ Given $h \in \mathcal{A}^n$, for the corresponding process N_t one has *(recall that N_t is constant on $[\tau_k, \tau_{k+1})$)*

$$N_{n+k} = N_{n+k-1} = N_n$$

→ $\mathcal{A}^0 \subset \mathcal{A}^1 \subset \cdots \mathcal{A}^n \subset \mathcal{A}^{n+1} \cdots \subset \mathcal{A}$.

Investment strategies, portfolios

- Recalling the dynamics of a self financing portfolio we have

$$\begin{aligned}
\log V_T &= \log v_0 + \int_0^T h_t^* \sigma(\theta_t) dB_t \\
&\quad + \int_0^T [r_0 + h_t^* \{r(\theta_t) - r_0 \mathbf{1}\} - \tfrac{1}{2} h_t^* \sigma\sigma^*(\theta_t) h_t] dt \\
&= \log v_0 + \int_0^T h_t^* \sigma(\theta_t) dB_t + \int_0^T f(\theta_t, h_t)\, dt
\end{aligned}$$

having put

$$f(\theta, h) := r_0 + h^* \{r(\theta) - r_0 \mathbf{1}\} - \frac{1}{2} h^* \sigma\sigma^*(\theta) h$$

$\longrightarrow$ *Our problem can now be formulated as follows*

The problem

Problem: Given a finite planning horizon $T > 0$, determine the optimal value

$$\sup_{h\in\mathcal{A}} E\left\{\log V_T \middle| \tau_0 = 0, p_0 = p\right\}$$

$$= \log v_0 + \sup_{h\in\mathcal{A}} E\left\{\int_0^T f(\theta_t, h_t)dt \middle| \tau_0 = 0, p_0 = p\right\}$$

as well as an optimal maximizing strategy

$$\hat{h} \in \mathcal{A}$$

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- Main result.

Remarks on problem setup

- Our problem is a stochastic control problem under incomplete information. The standard approach to such problems is to transform them into a complete information problem, the so-called ”separated problem”, where instead of the unobservable quantities one considers their distributions, conditional on the observations.

- This requires:
  - i) solving the associated filtering problem;
  - ii) formulating the separated problem so that its solution is indeed a solution of the original incomplete information problem.

Remarks on problem setup

- The associated filtering problem has been solved in work by Cvitanic, Liptser, Rozovskii and it was found that *"the given problem does not fit into a standard diffusion or point process filtering framework"*.
- Not only the filtering problem, but also the control part of the problem does not fit into any standard framework and so there remained the task to find an approach also for the control part.

  $\longrightarrow$ Here we do it for a log-utility function. *(For power utility a different approach had to be derived: FNR in AMO (2013))*

Remarks on the approach

- We show that also in our setup one can obtain results that are analogous to the classical ones, in particular, we also obtain a myopic optimal policy for this log-utility problem.

This can however not be shown directly as in the classical cases and so we derive:

i) an approximation result leading to a ”value iteration”-type algorithm;

ii) a general dynamic programming principle

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Filtering

The filtering problem associated to our incomplete information stochastic control problem has been studied in Cvitanic, Liptser, Rozovskii (2006).

- To summarize their results, recall the following:

$\mu(dt, dx) = \sum_k \mathbf{1}_{\{\tau_k<\infty\}}\delta_{\{\tau_k,\tilde{X}_k\}}(t,x)dtdx$ counting measure

$n(\theta_t)$: intensity of the Cox process $\Lambda_t := \int_0^t \int_{\mathbb{R}^m} \mu(dt, dx)$

$\rho^{\theta}_{\tau_k,t}(z) \sim N(z; m^{\theta}_k(t), \sigma^{\theta}_k(t))$: distribution of $\tilde{X}_t - \tilde{X}_k$

Filtering

- Put (for $f(\theta)$ given)

$$\phi^{\theta}(\tau_k, t) = n(\theta_t)e^{-\int_{\tau_k}^{t} n(\theta_s)ds} \quad : \quad \text{distribution of inter-jump times}$$

$$\psi_k(f; t, x) := E\left\{f(\theta_t)\rho^{\theta}_{\tau_k,t}(x - \tilde{X}_k)\phi^{\theta}(\tau_k, t)|\sigma\{\theta_{\tau_k}\} \vee \mathcal{G}_k\right\}$$

$$\pi_t(\varphi(\theta_t, t, x)) := E\{\varphi(\theta_t, t, x) \mid \mathcal{G}_t\} \quad \text{(expectation w.r.to } \theta_t)$$

$$\tilde{\mathcal{P}}(\mathcal{G}) := \mathcal{P}(\mathcal{G}) \otimes \mathcal{B}(R^m) \quad \text{with} \quad \mathcal{P}(\mathcal{G})$$

the predictable σ – algebra on $\Omega \times [0, \infty)$ with respect to $\mathcal{G}$

Filtering

Lemma: The compensator of $\mu(dt, dx)$ w.r.to $\tilde{\mathcal{P}}(\mathcal{G})$ is

$$\nu(dt, dx) = \sum_k \mathbf{1}_{(\tau_k, \tau_{k+1}]}(t) \frac{\pi_{\tau_k}(\psi_k(1, t, x))}{\int_t^\infty \int_{\mathbb{R}^m} \pi_{\tau_k}(\psi_k(1, s, y)) dy ds} dt dx$$

Theorem: Given $f(\theta)$, the filter process $\pi_t(f) := E\{f(\theta_t) \mid \mathcal{G}_t\}$ satisfies (recall $\pi_0(\cdot) = p_0(\cdot)$)

$$d\pi_t(f) = \pi_t(Qf)dt$$

$$+ \int \sum_k \mathbf{1}_{(\tau_k, \tau_{k+1}]}(t) \left[\frac{\pi_{\tau_k}(\psi_k(f;t,x))}{\pi_{\tau_k}(\psi_k(1;t,x))} - \pi_{t-}(f) \right] (\mu - \nu)(dt, dx)$$

Filtering

- Since the observations take place only along $\tau_1, \tau_2, \cdots,$ useful information also arrives only along that sequence and we have

Corollary: At the generic jump time τ_{k+1}, noticing that $d\pi_t(f)|_{t=\tau_{k+1}} = \pi_{\tau_{k+1}}(f) - \pi_{\tau_{k+1}-}(f)$, we have then

$$\pi_{\tau_{k+1}}(f) = \left. \frac{\pi_{\tau_k}(\psi_k(f; t, x))}{\pi_{\tau_k}(\psi_k(1; t, x))} \right|_{t=\tau_{k+1}, x=\tilde{X}_{k+1}}$$

Filtering

- Being $\theta_t \in \{e_1, \cdots, e_N\}$, we have $f(\theta_t) = \sum_i f(e_i)\mathbf{1}_{e_i}(\theta_t)$. It thus suffices to consider $\pi_t^i = \pi\left(\mathbf{1}_{e_i}(\theta_t)\right)$ and it results that

$$\pi^i_{\tau_{k+1}} = M^i\left(\tau_{k+1} - \tau_k,\ \tilde{X}_{\tau_{k+1}} - \tilde{X}_{\tau_k}, \pi_{\tau_k}\right)$$

for suitable functions $M^i(\cdot)$ and with $\pi_{\tau_k} := \left(\pi^1_{\tau_k}, \cdots, \pi^N_{\tau_k}\right)$

$\longrightarrow$ *Putting $\pi_k = \pi_{\tau_k}$, we obtain the Markov process $\left\{\tau_k, \pi_{\tau_k}, \tilde{X}_{\tau_k}\right\}_{k=1}^{\infty}$ with respect to $\mathcal{G}_k$ that will turn out to be the state process for the "separated" (completely observed) control problem.*

A contraction operator

- Recall
  $\mathcal{S}_N := \left\{ p \in \mathbb{R}^N \mid \sum_{i=1}^N p^i = 1 >;\ 0 \le p^i\ ,\ i = 1, \cdots, N \right\}$
  with the Hilbert metric and let $\Sigma := [0, \infty) \times \mathcal{S}_N$

  $\longrightarrow$ Also the filter values $\pi_t = (\pi_t^1, \cdots, \pi_t^N) \in \mathcal{S}_N$

- Let $C_b(\Sigma)$ be the set of bounded continuous functions $g:\ \Sigma\ \rightarrow\ \mathbb{R}$ with norm $\|g\| := \max_{x\in\Sigma} \mid g(x) \mid$.
- Let $C_{b,lip}(\Sigma)$ be the set of bounded and Lipschitz continuous functions $g:\ \Sigma\ \rightarrow\ \mathbb{R}$ with norm $N^\lambda(g) := \lambda\|g\| + [g]_{lip}$

$\longrightarrow$ $C_{b,lip}(\Sigma)$ is a Banach space with norm $N^\lambda(g),\ \forall\lambda > 0.$

A contraction operator

Definition: Let $J: C_b(\Sigma) \to C_b(\Sigma)$ be the operator

$$Jg(\tau, \pi) = E\left\{g(\tau_1, \pi_1) 1_{\{\tau_1 < T\}} | \tau_0 = \tau, \pi_0 = \pi\right\}$$

Lemma 1: J is a contraction operator on $C_b(\Sigma)$ with contraction constant $c := 1 - e^{-\bar{n}T} < 1$, where $\bar{n} := \max n(\theta) = \max_i n(e_i)$.

Lemma 2: J is a contraction operator on $C_{b,lip}(\Sigma)$ having contraction constant $c' := (c + \max(\bar{n}, \frac{2}{\log 3})\frac{1}{\lambda})$ with λ large enough so that $c' < 1$.

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- *Remarks on the problem setup and on the approach also in relation to classical approaches to portfolio optimization.*
- Filtering and an ensuing contraction operator.
- *Preliminary results:*
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- Main result.

Preliminary to the optimal strategy

- Recall

$$E\{\log V_T | \tau_0 = t, p_0 = p\}$$
$$= \log V_t + E\left\{ \int_t^T f(\theta_s, h_s) ds | \tau_0 = t, p_0 = p \right\}$$

Definition: Let

$$\hat{C}(\tau, \pi, h) = E\left\{ \int_\tau^{T \wedge \tau_1} f(\theta_s, h_s) ds | \tau_0 = \tau, \pi_0 = \pi \right\}$$

Preliminary to the optimal strategy

Lemma: We have

i) $$E\left\{\int_t^T f(\theta_s, h_s)ds|\tau_0 = t, \pi_0 = \pi\right\} = E\left\{\sum_k \hat{C}(\tau_k, \pi_k, h_k)1_{\{\tau_k<T\}}|\tau_0 = t, \pi_0 = \pi\right\}$$

ii) $\hat{C}$ is bounded and continuous on $[0, T] \times \mathcal{S}_N \times \bar{H}_m$

iii) $\exists\ \hat{h}(\tau, \pi)$ s.t. $\sup_{h\in\bar{H}_m} \hat{C}(\tau, \pi, h) = \hat{C}(\tau, \pi, \hat{h}(\tau, \pi)) := C(t, \pi)$

iv) $C(t, \pi)$ is Lipschitz for the metric

$$|t - \bar{t}| + d_H(\pi, \bar{\pi}) + \sum_{i=1}^m |h^i - \bar{h}^i|$$

Preliminary to value function

The sum on the RHS in i) has to be considered to be infinite: although the number of observation times τ_k up to T is a.s. finite, it depends on ω.

$\longrightarrow$ *The optimal strategy will turn out to be myopic and given by a maximizer of the individual terms in the sum on the RHS in i). Due to the infinite sum, this however does not follow directly.*

Preliminary to value function

- The reformulation of the control problem by means of the functions $\hat{C}(\tau_k, \pi_k, h_k)$ is rather crucial by considering that we choose the strategy h_k only at the instants τ_k, while the portfolio proportion $h_t = \gamma(\tilde{X}_t - \tilde{X}_k, h_k)$ depends on the evolution of the security prices that, on each of the open intervals (τ_k, τ_{k+1}), is unobserved.
- *Furthermore, our criterion depends also on the unobservable state process θ_t.*

Preliminary to value function

- For $E\{\log V_T|\tau_0 = t, \pi_0 = \pi\}$ what matters is $E\left\{\int_t^T f(\theta_s, h_s)ds|\tau_0 = t, \pi_0 = \pi\right\}$.

Definition:

$$
\begin{aligned}
W(t,\pi,h.) &:= E\left\{\int_t^T f(\theta_s, h_s)ds|\tau_0 = t,\ \pi_0 = \pi\right\} \\
&= E\left\{\sum_{k=0}^{\infty} \hat{C}(\tau_k, \pi_k, h_k)1_{\{\tau_k<T\}}|\tau_0 = t,\ \pi_0 = \pi\right\}
\end{aligned}
$$

$$W(t,\pi) := \sup_{h\in\mathcal{A}} W(t,\pi,h)$$

$$W^n(t,\pi) := \sup_{h\in\mathcal{A}^n} W(t,\pi,h)$$

Preliminary to value function

- *Working directly with the above value function leads to various difficulties*

  $\longrightarrow$ Need an auxiliary value function

Recall that J is a contraction operator on $C_{b,lip}(\Sigma)$ with norm $N^{\lambda}(\cdot)$.

$\longrightarrow$ $\lim_{n\to\infty} \sum_{k=0}^{n} J^k\, C(t,\pi)$ exists where, we recall, $C(t,\pi) = \sup_{h\in \bar{H}_m} \hat{C}(\tau,\pi,h)$

Preliminary to value function

Definition: Let

$$\bar{W}(t,\pi) := \sum_{k=0}^{\infty} J^k C(t,\pi)$$

Lemma:

$$\bar{W}(t,\pi) = C(t,\pi) + J\bar{W}(t,\pi)$$

"Value iteration" for the auxiliary value function

- Recall that, for $t \in [\tau_k, \tau_{k+1})$ we have $h_t^i = \gamma^i(\tilde{X}_t - \tilde{X}_k, h_k)$

Definition: Let

$$\bar{W}^0(t,\pi,h) := E\left\{ \int_t^T f(\theta_s, \gamma(\tilde{X}_s - \tilde{X}_t, h))ds | \tau_0 = t,\ \pi_0 = \pi \right\}$$

which is bounded and continuous and define, recursively,

$$\begin{aligned}
\bar{W}^0(t,\pi) &:= \max_{h \in \bar{H}_m} \bar{W}^0(t,\pi,h) \\
\bar{W}^n(t,\pi) &:= C(t,\pi) + J\bar{W}^{n-1}(t,\pi) \\
&= \sum_{k=0}^{n-1} J^k C(t,\pi) + J^n \bar{W}^0(t,\pi)
\end{aligned}$$

"Value iteration" for the auxiliary value function

Lemma: We have

1. *(Will lead to the DP principle)*

$$\bar{W}^n(t,\pi) = E\left\{\sum_{k=0}^{n-1} C(\tau_k,\pi_k)1_{\{\tau_k<T\}} + \bar{W}^0(\tau_n,\pi_n)1_{\{\tau_n<T\}} \mid \tau_0 = t, \pi_0 = \pi\right\}$$

2. *(Will lead to the approximation result)*
   Recalling the Lipschitz constant c' in $C_{b,lip}(\Sigma)$, given $\epsilon > 0$, let $n_\epsilon := (\log(1-c') + \log\varepsilon - \log N^\lambda(\bar{W}^1 - \bar{W}^0))/\log c'$. Then

$$N^\lambda(\bar{W} - \bar{W}^n) < \epsilon \quad \forall\, n \geq n_\epsilon$$

Proposition: For all $n \geq 0$ we have

$$W^n(t,\pi) = \bar{W}^n(t,\pi)$$

Main theorem

1. "Approximation theorem". Given $\epsilon > 0$, let n_ϵ be as defined previously. Then

$$N^\lambda(W - \bar{W}^n) < \epsilon \quad \forall\, n \geq n_\epsilon$$

i.e. the recursive algorithm for computing $\bar{W}^n$ is a "value iteration algorithm" for the actual optimal value function W.

2. "Dynamic Programming Principle". For any $n > 0$

$$W(t,\pi) = \sup_{h \in \mathcal{A}^n} E\left\{ \sum_{k=0}^{n} \hat{C}(\tau_k, \pi_k, h_k) 1_{\{\tau_k < T\}} + W(\tau_{n+1}, \pi_{n+1}) 1_{\{\tau_{n+1} < T\}} \Big| \tau_0 = t, \pi_0 = \pi \right\}$$

Main theorem (contd.)

3. Optimal value and optimal strategy

- Given $V_0 = v_0, \tau_0 = 0, \pi_0 = \pi$ we have

$$\sup_{h \in \mathcal{A}} E\left\{\log V_T | \tau_0 = 0, \pi_0 = \pi\right\}$$

$$= \log v_0 + \sup_{h \in \mathcal{A}} E\left\{\int_0^T f(\theta_t, h_t)dt | \tau_0 = 0, \pi_0 = \pi]\right\}$$

$$= \log v_0 + C(0, \pi) + \sum_{k=1}^{\infty} E\left\{C(\tau_k, \pi_k)1_{\{\tau_k < T\}} | \tau_0 = 0, \pi_0 = \pi\right\}$$

Main theorem (contd.)

- The optimal strategy is given by

  i) for $t = \tau_k$: $\hat{h}_k = \hat{h}(\tau_k, \pi_{\tau_k})$ such that

  $$C(t,\pi) = \sup_{h \in \bar{H}_m} \hat{C}(\tau,\pi,h) = \hat{C}(\tau,\pi,\hat{h}(\tau,\pi))$$

  ii) for $t \in [\tau_k, \tau_{k+1})$: $\hat{h}^i_t = \gamma^i(\tilde{X}_t - \tilde{X}_k, \hat{h}_k)$

Thank you for your attention