A moment matching market implied calibration

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Motivation I

“A derivative pricing model is said to be calibrated to a set of benchmark instruments if the value of these instruments, computed in the model, correspond to their market prices.” (Encyclopedia of Quantitative Finance)

Google Inc (NASDAQ:GOOG) Add to portfolio

View options by expiration Nov 30, 2012

Layout: Stackec

Calls							Puts					
Price	Change	Bid	Ask	Volume	Open Int	Strike	Price	Change	Bid	Ask	Volume	Open Int
-	-	45.30	48.00	-	-	620.00	0.30	-0.05	0.20	0.40	78	-
-	-	40.30	42.70	-	-	625.00	0.40	-0.20	0.30	0.50	54	-
-	-	35.50	37.50	-	-	630.00	0.61	-0.19	0.45	0.70	166	-
-	-	30.70	32.70	-	-	635.00	0.84	-0.01	0.80	0.95	142	-
26.17	0.00	25.80	28.10	1	-	640.00	1.22	+0.02	1.10	1.25	246	-
-	-	22.50	23.60	-	-	645.00	1.78	-0.12	1.65	1.95	152	-
18.77	-0.23	18.00	19.00	81	-	650.00	2.60	0.00	2.55	2.85	128	-
14.97	+3.07	14.70	15.40	36	-	655.00	3.60	-1.00	3.50	3.90	358	-
11.50	0.00	11.10	11.80	328	-	660.00	5.20	-0.60	5.00	5.20	498	-
8.50	-0.20	8.20	8.80	251	-	665.00	7.00	-0.90	7.30	7.70	240	-
5.97	-0.03	6.00	6.40	718	-	670.00	9.80	+0.40	9.60	10.20	315	-
4.30	-0.10	4.10	4.50	446	-	675.00	12.90	-2.90	12.80	13.50	67	-
3.10	0.00	2.75	3.00	178	-	680.00	18.52	-1.08	16.50	17.40	7	-
2.00	+0.20	1.85	2.15	354	-	685.00	22.62	+0.55	20.20	21.30	1	-
1.38	-0.27	1.25	1.50	413	-	690.00	-	-	24.50	25.60	-	-
0.96	-0.19	0.85	1.00	146	-	695.00	34.00	0.00	29.00	30.20	1	-
0.60	-0.10	0.55	0.75	89	-	700.00	-	-	33.50	34.90	-	-
0.55	-0.05	0.40	0.60	59	-	705.00	-	-	38.30	39.70	-	-
0.39	-0.11	0.30	0.50	2	-	710.00	44.00	0.00	43.50	44.60	2	-
0.36	-0.07	0.25	0.40	6	-	715.00	-	-	48.30	50.10	-	-
0.28	-0.07	0.20	0.25	1	-	720.00	-	-	53.10	55.00	-	-

P_i

GOOG
665.87 -4.10 (-0.61%) 4:00PM EST
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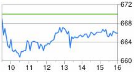

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$$\mathbf{p}^* : f(\{P_i\}, \{\hat{P}_i\}, \mathbf{p}^*) \leq f(\{P_i\}, \{\hat{P}_i\}, \mathbf{p}')$$

Motivation II

- Standard calibration problem: minimize distance f between market $\{P_i,\ i = 1, \dots, M\}$ and model prices $\{\hat{P}_i,\ i = 1, \dots, M\}$ of liquid derivatives
- typically perfect match not possible
- **optimal match**:

$$\mathbf{p}^* : f(\{P_i\}, \{\hat{P}_i\}, \mathbf{p}^*) \leq f(\{P_i\}, \{\hat{P}_i\}, \mathbf{p}'), \quad \mathbf{p}^*, \mathbf{p}' \in \mathbf{p}; \tag{1}$$

  where $\mathbf{p}$ = model parameter set
- common choice of f:

$$f(\{P_i\}, \{\hat{P}_i\}, \mathbf{p}) = \mathrm{RMSE}(\{P_i\}, \{\hat{P}_i\}, \mathbf{p}) = \sqrt{\sum_{i=1}^{M} \frac{\left(P_i - \hat{P}_i(\mathbf{p})\right)^2}{M}}$$

Motivation III

- use $\mathbf{p}^*$ to price exotic (illiquid) derivatives
- different $\mathbf{p}^*$ $\Rightarrow$ different prices for exotics and different hedge ratios

Sources of calibration risk:

- Which error function f?

> *'For complex products [...] banks must explicitly assess the need for valuation adjustments to reflect two forms of model risk: the model risk associated with using a possibly incorrect valuation methodology; and the* ***risk associated with using unobservable (and possibly incorrect) calibration parameters in the valuation model****." (Basel Committee on Banking Supervision)*

Motivation IV

- inverse problem (1)
  - ill-posed
  - instable solution for small changes in $\{P_i,\ i = 1, \ldots, M\} \Rightarrow$ significant impact on price of exotics/structured products
  - typically RMSE = non-convex function of **p** $\Rightarrow$ solution of (1) depends on initial values of **p**
  - computation time significant (need for numerical methods to compute model prices)
- potential solutions: relative entropy with respect to a prior model as
  - selection criterion
  - regularization of (1)

  But solution still depends on choice of the prior and on starting values of **p**

$\Rightarrow$ alternative methodology: **moment matching market implied calibration**

Moment matching market implied calibration: concept I

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0.60	-0.10	0.55	0.75	89	-	700.00	-	-	33.50	34.90	-	-
0.55	-0.05	0.40	0.60	59	-	705.00	-	-	38.30	39.70	-	-
0.39	-0.11	0.30	0.50	2	-	710.00	44.00	0.00	43.50	44.60	2	-
0.36	-0.07	0.25	0.40	6	-	715.00	-	-	48.30	50.10	-	-
0.28	-0.07	0.20	0.25	1	-	720.00	-	-	53.10	55.00	-	-

P(K,T)

C(K,T)

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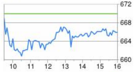

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$$\mathbb{E}\left[g(S_T)\right] = g(\kappa) + g'(\kappa)\left(\exp((r-q)T)S_0 - \kappa\right) + \exp(rT)\left(\int_0^{\kappa} g''(K)P(K,T)dK + \int_{\kappa}^{\infty} g''(K)C(K,T)dK\right) \Rightarrow \begin{cases} \text{variance} \\ \text{skewness} \\ \text{kurtosis} \\ \ldots \end{cases}$$

Moment matching market implied calibration: concept II

- $\mathbf{p}^* = \mathbf{p}$ compatible with market implied moments at one T
- market implied moments: vanilla option payoffs spanning formula (VIX, SKEW methodology)
- generalize CBOE methodology to extract 2nd to $(N+1)$th moments from option surface (N = size of $\mathbf{p}$)
- match 2nd to $(N+1)$th model and market implied moments
- $\Rightarrow$ moment matching calibration problem $\equiv$ system of N algebraic equations which gives directly $\mathbf{p}^*$ in terms of the moments observed in the market
- **Applications:**
    - calibration on one maturity
    - deriving starting values for standard calibration problem
    - deriving prior model

Extracting moments from option surface I

- any twice differentiable payoff can be expanded as a weighted sum of vanilla option payoffs:

$$\mathbb{E}\left[g(S_T)\right] = \quad g(\kappa) + g'(\kappa)\exp(rT)\left(C(\kappa, T) - P(\kappa, T)\right) + \exp(rT) \left(\int_0^\kappa g''(K)P(K,T)dK + \int_\kappa^\infty g''(K)C(K,T)dK\right), \quad (2)$$

  where $C(K, T)$ and $P(K, T)$ are prices of European call and put options with maturity T and strike K
- κ is some strike level
- $\Rightarrow$ closed-form expression for Nth moment of $X_T = \log\left(\frac{S_T}{S_0}\right)$:

$$g(S_T) = \left(\log\left(\frac{S_T}{S_0}\right)\right)^N$$

Extracting moments from option surface II

- For $N \geq 2$

$$\mathbb{E}\left[\left(\log\left(\frac{S_T}{S_0}\right)\right)^N\right] = \left(\log\left(\frac{K_0}{S_0}\right)\right)^N + N\left(\log\left(\frac{K_0}{S_0}\right)\right)^{N-1}\left(\frac{F_0}{K_0}-1\right) + \exp(rT) \left(\int_0^{K_0} \frac{N}{K^2}\left((N-1)\left(\log\left(\frac{K}{S_0}\right)\right)^{N-2} - \left(\log\left(\frac{K}{S_0}\right)\right)^{N-1}\right)P(K,T)dK + \int_{K_0}^{\infty} \frac{N}{K^2}\left((N-1)\left(\log\left(\frac{K}{S_0}\right)\right)^{N-2} - \left(\log\left(\frac{K}{S_0}\right)\right)^{N-1}\right)C(K,T)dK\right)$$

- Vanilla options only traded for a discrete set of strikes

$$\mathbb{E}\left[\left(\log\left(\frac{S_T}{S_0}\right)\right)^N\right] = \left(\log\left(\frac{K_0}{S_0}\right)\right)^N + N\left(\log\left(\frac{K_0}{S_0}\right)\right)^{N-1}\left(\frac{F_0}{K_0}-1\right) + \exp(rT)N \sum_{i=1}^{M} \frac{\Delta K_i}{K_i^2}\left((N-1)\left(\log\left(\frac{K_i}{S_0}\right)\right)^{N-2} - \left(\log\left(\frac{K_i}{S_0}\right)\right)^{N-1}\right)Q(K_i), \quad (3)$$

Matching of standardized moments I

- Market implied variance v, skewness s and kurtosis k:

$$\begin{cases} v &= \mathbb{E}\left[X_T^2\right] - \left(\mathbb{E}\left[X_T\right]\right)^2 \\ s &= \frac{\mathbb{E}\left[X_T^3\right] - 3\mathbb{E}[X_T]\mathbb{E}\left[X_T^2\right] + 2(\mathbb{E}[X_T])^3}{(\mathrm{Var}(X_T))^{3/2}} \\ k &= \frac{\mathbb{E}\left[X_T^4\right] - 4\mathbb{E}[X_T]\mathbb{E}\left[X_T^3\right] + 6(\mathbb{E}[X_T])^2\mathbb{E}\left[X_T^2\right] - 3(\mathbb{E}[X_T])^4}{(\mathrm{Var}(X_T))^2}, \end{cases}$$

- closed-form formula for the model moments if characteristic function of $X_T = \log\left(\frac{S_T}{S_0}\right)$ known in closed-form:

$$\mathbb{E}\left[X_T^N\right] = \mathrm{i}^{-N} \frac{d^N}{du^N} \phi_{X_T}(u)|_{u=0} \tag{4}$$

- $\phi_{X_T}(u)$ available for a wide range of asset pricing models

Matching of standardized moments II

- infer $\mathbf{p}^*$ by matching 2nd to $(N+1)$th model and market implied moments/standardized moments
- mean of S_T adjusted beforehand

Examples: Lévy driven models

- Stock price model

$$S_t = \frac{S_0 \exp((r-q)t + X_t)}{\mathbb{E}_{\mathbb{Q}}[\exp(X_t)]} = S_0 \exp((r-q+\omega)t + X_t) \quad (5)$$

  where $X = \{X_t, t \geq 0\}$ is a Lévy process
- $\omega = -\log(\phi_X(-\mathrm{i}))$ (convexity correction)
- characteristic function of X_T known in closed-form for many popular Lévy processes
- Numerical study for Meixner model (VG and NIG models: see paper)

Moment calibration problem

- Exponential VG, NIG and Meixner models: 3 parameters calibrated by solving

$$\begin{cases} \mathrm{Var}^{\mathrm{Market}}\left(X_T\right) & = \mathrm{Var}^{\mathrm{Model}}\left(X_T\right) \\ \mathrm{Skewness}^{\mathrm{Market}}\left(X_T\right) & = \mathrm{Skewness}^{\mathrm{Model}}\left(X_T\right) \\ \mathrm{Kurtosis}^{\mathrm{Market}}\left(X_T\right) & = \mathrm{Kurtosis}^{\mathrm{Model}}\left(X_T\right) \end{cases} \tag{6}$$

  if (6) admits a solution which satisfies the domain conditions of the model parameter set **p**
- If (6) admits no solution $\Rightarrow$ modified moment calibration problem (adjust market implied skewness and kurtosis)

Meixner model I

Characteristic function of the Meixner distribution **Meixner**(α, β, δ) with parameters $\alpha > 0$, $\beta \in]-\pi, \pi[$ and $\delta > 0$

$$\phi_{\text{Meixner}}(u; \alpha, \beta, \delta) = \left(\frac{\cos\left(\frac{\beta}{2}\right)}{\cosh\left(\frac{\alpha u - \mathrm{i}\beta}{2}\right)} \right)^{2\delta}$$

	$X_1 \sim$ **Meixner**(α, β, δ)	$X_T \sim$ **Meixner**$(\alpha, \beta, \delta T)$
mean	$\alpha\delta \tan\left(\frac{\beta}{2}\right)$	$\alpha\delta T \tan\left(\frac{\beta}{2}\right)$
variance	$\frac{\alpha^2\delta}{2\cos^2\left(\frac{\beta}{2}\right)}$	$\frac{\alpha^2\delta T}{2\cos^2\left(\frac{\beta}{2}\right)}$
skewness	$\sin\left(\frac{\beta}{2}\right)\sqrt{\frac{2}{\delta}}$	$\sin\left(\frac{\beta}{2}\right)\sqrt{\frac{2}{\delta T}}$
kurtosis	$3 + \frac{2-\cos(\beta)}{\delta}$	$3 + \frac{2-\cos(\beta)}{\delta T}$

Meixner model II

system (6) reduces to

$$\begin{cases} \beta = 2\arctan\left(\operatorname{sign}(s)\sqrt{\frac{s^2}{2(k-3)-3s^2}}\right) \\ \alpha = \sqrt{v}s\cot\left(\frac{\beta}{2}\right) \\ \delta = \frac{2-\cos(\beta)}{T(k-3)}. \end{cases} \tag{7}$$

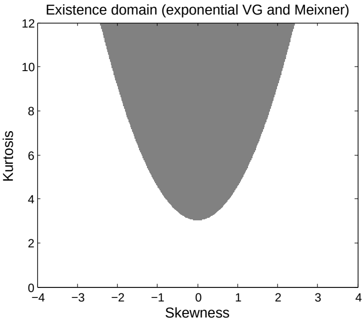
- Meixner system (7) admits a solution $\{\alpha > 0, \beta \in \left]-\pi, \pi\right[, \delta > 0\}$ iff

$$\mathbf{6 + 3s^2 - 2k < 0.} \tag{8}$$

- existence domain of the Meixner moment calibration problem independent of the market implied variance
- only depends on the market implied skewness and kurtosis

Moment calibration problem: existence domain

$$6 + 3s^2 - 2k < 0 \quad \supset \quad 9 + 5s^2 - 3k < 0$$



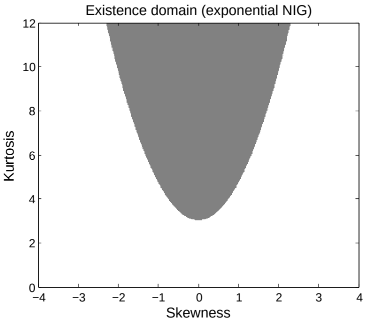


Figure : Existence domain for the moment calibration problem (6) under the VG and Meixner exponential models (left) and under the NIG exponential model (right).

Modified moment calibration problem I

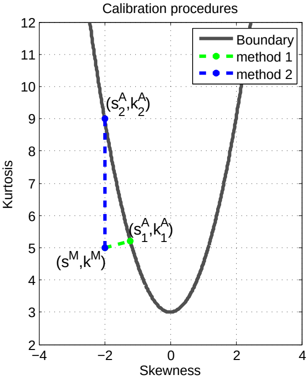


Figure : Calibration methods.

Method 1: least squares moment calibration problem

- replace (s^M, k^M) by $(s_1^A, k_1^A + \epsilon)$
- minimize distance between adjusted and market implied couples skewness-kurtosis
- $\equiv$ allocating same importance to skewness and kurtosis matching in a standard calibration optimizer

Modified moment calibration problem II

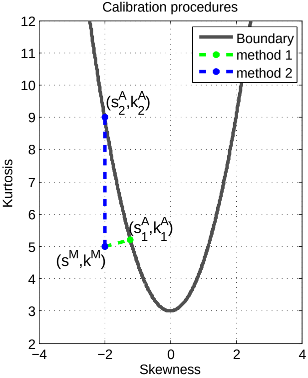


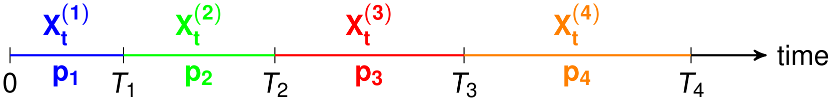
Figure : Calibration methods.

Method 2: matching the skew

- replace (s^M, k^M) by $(s_2^A = s^M, k_2^A + \epsilon)$
- match market implied skewness
- adjust kurtosis

Bootstrapping market implied moment matching calibration I

- Calibration of Markov models with piecewise constant parameters between successive quoted option maturities
- exploit additive property of cumulants of independent random variables
- solve M independent moment matching systems of N equations, where M is the number of maturities
- Examples: Lévy models with piecewise constant parameters



Examples: term structure Lévy models I

- Stock price model:

$$S_t = S_{T_{i-1}} \exp\left((r - q + \omega_i)(t - T_{i-1}) + X^{(i)}_{t-T_{i-1}}\right), \quad t \in [T_{i-1}, T_i)$$

  where $X^{(i)} = \left\{X^{(i)}_t, t \geq 0\right\}, \; i = 1, \ldots, M$ are independent Lévy processes

- $T_0 \equiv 0$
- $\omega_i = -\log\left(\phi_{X^{(i)}_1}(-\mathrm{i})\right), \; i = 1, \ldots M$ (convexity corrections)

$$\begin{aligned} S_t = \quad & S_0 \prod_{j=1}^{i-1} \exp\left((r - q + \omega_j)(T_j - T_{j-1}) + X^{(j)}_{T_j - T_{j-1}}\right) \\ & \exp\left((r - q + \omega_i)(t - T_{i-1}) + X^{(i)}_{t-T_{i-1}}\right), \quad t \in [T_{i-1}, T_i] \,. \end{aligned}$$

Matching of the subprocesses standardized moments I

- match market implied and model moments of the subprocesses $X_t^{(i)},\ t \in [T_{i-1},\ T_i)\,,\ i = 1, 2, \ldots, M$
- market implied formula (3): approximation of the moments of X_t over the periods $[0, T_i]$
- need for similar approximations for the moments of X_t over the subperiods $[T_{i-1}, T_i], i = 1, \ldots, M$
- $\Rightarrow$ consider cumulants as latent variables
- compute recursively the cumulants of $X^{(i)},\ i = 1, \ldots, M$:

$$\kappa_n\left(X^{(i)}_{T_i - T_{i-1}}\right) = \kappa_n\left(X_{T_i}\right) - \sum_{j=1}^{i-1} \kappa_n\left(X^{(j)}_{T_j - T_{j-1}}\right), \quad n = 2, \ldots, N+1$$

Matching of the subprocesses standardized moments II

- Market implied variance v_i, skewness s_i and kurtosis k_i, $i = 1, \ldots, M$:

$$\begin{cases} v_i = \kappa_2\left(X^{(i)}_{T_i - T_{i-1}}\right) \\ s_i = \frac{\kappa_3\left(X^{(i)}_{T_i - T_{i-1}}\right)}{v_i^{3/2}} \\ k_i = \frac{\kappa_4\left(X^{(i)}_{T_i - T_{i-1}}\right)}{v_i^2} + 3, \end{cases}$$

Matching of the subprocesses standardized moments III

- solve successively the moment system

$$\begin{cases} \mathrm{Var}^{\mathrm{Model}}\left(X^{(i)}_{T_i - T_{i-1}}\right) & = v_i \\ \mathrm{Skewness}^{\mathrm{Model}}\left(X^{(i)}_{T_i - T_{i-1}}\right) & = s_i \\ \mathrm{Kurtosis}^{\mathrm{Model}}\left(X^{(i)}_{T_i - T_{i-1}}\right) & = k_i \end{cases} \qquad i = 1, \ldots, M \qquad (9)$$

  if (9) admits a solution which satisfies the domain conditions of the parameter set $\mathbf{p_i}$ of the subprocess $X^{(i)}$

- if (9) admits no solution $\Rightarrow$ modified moment calibration problem (i.e. replace s_i by $s_i^{A_1}$ (or $s_i^{A_2}$) and k_i by $k_i^{A_1}$ (or $k_i^{A_2}$))

Numerical study: calibration on one maturity I

- Comparison with standard calibration problem: Carr-Madan formula

$$\hat{C}(K,T) = \frac{\exp(-\alpha \log(K))}{\pi} \int_0^{+\infty} \exp\big(-\mathrm{i}v \log(K)\big)\varrho(v)\mathrm{d}v,$$

where

$$\varrho(v) = \frac{\exp(-rT)\mathbb{E}[\exp(\mathrm{i}(v-(\alpha+1)\mathrm{i})\log(S_T))]}{\alpha^2+\alpha-v^2+\mathrm{i}(2\alpha+1)v}$$

- calibration of VG, NIG and Meixner models on each T-set of S&P 500 options quoted on 30/10/09

Numerical study: calibration on one maturity II

Table : Standardized moments - Meixner exponential model (30/10/2009)

	Market			Moment matching (1)			Moment matching (2)			Carr-Madan		
T	Var	Skew	Kurt	Var	Skew	Kurt	Var	Skew	Kurt	Var	Skew	Kurt
0.01918	0.002520	-1.6507	7.4833	0.002520	-1.6507	7.4833	0.002520	-1.6507	7.4833	0.002405	-1.6803	9.2223
0.06027	0.006019	-2.1681	11.1906	0.006019	-2.1681	11.1906	0.006019	-2.1681	11.1906	0.005911	-2.0472	11.5531
0.13699	0.012751	-2.0766	11.2002	0.012751	-2.0766	11.2002	0.012751	-2.0766	11.2002	0.012356	-1.8071	9.3359
0.16986	0.015881	-1.7732	8.4249	0.015881	-1.7732	8.4249	0.015881	-1.7732	8.4249	0.015455	-1.7920	9.1876
0.21370	0.019710	-2.1268	11.7858	0.019710	-2.1268	11.7858	0.019710	-2.1268	11.7858	0.018796	-1.7759	8.9225
0.30959	0.027764	-1.6737	7.2763	0.027764	-1.6879	7.3735	0.027764	-1.6737	7.3020	0.027713	-1.7469	8.5468
0.38630	0.037422	-2.1225	11.5692	0.037422	-2.1225	11.5692	0.037422	-2.1225	11.5692	0.035498	-1.7840	8.7435
0.41644	0.039691	-1.7508	7.7691	0.039691	-1.7508	7.7691	0.039691	-1.7508	7.7691	0.038818	-1.8079	8.8607
0.63562	0.064212	-1.9973	9.8895	0.064212	-1.9973	9.8895	0.064212	-1.9973	9.8895	0.061368	-1.7922	8.6016
0.66575	0.065620	-1.6770	6.9391	0.065620	-1.6228	7.0502	0.065620	-1.6770	7.3183	0.065325	-1.8128	8.6972
0.88493	0.092286	-1.9473	9.2353	0.092286	-1.9473	9.2353	0.092286	-1.9473	9.2353	0.089373	-1.8171	8.6485
0.91781	0.085001	-1.2456	4.2732	0.085001	-0.9570	4.4737	0.085001	-1.2456	5.4275	0.093292	-1.8115	8.5731
1.13425	0.115564	-1.6214	6.6959	0.115564	-1.5719	6.8064	0.115564	-1.6214	7.0433	0.118806	-1.8296	8.6449
1.63288	0.172688	-1.6207	6.6222	0.172688	-1.5569	6.7358	0.172688	-1.6207	7.0398	0.177114	-1.8041	8.4365
2.13151	0.249946	-2.1216	11.7481	0.249946	-2.1216	11.7481	0.249946	-2.1216	11.7481	0.239557	-1.7952	8.3330

Numerical study: calibration on one maturity III

Table : Optimal parameter set - Meixner exponential model (30/10/2009)

	Moment matching (1)				Moment matching (2)				Carr-Madan			
T	α	β	δ	RMSE	α	β	δ	RMSE	α	β	δ	RMSE
0.01918	0.0447	-2.1523	29.6489	0.7277	0.0447	-2.1523	29.6489	0.7277	0.0978	-1.4006	15.3403	0.3260
0.06027	0.1171	-1.9251	4.7539	0.6017	0.1171	-1.9251	4.7539	0.6017	0.1637	-1.5316	3.8035	0.3432
0.13699	0.2102	-1.6801	1.8775	0.5381	0.2102	-1.6801	1.8775	0.5381	0.1885	-1.6345	2.3777	0.2808
0.16986	0.1500	-1.9593	2.5814	0.8985	0.1500	-1.9593	2.5814	0.8985	0.2058	-1.6499	1.9780	0.2694
0.21370	0.2809	-1.6319	1.0978	0.7772	0.2809	-1.6319	1.0978	0.7772	0.2117	-1.7103	1.6901	0.3269
0.30959	0.0745	-2.6236	2.1188	1.2492	0.0745	-2.6194	2.1524	1.2647	0.2318	-1.7959	1.2947	0.2859
0.38630	0.3682	-1.6795	0.6370	1.0458	0.3682	-1.6795	0.6370	1.0458	0.2624	-1.8160	1.0108	0.3174
0.41644	0.1166	-2.4964	1.4093	1.5078	0.1166	-2.4964	1.4093	1.5078	0.2727	-1.8349	0.9264	0.3255
0.63562	0.3410	-1.9557	0.5425	0.8997	0.3410	-1.9557	0.5425	0.8997	0.3101	-1.9222	0.6584	0.3617
0.66575	0.1146	-2.6038	1.0602	2.0610	0.1146	-2.6204	0.9973	1.8868	0.3167	-1.9423	0.6230	0.4185
0.88493	0.3178	-2.1556	0.4625	1.1760	0.3178	-2.1556	0.4625	1.1760	0.3526	-1.9902	0.4816	0.3684
0.91781	0.1304	-2.2673	1.9529	6.5955	0.1304	-2.4522	1.2440	3.4699	0.3485	-2.0174	0.4754	0.4654
1.13425	0.1520	-2.5872	0.6602	2.2549	0.1520	-2.6033	0.6233	1.9455	0.3850	-2.0455	0.3838	0.3490
1.63288	0.1858	-2.5822	0.4668	2.8691	0.1858	-2.6031	0.4333	2.2391	0.4430	-2.0852	0.2807	0.5603
2.13151	0.9989	-1.6308	0.1105	4.7591	0.9989	-1.6308	0.1105	4.7591	0.4889	-2.1260	0.2223	0.5409

Numerical study: calibration on one maturity IV

- v can always be reproduced under moment calibration problem (existence domain independent of v)
- standard calibration: difference between model and market implied var per annum of up to > 15 % (max $\frac{\Delta\sigma}{\sigma} = 0.07336$)
- v, s and k can be matched for a wider set of T's under VG and Meixner models
- if $(s, k) \notin$ existence domain, s and k still fitted relatively well
- standard calibration: significant discrepancies between model and market implied moments
- if $\nexists$ solution for moment calibration problem (6), method 2 for the modified moment calibration problem gives a better fit of the volatility curve than method 1
- moment calibration procedure: more than 5000 x faster

Numerical results: term structure Lévy models I

Table : Optimal parameter set - Meixner term structure model

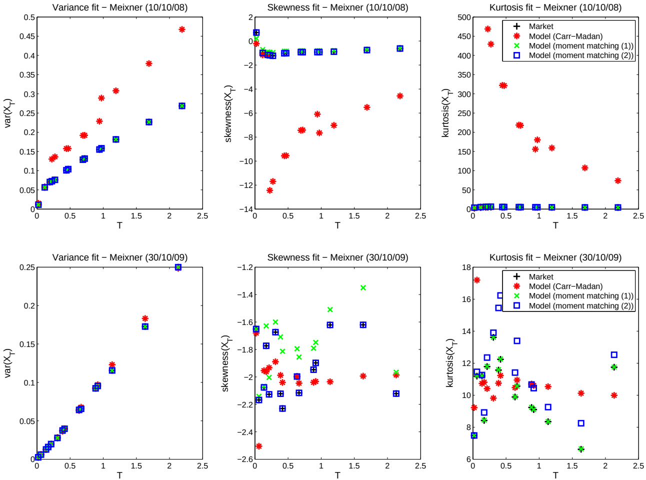
	(10/10/2008)											
	Moment matching (1)				Moment matching (2)				Carr-Madan			
T_i	α_i	β_i	δ_i	RMSE(T_i)	α_i	β_i	δ_i	RMSE(T_i)	α_i	β_i	δ_i	RMSE(T_i)
0.02192	0.0477	0.8889	371.9023	3.2043	0.0477	2.0103	131.0694	3.1049	3.8389e-06	-3.1413	1858.2444	0.9678
0.11781	0.0950	-2.3265	16.3814	3.1087	0.0950	-2.5529	8.7770	1.8415	5.7077e-06	-3.1416	6.6035	1.2898
0.19452	0.0529	-2.9396	1.3250	3.4731	0.0529	-2.9644	1.0203	1.8484	0.00019	-3.1407	1.4830	1.3635
0.22466	0.0210	-3.0698	0.4274	3.6241	0.0210	-3.0860	0.2565	2.1666	0.0117	-3.1391	0.0459	1.5801
0.27123	0.0261	-3.0503	0.4472	2.7271	0.0261	-3.0719	0.2606	2.1430	8.5556e-05	-3.1413	0.9918	1.2528
0.44384	0.5148	-0.9929	0.8394	4.6613	0.5148	-0.9929	0.8394	3.2051	5.9155e-09	-3.1416	1.1745	1.4177
0.47123	0.4138	-1.5291	0.6777	7.1605	0.4138	-1.5291	0.6777	4.8147	0.00017	3.1397	0.3577	1.6839
0.69315	0.5182	-1.2038	0.5592	6.6668	0.5182	-1.2038	0.5592	5.1796	2.1703e-06	-3.1416	0.4780	1.1667
0.72055	0.0242	-3.0392	0.9565	8.5937	0.0242	-3.0531	0.7138	6.6603	1.1912e-05	-3.1414	2.4245	1.2551
0.94247	0.0688	-2.8534	0.9291	7.2809	0.0688	-2.8768	0.7853	5.6085	2.3557e-05	-3.1416	0.2465	1.4430
0.97260	0.0253	-3.0351	0.9390	8.6631	0.0253	-3.0375	0.8988	6.5982	0.00239	-3.1410	0.0532	1.6346
1.19178	0.2454	-2.1509	0.7925	8.3192	0.2454	-2.1509	0.7925	6.7103	1.8079e-07	-3.1416	0.5224	1.4242
1.69041	0.7415	-0.7305	0.2876	11.1553	0.7415	-0.7305	0.2876	9.8979	0.00016	-3.1413	0.1836	1.2781
2.18904	0.8963	-0.2863	0.2049	14.9862	0.8963	-0.2863	0.2049	13.8745	0.9077	-2.2778	0.0757	1.0313

Numerical results: term structure Lévy models II

Table : Optimal parameter set - Meixner term structure model

	(30/10/2009)											
	Moment matching (1)				Moment matching (2)				Carr-Madan			
T_i	α_i	β_i	δ_i	RMSE(T_i)	α_i	β_i	δ_i	RMSE(T_i)	α_i	β_i	δ_i	RMSE(T_i)
0.01918	0.0447	-2.1523	29.6489	0.7277	0.0447	-2.1523	29.6489	0.7277	0.0978	-1.4006	15.3403	0.3260
0.06027	0.0265	-2.9087	3.2854	0.7523	0.0265	-2.9123	3.1855	0.7398	0.2225	-1.7623	1.4978	0.2306
0.13699	0.2215	-1.8492	1.2966	0.4741	0.2215	-1.8492	1.2966	0.4781	0.1983	-1.7627	1.7163	0.1936
0.16986	0.0250	-2.6005	21.7287	1.4148	0.0250	-2.8632	5.8566	1.0197	0.2742	-1.8224	0.9737	0.1716
0.21370	0.0277	-3.0489	0.4895	0.6365	0.0277	-3.0510	0.4682	0.5748	0.2389	-1.9855	0.8130	0.2270
0.30959	1.0124	-0.4481	0.1558	1.8192	1.0124	-0.4481	0.1558	1.6482	0.2481	-2.0804	0.7869	0.1848
0.38630	0.0439	-2.9711	0.9447	1.4873	0.0439	-3.0303	0.4030	1.3052	0.3605	-2.0493	0.4455	0.2074
0.41644	0.0213	-3.0924	0.2004	1.3591	0.0213	-3.0990	0.1504	1.8268	0.3774	-2.1533	0.3762	0.2077
0.63562	0.1655	-2.6087	0.5668	1.2822	0.1655	-2.6087	0.5668	0.7638	0.3400	-2.1966	0.3815	0.2714
0.66575	0.0168	-3.1177	0.0472	1.0700	0.0168	-3.1262	0.0196	1.6041	0.4212	-2.3171	0.2614	0.2838
0.88493	0.1135	-2.8259	0.4664	1.4475	0.1135	-2.8259	0.4664	0.5182	0.3907	-2.2903	0.2552	0.3055
0.91781	1.0154	-0.7865	0.1554	1.7364	1.0154	-0.7865	0.1554	0.9917	0.2987	-2.4977	0.2868	0.3507
1.13425	1.0612	-0.6984	0.1463	4.0894	1.0612	-0.6984	0.1463	3.4946	0.4263	-2.3457	0.1991	0.4186
1.63288	0.1069	-2.8193	0.5163	6.4989	0.1069	-2.9105	0.2666	3.1357	0.5124	-2.2961	0.1546	0.6294
2.13151	0.4695	-2.6635	0.0788	4.0452	0.4695	-2.6635	0.0788	1.6533	0.5002	-2.4426	0.1227	0.6334

Numerical results: term structure Lévy models III



Numerical results: term structure Lévy models IV

Table : Average precision and computation time for the different calibration methodologies.

	Meixner		
	Carr-Madan	moment matching (1)	moment matching (2)
RMSE	0.4209	3.9842	2.3644
variance RMSE	0.0869	0	0
skewness RMSE	1.1108	0.1876	0
kurtosis RMSE	22.3479	0.0563	2.3616
cpu (sec)	77.7178	0.000425	0.000098

Conclusion

- market implied calibration based on moment matching
- extracting moments from option surface
- calibration in terms of closed-form formulae only $\Rightarrow$
  - no need of starting value for the model parameters
  - avoids to be stuck in local minima
  - almost instantaneous calibration
- applications
  - calibration on one maturity curve
  - calibration on whole set of maturities: bootstrapping calibration
  - providing starting values/prior model for standard calibration problems

papers available at http://ssrn.com/abstract=2021466 and http://ssrn.com/abstract=2255558

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