

Funding, Credit, Collateral and Hedging: Arbitrage Free Valuation under non-separable risks

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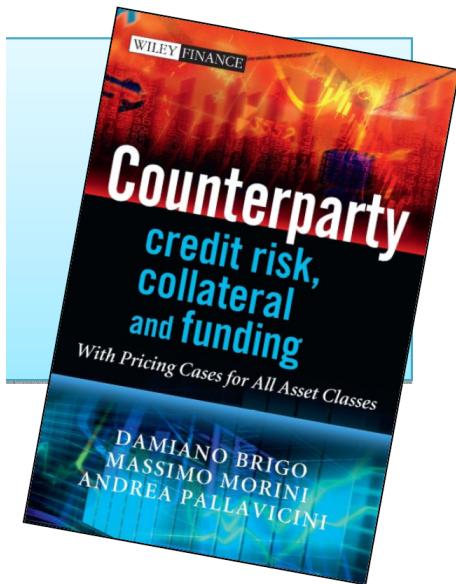
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Agenda I

- 1 Pre-funding subtleties and Payout risk
 - Bilateral Counterparty Risk: CVA and DVA
 - DVA Hedging? DVA as Funding Benefit?
 - Risk Free Closeout or Replacement Closeout?
 - Can we neglect first to default risk?
 - Payoff Risk
- 2 Counterparty Credit Risk and Collateral Margining
 - Collateralization, Gap Risk and Re-Hypothecation
- 3 Adding Collateral Margining Costs and Funding rigorously
 - Risk-Neutral Modelling of Bilateral CVA with Margining
 - The recursive non-decomposable nature of adjusted prices
 - Funding Costs, CVA Desk and Bank Structure
- 4 Conclusions and References

Presentation based on the Forthcoming Book



An online colloquial survey

For an introductory dialogue on Counterparty Risk, illustrating the themes of the book, see

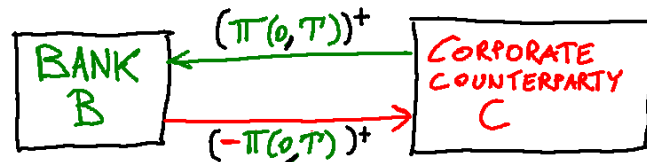
CVA Q&A

D. Brigo (2012). Counterparty Risk FAQ: Credit VaR, CVA, DVA, Closeout, Netting, Collateral, Re-hypothecation, Wrong Way Risk, Basel, Funding, and Margin Lending. SSRN.com, arXiv.org.

See also References at the end of this presentation.

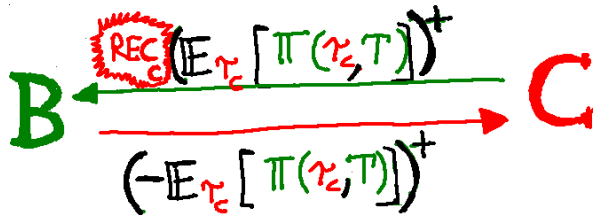
Let's start by introducing COUNTERPARTY CREDIT RISK

Context



π :
PORTFOLIO
CASH FLOWS
TO B

τ_C
DEFAULT OF C
CLOSEOUT:



The case of symmetric counterparty risk

$$\mathbb{E}_t \left\{ \Pi_B^D(t, T) \right\} = \mathbb{E}_t \left\{ \Pi_B(t, T) \right\} + \text{DVA}_B(t) - \text{CVA}_B(t)$$

$$\text{DVA}_B(t) = \mathbb{E}_t \left\{ \text{LGD}_B \cdot \mathbf{1}(t < \tau^{\text{1st}} = \tau_B < T) \cdot D(t, \tau_B) \cdot [-\text{NPV}_B(\tau_B)]^+ \right\}$$

$$\text{CVA}_B(t) = \mathbb{E}_t \left\{ \text{LGD}_C \cdot \mathbf{1}(t < \tau^{\text{1st}} = \tau_C < T) \cdot D(t, \tau_C) \cdot [\text{NPV}_B(\tau_C)]^+ \right\}$$

- Obtained simplifying a first principles accounting cash flows formula and taking expectation. Almost no assumption.
- 2nd term : adj due to scenarios $\tau_B < \tau_C$. This is positive to the investor/ Bank B and is called "Debit Valuation Adjustment" (DVA)
- 3d term : Counterparty risk adj due to scenarios $\tau_C < \tau_B$
- Bilateral Valuation Adjustment as seen from B :

$$\text{BVA}_B = \text{DVA}_B - \text{CVA}_B.$$
- If computed from the opposite point of view of "C" having counterparty "B", $\text{BVA}_C = -\text{BVA}_B$. Symmetry.

The case of symmetric counterparty risk

Strange consequences of the formula new mid term, i.e. DVA

- credit quality of investor WORSENS \Rightarrow books POSITIVE MARK TO MKT
- credit quality of investor IMPROVES \Rightarrow books NEGATIVE MARK TO MKT
- Citigroup in its press release on the first quarter revenues of 2009 reported a *positive* mark to market due to its *worsened* credit quality: “Revenues also included [...] a net 2.5\$ billion positive CVA on derivative positions, excluding monolines, mainly due to the widening of Citi’s CDS spreads”

The case of symmetric counterparty risk: DVA?

October 18, 2011, 3:59 PM ET, WSJ. Goldman Sachs Hedges Its Way to Less Volatile Earnings.

Goldman's DVA gains in the third quarter totaled \$450 million [...] \$1.9 billion in DVA gains that J.P. Morgan Chase and Citigroup each recorded for the third quarter. Bank of America reported \$1.7 billion of DVA gains in its investment bank [...]

Is DVA real? **DVA Hedging**. Buying back bonds? Proxying?

DVA hedge? One should sell protection on oneself, buying back bonds? Difficult.

Most times: proxying. Sell protection on a number of names highly correlated to oneself (*above WSJ interview, systemic risk problem*)

Even if DVA can be partly unreal to us because we can't hedge it, it is REAL FOR THE OTHER PARTY, since it's the other party's CVA. Price Reality becomes a matter of PERSPECTIVE.

DVA or no DVA? Accounting VS Capital Requirements

NO DVA: Basel III, page 37, July 2011 release

This CVA loss is calculated without taking into account any offsetting debit valuation adjustments which have been deducted from capital under paragraph 75. Stefan Walter spoke about "*perverse incentives*"

YES DVA: FAS 157

Because nonperformance risk (the risk that the obligation will not be fulfilled) includes the reporting entity's credit risk, the reporting entity should consider the effect of its credit risk (credit standing) on the fair value of the liability in all periods in which the liability is measured at fair value under other accounting pronouncements FAS 157 (see also IAS 39)

Funding and DVA

DVA a component of FVA?

DVA is related to funding costs when the payout is uni-directional, eg shorting/issuing a bond, borrowing in a loan, or going short a call option.

Indeed, if we are short simple products that are uni-directional, we are basically borrowing.

As we shorted a bond or option, for example, we received cash V_0 in the beginning, and we will have to pay the product payout in the end.

This cash can be used by us to fund other activities, and allows us to spare the costs of funding this cash V_0 from our treasury.

Funding and DVA

Our treasury usually funds in the market, and the market charges our treasury a cost of funding that is related to the borrowed amount V_0 , to the period T and to our own bank credit risk $\tau_B < T$.

In this sense the funding cost we are sparing when we avoid borrowing looks similar to DVA: it is related to the price of the object we are shorting and to our own credit risk.

However quite a number of assumptions is needed to identify DVA with a *pure funding benefit*, as we will see below.

Closeout: Replacement (ISDA?) VS Risk Free

$$DVA_B(t) = \mathbb{E}_t \left\{ LGD_B \cdot \mathbf{1}(t < \tau^{1st} = \tau_B < T) \cdot D(t, \tau_B) \cdot [-NPV_B(\tau_B)]^+ \right\}$$

$$CVA_B(t) = \mathbb{E}_t \left\{ LGD_C \cdot \mathbf{1}(t < \tau^{1st} = \tau_C < T) \cdot D(t, \tau_C) \cdot [NPV_B(\tau_C)]^+ \right\}$$

NPV_B in the CVA/DVA formulas we have seen before, is the **(credit-) Risk Free** Net Present Value (residual value) at the first default. Risk free means that this residual value is computed without taking into account any residual credit quality. This is called **RISK FREE CLOSEOUT**

In other terms, if we replace the defaulted deal with a new, equivalent deal with a *new counterparty*, we are not accounting for the CVA of the new counterparty. And what if that counterparty defaults too before the end of the deal? *We could enter into a recursive/infinately regressing boundary condition here.*

Closeout: Replacement (ISDA?) VS Risk Free

We are not accounting for our DVA either when pricing the NPV at the default of the counterparty (assuming this happens first).

RISK-FREE CLOSEOUT: The very fact that we have been using a formula with CVA and DVA previously at time $t < \tau^1$, makes our valuation AT default τ^1 inconsistent, or at least **discontinuous**, if we remove CVA and DVA all of a sudden at default.

REPLACEMENT CLOSEOUT: If we DO include residual DVA (and maybe CVA) at τ^1 , we have REPLACEMENT CLOSEOUT, ie the cost of replacing the defaulted deal with an equivalent one in the mkt.

On the other hand, if we are closing the position and liquidating it NOW, why should we account for any residual credit risk?

ISDA: "soft" suggestions in favour of the REPLACEMENT closeout (unfortunately a computational nightmare)

Closeout: Replacement (ISDA?) VS Risk Free

We can study RISK FREE vs REPLACEMENT CLOSEOUT on CVA/DVA for a LOAN. There should be **NO Impact** of an early default of the **Lender** on the loan price adjustment. Instead:

Statistical Dependence $(\tau_B, \tau_C) \rightarrow$	independence	co-monotonicity
Closeout \downarrow		
Risk Free	Negatively Impacts Borrower	No Impact
Replacement	No Impact	Further Negatively Impacts Lender

For a numerical case study and more details see Brigo and Morini (2010, 2011). The effect may be quite sizeable.

A simplified formula without τ^{1st} for bilateral VA

There is another source of confusion on CVA/DVA. Some market players take τ^1 out.

- The simplified formula is only a simplified representation of bilateral risk and ignores that upon the first default closeout proceedings are started, thus involving a degree of double counting
- It is attractive because it allows for the construction of a bilateral counterparty risk pricing system based only on a unilateral one.
- The correct formula involves default dependence between the two parties through τ^{1st} and allows no such incremental construction

A simplified formula without τ^{1st} for bilateral VA

One can show easily that the difference between the full correct formula and the simplified formula is

$$\begin{aligned}
 D^{BC} &= E_0[1_{\{\tau_B < \tau_C < T\}} LGD_C D(0, \tau_C) (E_{\tau_C}(\Pi(\tau_C, T)))^+] \quad (1) \\
 &- E_0[1_{\{\tau_C < \tau_B < T\}} LGD_B D(0, \tau_B) (-E_{\tau_B}(\Pi(\tau_B, T)))^+].
 \end{aligned}$$

For an equity forward, we compute the difference D^{BC} between the correct bilateral risk free closeout formula and the simplified one.

CVA difference as a function of Kendall's tau

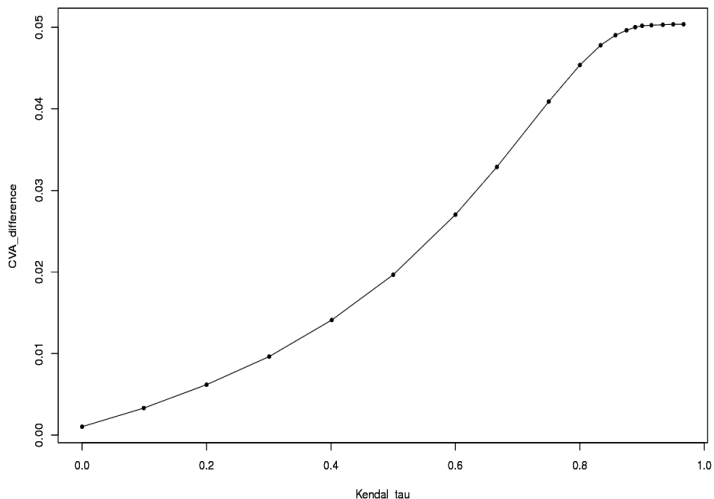


Figure: D^{BC} plotted against Kendall's tau between τ_B and τ_C , all other quantities being equal: $S_0 = 1$, $T = 5$, $\sigma = 0.4$, $K = 1$, $\lambda_B = 0.1$, $\lambda_C = 0.05$.

PAYOFF RISK

The exact payout corresponding with the Credit and Debit valuation adjustment is not clear.

- DVA or not?
- Which Closeout?
- First to default risk or not?
- How are collateral and funding accounted for? (Next part)

Worse than model risk: Payout risk. WHICH PAYOUT?

At a recent industry panel (WBS) on CVA it was stated that 5 banks might compute CVA in 15 different ways.

Collateral Management and Gap Risk

Collateral (CSA) is considered to be the solution to counterparty risk.

Periodically, the position is re-valued ("marked to market") and a quantity related to the change in value is posted on the collateral account from the party who is penalized by the change in value.

This way, the collateral account, at the periodic dates, contains an amount that is close to the actual value of the portfolio and if one counterparty were to default, the amount would be used by the surviving party as a guarantee (and viceversa).

Gap Risk is the residual risk that is left due to the fact that the realignment is only periodical. If the market were to move a lot between two realigning ("margining") dates, a significant loss would still be faced.

Folklore: Collateral completely kills CVA and gap risk is negligible.

Collateral Management and Gap Risk I

Folklore: Collateral completely kills CVA and gap risk is negligible.

We are going to show that there are cases where this is not the case at all (B. Capponi and Pallavicini 2012, Mathematical Finance)

- Risk-neutral evaluation of counterparty risk in presence of collateral management can be a difficult task, due to the complexity of clauses.
- Only few papers in the literature deal with it. Among them we cite Cherubini (2005), Alavian *et al.* (2008), Yi (2009), Assefa *et al.* (2009), Brigo *et al.* (2011) and citations therein.
- Example: Collateralized bilateral CVA for a netted portfolio of IRS with 10y maturity and 1y coupon tenor for different default-time correlations with (and without) collateral re-hypothecation. See B, Capponi, Pallavicini and Papatheodorou (2011)

Collateral Management and Gap Risk II

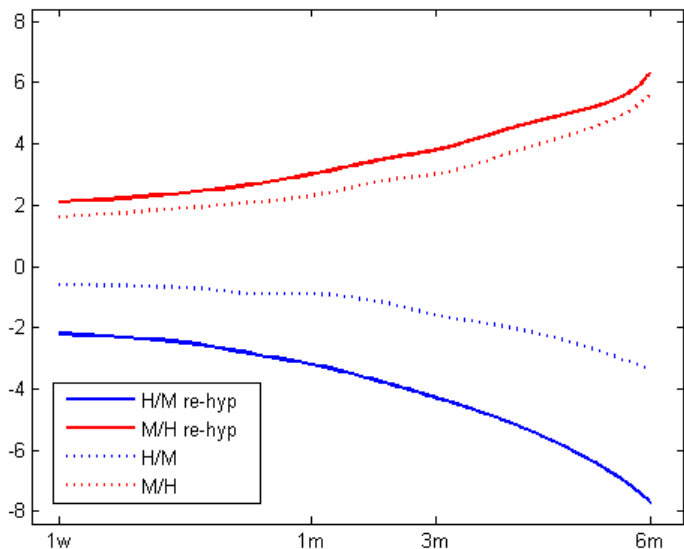


Figure explanation

Bilateral valuation adjustment, margining and rehypotecation

The figure shows the BVA(DVA-CVA) for a ten-year IRS under collateralization through margining as a function of the update frequency δ with zero correlation between rates and counterparty spread, zero correlation between rates and investor spread, and zero correlation between the counterparty and the investor defaults. The model allows for nonzero correlations as well.

Continuous lines represent the re-hypothecation case, while **dotted lines** represent the opposite case. The *red line* represents an investor riskier than the counterparty, while the *blue line* represents an investor less risky than the counterparty. All values are in basis points.

See the full paper by Brigo, Capponi, Pallavicini and Papatheodorou ‘Collateral Margining in Arbitrage-Free Counterparty Valuation Adjustment including Re-Hypothecation and Netting’ available at <http://arxiv.org/abs/1101.3926>

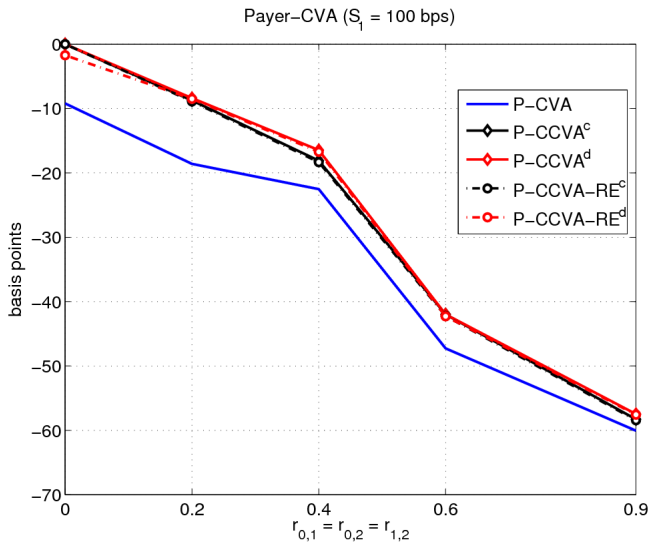
Figure explanation

From the fig, we see that the case of an investor riskier than the counterparty (M/H) leads to positive value for DVA-CVA, while the case of an investor less risky than the counterparty has the opposite behaviour. If we inspect the DVA and CVA terms as in the paper we see that when the investor is riskier the DVA part of the correction dominates, while when the investor is less risky the counterparty has the opposite behaviour.

Re-hypothecation enhances the absolute size of the correction, a reasonable behaviour, since, in such case, each party has a greater risk because of being unsecured on the collateral amount posted to the other party in case of default.

Let us now look at a case with more contagion: a CDS.

Collate



Collateral Management and Gap Risk II

The figure refers to a payer CDS contract as underlying. See the full paper by Brigo, Capponi and Pallavicini (2011) for more cases.

If the investor holds a payer CDS, he is buying protection from the counterparty, i.e. he is a protection buyer.

We assume that the spread in the fixed leg of the CDS is 100 while the initial equilibrium spread is about 250.

Given that the payer CDS will be positive in most scenarios, when the investor defaults it is quite unlikely that the net present value be in favor of the counterparty.

We then expect the CVA term to be relevant, given that the related option will be mostly in the money. This is confirmed by our outputs.

Collateral Management and Gap Risk III

We see in the figure a relevant CVA component (part of the bilateral DVA - CVA) starting at 10 and ending up at 60 bps when under high correlation.

We also see that, for zero correlation, collateralization succeeds in completely removing CVA, which goes from 10 to 0 basis points.

However, collateralization seems to become less effective as default dependence grows, in that collateralized and uncollateralized CVA become closer and closer, and for high correlations we still get 60 basis points of CVA, even under collateralization.

The reason for this is the instantaneous default contagion that, under positive dependency, pushes up the intensity of the survived entities, as soon as there is a default of the counterparty.

Collateral Management and Gap Risk IV

Indeed, the term structure of the on-default survival probabilities (see paper) lies significantly below the one of the pre-default survival probabilities conditioned on $\mathcal{G}_{\tau-}$, especially for large default correlation.

The result is that the default leg of the CDS will increase in value due to contagion, and instantaneously the Payer CDS will be worth more. This will instantly increase the loss to the investor, and most of the CVA value will come from this jump.

Given the instantaneous nature of the jump, the value at default will be quite different from the value at the last date of collateral posting, before the jump, and this explains the limited effectiveness of collateral under significantly positive default dependence.

Basic Payout plus Credit and Collateral: Cash Flows I

- We calculate prices by discounting cash-flows under the pricing measure. Collateral and funding are modeled as additional cashflows (as for CVA and DVA)
- We start from derivative's basic cash flows without credit, collateral or funding risks

$$\bar{V}_t(C; F) := \mathbb{E}_t[\Pi(t, T \wedge \tau) + \dots]$$

where

→ $\tau := \tau_C \wedge \tau_I$ is the first default time, and

→ $\Pi(t, u)$ is the sum of all discounted payoff terms up from t to u ,

Cash flows are stopped either at the first default or at portfolio's expiry if defaults happen later.

Basic Payout plus Credit and Collateral: Cash Flows II

- As second contribution we consider the collateralization procedure and we add its cash flows.

$$\begin{aligned}\bar{V}_t(C; F) &:= \mathbb{E}_t[\Pi(t, T \wedge \tau)] \\ &+ \mathbb{E}_t[\gamma(t, T \wedge \tau; C) + \mathbf{1}_{\{\tau < T\}} D(t, \tau) C_{\tau^-} + \dots]\end{aligned}$$

where

- C_t is the collateral account defined by the CSA,
- C_{τ^-} is the pre-default value of the collateral account, and
- $\gamma(t, u; C)$ are the collateral margining costs up to time u .

- Notice that when applying close-out netting rules, first we will net the exposure against C_{τ^-} , then we will treat any remaining collateral as an unsecured claim.
- If $C > 0$ collateral has been overall posted by the counterparty to protect us, and we have to pay interest c^+ .
- If $C < 0$ we posted collateral for the counterparty (and we are remunerated at interest c^-).

Basic Payout plus Credit and Collateral: Cash Flows III

- The cash flows due to the margining procedure on the time grid $\{t_k\}$ are equal to

$$\gamma(t, u; C) := \sum_{k=1}^{n-1} 1_{\{t \leq t_k < u\}} D(t, t_k) C_{t_k} (1 - P_{t_k}(t_{k+1}) (1 + \alpha_k \tilde{c}_{t_k}(t_{k+1})))$$

where $\alpha_k = t_{k+1} - t_k$ and the collateral accrual rates are given by

$$\tilde{c}_t := c_t^+ 1_{\{C_t > 0\}} + c_t^- 1_{\{C_t < 0\}}$$

- Then, according to CSA, we introduce the pre-default value of the collateral account C_{τ^-} as

$$C_{\tau^-} := \sum_{k=1}^{n-1} 1_{\{t_k < \tau < t_{k+1}\}} C_{t_k} (1 + \alpha_k \tilde{c}_{t_k}(t_{k+1})) P_{\tau}(t_{k+1})$$

Close-Out: Trading-CVA/DVA under Collateral – I

- As third contribution we consider the cash flow happening at 1st default, and we have

$$\begin{aligned}\bar{V}_t(C; F) &:= \mathbb{E}_t[\Pi(t, T \wedge \tau)] \\ &+ \mathbb{E}_t[\gamma(t, T \wedge \tau; C) + \mathbf{1}_{\{\tau < T\}} D(t, \tau) C_{\tau-}] \\ &+ \mathbb{E}_t[\mathbf{1}_{\{\tau < T\}} D(t, \tau) (\theta_\tau(C, \varepsilon) - C_{\tau-}) + \dots]\end{aligned}$$

where

- ε_τ is the amount of losses or costs the surviving party would incur on default event (close-out amount), and
- $\theta_\tau(C, \varepsilon)$ is the on-default cash flow.
- θ_τ will contain collateral adjusted CVA and DVA payouts for the instrument cash flows
- We define θ_τ including the pre-default value of the collateral account since it is used by the close-out netting rule to reduce exposure

Close-Out: Trading-CVA/DVA under Collateral – II

- The close-out amount is not a symmetric quantity w.r.t. the exchange of the role of two parties, since it is valued by one party after the default of the other one.

$$\varepsilon_{\tau} := \mathbf{1}_{\{\tau=\tau_C\}}\varepsilon_{I,\tau} + \mathbf{1}_{\{\tau=\tau_I\}}\varepsilon_{C,\tau}$$

- Without entering into the detail of close-out valuation we can assume a close-out amount equal to the risk-free price of remaining cash flows inclusive of collateralization and funding costs. More details in the examples.
 - See ISDA document “Market Review of OTC Derivative Bilateral Collateralization Practices” (2010).
 - See, for detailed examples, Parker and McGarry (2009) or Weeber and Robson (2009)
 - See, for a review, Brigo, Morini, Pallavicini (2013).

Close-Out: Trading-CVA/DVA under Collateral – III

- At transaction maturity, or after applying close-out netting, the originating party expects to get back the remaining collateral.
- Yet, prevailing legislation's may give to the Collateral Taker some rights on the collateral itself.
 - In presence of re-hypothecation the collateral account may be used for funding, so that cash requirements are reduced, but counterparty risk may increase.
 - See Brigo, Capponi, Pallavicini and Papatheodorou (2011).
- In case of collateral re-hypothecation the surviving party must consider the possibility to recover only a fraction of his collateral.
 - We name such recovery rate REC'_I , if the investor is the Collateral Taker, or REC'_C in the other case.
 - In the worst case the surviving party has no precedence on other creditors to get back his collateral, so that

$$REC_I \leq REC'_I \leq 1, \quad REC_C \leq REC'_C \leq 1$$

Close-Out: Trading-CVA/DVA under Collateral – IV

- The on-default cash flow $\theta_\tau(\mathbf{C}, \varepsilon)$ can be calculated by following ISDA documentation. We obtain

$$\begin{aligned} \theta_\tau(\mathbf{C}, \varepsilon) &:= \mathbf{1}_{\{\tau=\tau_C < \tau_I\}} \left(\varepsilon_{I,\tau} - \text{LGD}_C(\varepsilon_{I,\tau}^+ - \mathbf{C}_{\tau^-}^+)^+ - \text{LGD}'_C(\varepsilon_{I,\tau}^- - \mathbf{C}_{\tau^-}^-)^+ \right) \\ &\quad + \mathbf{1}_{\{\tau=\tau_I < \tau_C\}} \left(\varepsilon_{C,\tau} - \text{LGD}_I(\varepsilon_{C,\tau}^- - \mathbf{C}_{\tau^-}^-)^- - \text{LGD}'_I(\varepsilon_{C,\tau}^+ - \mathbf{C}_{\tau^-}^+)^- \right) \end{aligned}$$

where loss-given-defaults are defined as $\text{LGD}_C := 1 - \text{REC}_C$, and so on.

- If both parties agree on exposure, namely $\varepsilon_{I,\tau} = \varepsilon_{C,\tau} = \varepsilon_\tau$ then

$$\begin{aligned} \theta_\tau(\mathbf{C}, \varepsilon) &:= \varepsilon_\tau - \mathbf{1}_{\{\tau=\tau_C < \tau_I\}} \Pi_{\text{CVAcoll}} + \mathbf{1}_{\{\tau=\tau_I < \tau_C\}} \Pi_{\text{DVAcoll}} \\ \Pi_{\text{CVAcoll}} &= \text{LGD}_C(\varepsilon_\tau^+ - \mathbf{C}_{\tau^-}^+)^+ + \text{LGD}'_C(\varepsilon_\tau^- - \mathbf{C}_{\tau^-}^-)^+ \\ \Pi_{\text{DVAcoll}} &= \text{LGD}_I((-\varepsilon_\tau)^+ - (-\mathbf{C}_{\tau^-})^+)^+ + \text{LGD}'_I(\mathbf{C}_{\tau^-}^+ - \varepsilon_\tau^+)^+ \end{aligned}$$

Close-Out: Trading-CVA/DVA under Collateral – V

- In case of re-hypothecation, when $LGD_C = LGD'_C$ and $LGD_I = LGD'_I$, we obtain a simpler relationship

$$\begin{aligned} \theta_\tau(\mathbf{C}, \varepsilon) &:= \varepsilon_\tau \\ &- \mathbf{1}_{\{\tau = \tau_C < \tau_I\}} LGD_C(\varepsilon_{I, \tau} - C_{\tau^-})^+ \\ &- \mathbf{1}_{\{\tau = \tau_I < \tau_C\}} LGD_I(\varepsilon_{C, \tau} - C_{\tau^-})^- \end{aligned}$$

Funding and Hedging – I

- As fourth and last contribution we consider the funding and hedging procedures and we add their cash flows.

$$\begin{aligned}\bar{V}_t(C; F) &:= \mathbb{E}_t[\Pi(t, T \wedge \tau)] \\ &+ \mathbb{E}_t[\gamma(t, T \wedge \tau; C) + \mathbf{1}_{\{\tau < T\}} D(t, \tau) \theta_\tau(C, \varepsilon)] \\ &+ \mathbb{E}_t[\varphi(t, T \wedge \tau; F, H)]\end{aligned}$$

where

- F_t is the cash account for the hedging strategy of the trade,
 - H_t is the risky-asset account implementing the hedging strategy,
 - $\varphi(t, u; F, H)$ are the cash F and hedging H funding costs up to u .
- In classical Black Scholes on Equity, for a call option (no credit risk, no collateral, no funding costs),

$$\bar{V}_t^{\text{Call}} = \Delta_t S_t + \eta_t B_t =: H_t + F_t, \quad \tau = +\infty, \quad C = \gamma = \varphi = 0.$$

Funding and Hedging – II

- The cash flows due to the funding and hedging strategy on the time grid $\{t_j\}$ are equal to

$$\begin{aligned} \varphi(t, u; F, H) &:= \sum_{j=1}^{m-1} \mathbf{1}_{\{t \leq t_j < u\}} D(t, t_j) (F_{t_j} + H_{t_j}) \left(1 - P_{t_j}(t_{j+1}) (1 + \alpha_k \tilde{f}_{t_j}(t_{j+1})) \right) \\ &- \sum_{j=1}^{m-1} \mathbf{1}_{\{t \leq t_j < u\}} D(t, t_j) H_{t_j} \left(1 - P_{t_j}(t_{j+1}) (1 + \alpha_k \tilde{h}_{t_j}(t_{j+1})) \right) \end{aligned}$$

where the funding and lending rates for F and H are given by

$$\tilde{f}_t := f_t^+ \mathbf{1}_{\{F_t > 0\}} + f_t^- \mathbf{1}_{\{F_t < 0\}}, \quad \tilde{h}_t := h_t^+ \mathbf{1}_{\{H_t > 0\}} + h_t^- \mathbf{1}_{\{H_t < 0\}}$$

Funding and Hedging – III

Cash is borrowed $F > 0$ from the treasury at an interest f^+ (cost) or is lent $F < 0$ at a rate f^- (revenue)

Risky Hedge asset is worth H . Cash needed to buy $H > 0$ ie the risky hedge is borrowed at an interest f^+ from the treasury (cost); in this case H can be used for asset lending (Repo for example) at a rate h^+ (revenue);

On the other hand if risky hedge is worth $H < 0$, we may borrow from the repo market by posting the asset H as guarantee (rate h^- , cost), and lend the obtained cash to the treasury to be remunerated at a rate f^- (revenue).

It is possible to include the risk of default of the funder and funded, leading to CVA and DVA adjustments for the funding position, see PPB.

Recursive non-decomposable Nature of Pricing – I

$$(*) \quad \bar{V}_t = \mathbb{E}_t \left[\Pi(t, T \wedge \tau) + \gamma(t, T \wedge \tau) + 1_{\{\tau < T\}} D(t, \tau) \theta_\tau(C, \varepsilon) + \varphi(t, T \wedge \tau) \right]$$

Can we interpret:

$$\mathbb{E}_t \left[\Pi(t, T \wedge \tau) + 1_{\{\tau < T\}} D(t, \tau) \theta_\tau(C, \varepsilon) \right] : \text{RiskFree Price} + \text{DVA} - \text{CVA?}$$

$$\mathbb{E}_t \left[\gamma(t, T \wedge \tau) + \varphi(t, T \wedge \tau; F, H) \right] : \text{Funding adjustment FVA?}$$

Not really. This is not really a decomposition. In fact since

$$\bar{V}_t = F_t + H_t(+C_t) \quad (\text{re-hypo})$$

we see that the φ present value term depends on future

$F_t = \bar{V}_t - H_t(-C_t)$ and generally the closeout θ , via ε and C , depends on future \bar{V} too. All terms feed each other and there is no neat separation of risks. *Recursive pricing: Nonlinear PDE's / BSDEs for \bar{V}*

"FinalPrice = RiskFreePrice (+ DVA?) - CVA - FVA" not possible.

See Pallavicini Perini B. (2011, 2012) for \bar{V} equations and algorithms.

Recursive non-decomposable Nature of Pricing – II

We can obtain a valuation PDE by a number of further steps:

- Write the equation for \bar{V}_{t_j} starting from $\bar{V}_{t_{j+1}}$, backwards.
- Take the continuous time limit, where funding happens instantaneously and collateral is posted instantaneously (still gap risk, unless you assume NPV to be left continuous)
- Immersion hypothesis for credit risk: work under default-free filtration F_t , where λ is the first-to-default \mathcal{F} adapted intensity.
- Assume a Markovian vector of underlying assets S (pre- credit and funding) with generator \mathcal{L} , whose second order part is \mathcal{L}_2 .
- Use Ito's formula on \bar{V}
- Match dt terms

Recursive non-decomposable Nature of Pricing – III

This leads to the following PDE with terminal condition $\bar{V}_T = 0$.

$$(\partial_t - \tilde{f}_t - \lambda_t + \mathcal{L}_2)\bar{V}_t + \tilde{f}_t H_t + (\tilde{f}_t - \tilde{c}_t)C_t + \lambda_t \theta(C_t, \bar{V}_t) = 0,$$

$$H_t = S_t \frac{\partial \bar{V}_t}{\partial S} \text{ (delta hedging), } \quad \bar{V}_t = H_t + F_t + C_t$$

This PDE is NON-LINEAR not only because of θ , but especially because \tilde{f} depends on F , and hence on \bar{V} itself.

A way to probabilistically represent a nonlinear PDE, generalizing Feynman Kac, is through BACKWARDS SDEs.

Hence this problem can be studied via backwards SDEs (see for example B Pallavicini Perini 2011, Crepey 2012) or "nonlinear Feynman Kac".

Recursive non-decomposable Nature of Pricing – IV

- This recursive equation can be solved iteratively via LS MC techniques as in standard CVA calculations → See PPB (2011)
 - See Pallavicini, Perini, Brigo (2011) for a discrete time solution.
 - See Crépey et al. (2012a) for further examples and a comprehensive BSDEs approach.
- The recursive feature of pricing equations can be avoided by simplified approaches starting either from simplistic spreads in discounting, or from simplistic collateral/closeout rules and symmetric funding rates (FVA).

Explain Funding Rates: Trading vs. Funding DVA – I

- The funding rate \tilde{f}_t is determined by the party managing the funding account for the investor, eg the bank's treasury:
 - trading positions may be netted before funding on the mkt
 - a Funds Transfer Pricing (FTP) process may be implemented to gauge the performances of different business units;
 - a maturity transformation rule can be used to link portfolios to effective maturity dates;
 - sources of funding can be mixed into the internal funding curve ...
- In the literature the role of the treasury is usually neglected, leading to some controversial results particularly when the funding positions are not distinguished from the trading positions.
- In particular, partial claims “funding costs = DVA”, or “there are no funding costs”, are cited in the literature (Hull White, “FVA = 0”)

Conclusions on funding

The law of one price

Charging "FVA" to the counterparty is controversial, and FVA cannot be bilateral, since we do not know the funding policy of our counterparties. So even if DVA was giving us some hope to realign symmetry of prices, funding finally destroys the law of one price and makes prices a matter of perspective if included. Bid Ask? Equilibrium approach?

Is the funding inclusive "price" a real price? Price and Value

Each entity computes a different funding adjusted price for the same product. The funding adjusted "price" is not a price in the conventional sense. We may use it for cost/profitability analysis or to pay our treasury, but can we charge it to a client? Why should the client pay for our funding inefficiencies? It is more a "value" than a "price".

Funding structures inside a bank?

Funding implications on a Bank structure

Including funding costs into valuation, even via a simplistic FVA, involves methodological, organisational, and structural challenges.

Many difficulties are similar to CVA's and DVA's, so Funding can be integrated in the CVA effort typically.

- Reboot IT functions, analytics, methodology, by adopting a consistent global methodology including a consistent credit-debit-collateral-funding adjustment
- Very strong investment, discontinuity, and against the "internal competition" culture
- OR include separate and inconsistent CVA and FVA adjustments, accepting simplifications and double counting.
- It can be important to analyze the global funding implications of the whole trading activity of the bank.

Conclusions I

- Shift from complex products on single risk classes to simple products under complex interconnected risks
- Counterparty Risk adds exotic optionality even to vanilla portfolios.
- Highly specialized hybrid modeling framework. MODEL RISK
- Bilateral CVA brings in symmetry but also paradoxical statements
- The DVA term in bilateral CVA is hard to hedge, especially in the jump-to-default risk component.
- Bilateral CVA requires a choice of closeout? First to default risk? PAYOUT RISK.
- Gap risk in collateralization remains relevant in presence of strong contagion
- Funding costs can be included consistently but they break the law of one price...


Conclusions II

- ... Credit Debit and Funding costs are NOT separable...
- The Funding and Credit adjustment can be included implicitly in term structure models for multiple curve LIBOR vs OIS (Pallavicini and B., 2013)
- ... and can alter the structure of the bank organization and are politically sensitive
- Basel III will make CVA (FVA?) capital requirements rather severe
- Proper valuation and management of CVA/DVA/FVA requires a *Consistent Global Valuation approach* because these risks are all INTERCONNECTED.
- CCP will prompt for checking initial margins calculations and uncollateralized CVA gap risk MTM over the margin period.
- This is one of the Mathematical Finance challenges of our times
- **Thank you for your attention!**

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