Numerical approximation for a portfolio optimization problem under liquidity risk and costs.

presented by :

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Joint work with :

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Plan

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1. Motivation
2. Problem formulation
3. Discretized problem
4. Convergence of the numerical scheme
5. Numerical results

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- Control problem of portfolio optimization under liquidity risk and price impact.
- The value function is the unique continuous viscosity solution of some HJB equation.
- Numerical resolution of the impulse control problem under state constraints based on a probabilistic method.

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- Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space equipped with a filtration $(\mathcal{F}_t)_{0 \leq t \leq T}$ supporting an one-dimensional Brownian motion W on a finite horizon $[0, T]$, $T < \infty$.

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- We consider a continuous time financial market model. We denote by X_t the amount of money and by Y_t the number of shares in the stock held by the investor at time t. The price process of the risky asset is denoted by P_t.
- We model the investor's trades through an impulse control strategy $\alpha = (\tau_n, \zeta_n)_{n \geq 1}$, where the non-decreasing s.t. $\tau_1 \leq \ldots \tau_n \leq \ldots < T$ represent the intervention times and $(\zeta_n)_{n \geq}$ are $\mathcal{F}_{\tau_n}$-measurable real valued r.v. and represent the number of stock trade at these times.

Dynamics of Y

$$Y_s = Y_{\tau_n}, \qquad \tau_n \leq s < \tau_{n+1}$$

$$Y_{\tau_{n+1}} = Y_{\tau_{n+1}^-} + \zeta_{n+1}$$

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Dynamics of P

$$dP_s = P_s(bds + \sigma dW_s), \qquad \tau_n \leq s < \tau_{n+1}$$
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Dynamics of X

$$dX_s = rX_s ds \qquad \tau_n \le s < \tau_{n+1}$$
$$X_{\tau_{n+1}} = X_{\tau_{n+1}^-} - \zeta_{n+1} e^{\lambda \zeta_{n+1}} P_{\tau_{n+1}^-} - k$$

State process

$$Z_s^{\alpha,t,z} = (X_s^{\alpha,t,x}, Y_s^{\alpha,t,y}, P_s^{\alpha,t,p}) \quad \forall s \in [t,T]$$

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The investor's net wealth

$$L(z) = \max[L_0(z), L_1(z)]\mathbf{1}_{y\geq 0} + L_0(z)\mathbf{1}_{y<0}$$

where

$$L_0(z) \;=\; x + ype^{-\lambda y} - k, \quad and \quad L_1(z) \;=\; x.$$

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Solvency region

$$S = \{z = (x, y, p) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}_+^* \;\; : L(z) > 0\},$$

with

$$\partial S = \{z = (x, y, p) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}_+^* \;\; : L(z) = 0\} \;\; and \;\; \bar{S} = S \cup \partial S.$$

Value Function

$$v(t,z) = \sup_{\alpha \in \mathcal{A}(t,z)} \mathbb{E}\left[e^{-r(T-t)} U_L(Z_T^{\alpha,t,z}) \right], \qquad (t,z) \in [0,T] \times \bar{S}.$$

with

$$U_L(z) = U(L(z)) = K\big(L(z)\big)^{\gamma}, \quad \gamma \in]0,1[.$$

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HJB-QVI

$$\min\left[-\frac{\partial v}{\partial t} - \mathcal{L}v \ , \ v - \mathcal{H}v \right] = 0 \quad sur \quad [0,T) \times S$$

$$\mathcal{L}\varphi = rx\frac{\partial \varphi}{\partial x} + bp\frac{\partial \varphi}{\partial p} + \frac{1}{2}\sigma^2 p^2 \frac{\partial^2 \varphi}{\partial p^2} - r\varphi,$$

$$\mathcal{H}\varphi(t,z) = \sup_{\zeta \in \mathcal{C}(z)} \varphi(t, \Gamma(z,\zeta)), \quad (t,z) \in [0,T] \times \bar{S}$$

$$\Gamma(z,\zeta) = (x - \zeta p e^{\lambda\zeta} - k, y + \zeta, p e^{\lambda\zeta}).$$

Theorem [Ly Vath, Mnif and Pham]

The value function v is continuous on $[0,T)\times S$ and is the unique (in $[0,T)\times S$) constrained viscosity solution to HJB-QVI satisfying the boundary and terminal conditions :

$$\lim_{\substack{(t',z')\to(t,z)\\ z'\in S}} v(t',z') = 0, \qquad \forall (t,z)\in[0,T)\times D_0$$

$$\lim_{\substack{(t',z')\to(T,z)\\ z'\in S}} v(t',z') = \max[U_L(z),\mathcal{H}U_L(z)], \qquad \forall z\in\bar{S},$$

and

$$|v(t,z)| \leq K\left(1+\left(x+\frac{p}{\lambda}\right)\right)^{\gamma}, \quad \forall (t,z)\in[0,T)\times S$$

where $D_0 = \{(0,0)\}\times\mathbb{R}_+^*$ and $K<\infty$.

$$\mathcal{G}_\gamma([0,T]\times\bar{S}) = \left\{ v:[0,T]\times\bar{S}\to\mathbb{R};\ \sup_{[0,T]\times\bar{S}} \frac{|\,v(t,z)\,|}{(1+(x+\frac{p}{\lambda}))^{\gamma}} < \infty \right\}$$

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A classical way for a numerical approximation

Finite difference scheme

$$\frac{\partial \varphi}{\partial x}(x) \sim \frac{\varphi(x+\delta) - \varphi(x-\delta)}{2\delta}$$

$$\frac{\partial^2 \varphi}{\partial x^2}(x) \sim \frac{\varphi(x+\delta) - 2\phi(x) + \varphi(x-\delta)}{\delta^2}$$

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$$\varphi(t,z) = \max(\pounds_\delta \varphi(t,z); \sup_{\zeta \in \mathcal{C}_\delta(z)} \varphi(t, \Gamma(z,\zeta))$$

Discretization scheme

$$S^h(t,z,\psi,\varphi) := \begin{cases} min\Big[\psi - \mathbb{E}[\varphi(t+h, Z_{t+h}^{0,t,z})], \psi - \mathcal{H}\varphi(t,z)\Big]; & t \in [0, T-h] \\ min\Big[\psi - \mathbb{E}[\varphi(T, Z_T^{0,t,z})], \psi - \mathcal{H}\varphi(t,z)\Big]; & t \in (T-h, T) \\ min\Big[\psi - U_L(z), \psi - \mathcal{H}U_L(z)\Big]; & t = T \end{cases}$$

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$$v^h(T,z) = \max\Big[U_L(z), \mathcal{H}U_L(z)\Big]$$

$$v^h(t_i,z) = \max\Big[\mathbb{E}[v^h(t_i+h, Z^{0,t_i,z}_{t_i+h})], \mathcal{H}v^h(t_i,z)\Big],$$

where $h = T/m$ and $i \in \{0, .., m-1\}$.

Localized domain

$$\bar{S}_{loc} = \bar{S} \cap ([x_{min}, x_{max}] \times [y_{min}, y_{max}] \times [0, p_{max}])$$

$$R := \min \Big(\mid x_{min} \mid, \mid x_{max} \mid, \mid y_{min} \mid, \mid y_{max} \mid, \mid p_{max} \mid \Big).$$

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Space grid

$$\mathbb{Z}_l = \{z = (x, y, p) \in \mathbb{X}_l \times \mathbb{Y}_l \times \mathbb{P}_l; z \in \bar{S}_{loc}\}$$

where $\mathbb{X}_l$ is the uniform grid on $[x_{min}, x_{max}]$ of step $\frac{x_{max} - x_{min}}{l}$ and similarly for $\mathbb{Y}_l$ and $\mathbb{P}_l$.

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Grid of the admissible controls

$$\mathcal{C}_{M,R}(z) = \{\zeta_i = \zeta_{min} + \frac{i}{M}(\zeta_{max} - \zeta_{min}); 0 \leq i \leq M / \Gamma(z, \zeta_i) \in \bar{S}_{loc}\}$$

where $\zeta_{min} < \zeta_{max} \in \mathbb{R}$ and $M \in \mathbb{N}^*$ are fixed constants.

Functional Quantization

$$\mathcal{E}^{N,R}[v^h(t, Z_t^{0,s,z})] := \sum_{i_1=1}^{N_1} .. \sum_{i_{d(N)}=1}^{N_{d(N)}} \mathbb{P}_{i_1..i_{d(N)}} v^h(t, Z_{N,R}^{0,s,z}(t)) \quad \forall\, s \leq t$$

$$Z_{N,R}^{0,s,z}(t) := \Big(x, y, proj_{[0,p_{max}]}(p \exp\left\{(b - \frac{\sigma^2}{2})(t-s) + \sigma W^N_{i_1..i_{d(N)}}(t-s)\right\})\Big)$$

$$W^N_{i_1..i_{d(N)}}(t) = \sum_{n=1}^{d(N)} \sqrt{\lambda_n} x_{i_n} e_n(t) = \sum_{n=1}^{d(N)} \frac{\sqrt{2T}}{\pi(n-\frac{1}{2})} x_{i_n} \sin\big(\frac{\pi t}{T}(n - \frac{1}{2})\big)$$

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The optimal grid (x_{i_n}) and the associated weights $\mathbb{P}_{i_1..i_{d(N)}}$ are downloaded from the website : http ://www.quantize.maths-fi.com/downloads.

H. Luschgya and G. Pages.
Functional quantization of Gaussian processes.
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Discretization scheme

$$v^h(T,z) = \max\left[U_L(z), \sup_{\zeta\in\mathcal{C}_{M,R}(z)} U_L(\Gamma(z,\zeta))\right]$$

$$v^h(t_i,z) = \max\left[\mathcal{E}^{N,R}[v^h(t_{i+1}, Z^{0,t_i,z}_{t_{i+1}})], \mathcal{H}^{M,R}v^h(t_i,z)\right]$$

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Iterative scheme

We define the sequence $\varphi_n(t,z)$, solution of stopping time problems, as follows :

$$\begin{aligned} \varphi_{n+1}(t,z) &= \sup_{\tau \in \mathcal{S}_{t,T}} \mathbb{E}[\mathcal{H}\varphi_n(\tau, Z_\tau^{0,t,z})], \\ \varphi_0(t,z) &= v_0(t,z), \end{aligned}$$

and we show that

$$\varphi_n(t,z) = v_n(t,z).$$

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Theorem

v_n (hence φ_n) converges towards v when n goes to $+\infty$

Approximation scheme

$$v^h_{n+1}(T,z) = \max\left[U_L(z), \sup_{\zeta\in\mathcal{C}_{M,R}(z)} U_L(\Gamma(z,\zeta))\right]$$

$$v^h_{n+1}(t_i,z) = \max\left[\mathcal{E}^{N,R}[v^h_{n+1}(t_{i+1}, Z^{0,t_i,z}_{t_{i+1}})], \mathcal{H}^{M,R} v^h_n(t_i,z)\right]$$

for $i = 0, ..., m-1$; $z = (x,y,p) \in \mathbb{Z}_l$ and starting from

$$v^h_0(t,z) = \mathcal{E}^{N,R}[U_L(Z^{0,t,z}_T)]$$

Plan

Barles G. and P.E. Souganidis.
Convergence of approximation schemes for fully nonlinear second order equations.
Asymptotic analysis, 4, 271-283, 1991.

Monotonicity + Stability + Consistency $\rightarrow$ Convergence

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Monotonicity + Stability + Consistency → Convergence

Convergence

For all $(t, z) \in [0, T) \times S$ we have that

$$\lim_{\substack{(t',z')\to(t,z)\\(h,M,N,R)\to(0,+\infty)\\(t',z')\in\mathbb{T}_m\times\mathbb{Z}_l}} v^{h,M,N,R}(t',z') = v(t,z),$$

where $v^{h,R,N,M}$ is the solution of the discretized scheme and v is the solution of the HJB-QVI.

Monotonicity

$\forall\ h > 0$, $(t,z) \in [0,T] \times \bar{S}$, $g \in \mathbb{R}$ and $\varphi, \psi \in \mathcal{G}_\gamma$ s.t. $\varphi \le \psi$ we have that

$$S^{h,R,N,M}(t,z,g,\varphi) \ge S^{h,R,N,M}(t,z,g,\psi)$$

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$$S^{h,R,N,M}(t, z, g, \varphi) \ge S^{h,R,N,M}(t, z, g, \psi)$$

Stability

For all $h > 0$, there exists a unique solution $v_n^{h,R,N,M} \in \mathcal{G}_\gamma([0, T] \times \bar{S})$ to the discretized scheme and the sequence $(v_n^{h,R,N,M})_h$ is uniformly bounded in $\mathcal{G}_\gamma([0, T] \times \bar{S})$ i.e. there exists $w \in \mathcal{G}_\gamma([0, T] \times \bar{S})$ s.t. $|v_n^{h,R,N,M}| \le |w|$ for all $h > 0$.

Consistency

(i) $\forall (t,z) \in [0,T) \times \bar{S}$ and Lipschitz function $\phi \in C^{1,2}([0,T) \times \bar{S})$ we have

$$\limsup_{\substack{(h,t^{'},z^{'}) \to (0,t,z) \\ (M,N,R) \to +\infty}} \min \left\{ \frac{\phi(t^{'},z^{'}) - \mathcal{E}^{N,R}[\phi(t^{'}+h, Z^{0,t^{'},z^{'}}_{t^{'}+h})]}{h}, \phi(t^{'},z^{'}) - \mathcal{H}^{M,R}\phi(t^{'},z^{'}) \right\}$$

$$\leq \min \left\{ \Big(-\frac{\partial \phi}{\partial t} - \mathcal{L}\phi \Big)(t,z), \big(\phi(t,z) - \mathcal{H}\phi(t,z) \big) \right\}$$

and

$$\liminf_{\substack{(h,t^{'},z^{'}) \to (0,t,z) \\ (M,N,R) \to +\infty}} \min \left\{ \frac{\phi(t^{'},z^{'}) - \mathcal{E}^{N,R}[\phi(t^{'}+h, Z^{0,t^{'},z^{'}}_{t^{'}+h})]}{h}, \phi(t^{'},z^{'}) - \mathcal{H}^{M,R}\phi(t^{'},z^{'}) \right\}$$

$$\geq \min \left\{ \Big(-\frac{\partial \phi}{\partial t} - \mathcal{L}\phi \Big)(t,z), \big(\phi(t,z) - \mathcal{H}\phi(t,z) \big) \right\}$$

Consistency (sequel)

(ii) $\forall z \in \bar{S}$ and Lipschitz function $\phi \in C^{1,2}([0,T] \times \bar{S})$ we have

$$\limsup_{\substack{(h,t^{'},z^{'})\to(0,T,z)\\(M,N,R)\to+\infty}} \min\left\{\phi(t^{'},z^{'}) - U_L(z^{'}), \left(\phi(t^{'},z^{'}) - \mathcal{H}^{M,R}U_L(z^{'})\right)\right\}$$
$$\leq \min\left\{\phi(T,z) - U_L(z), \left(\phi(T,z) - \mathcal{H}U_L(z)\right)\right\}$$

and

$$\liminf_{\substack{(h,t^{'},z^{'})\to(0,T,z)\\(M,N,R)\to+\infty}} \min\left\{\phi(t^{'},z^{'}) - U_L(z^{'}), \left(\phi(t^{'},z^{'}) - \mathcal{H}^{M,R}U_L(z^{'})\right)\right\}$$
$$\geq \min\left\{\phi(T,z) - U_L(z), \left(\phi(T,z) - \mathcal{H}U_L(z)\right)\right\}$$

Plan

Parameter	Value	Parameter	Value
Maturity	1 year	x_{min}	-100
λ	5.00E(-07)	x_{max}	200
γ	0.5	y_{min}	-4
σ	0.25	y_{max}	20
b	0.1	p_{min}	0
k	1	p_{max}	50
		l	20
		m	40
		M	100
		N	96
		$\bar{\varepsilon}$	10^{-3}

Table 1 : *Parameters*

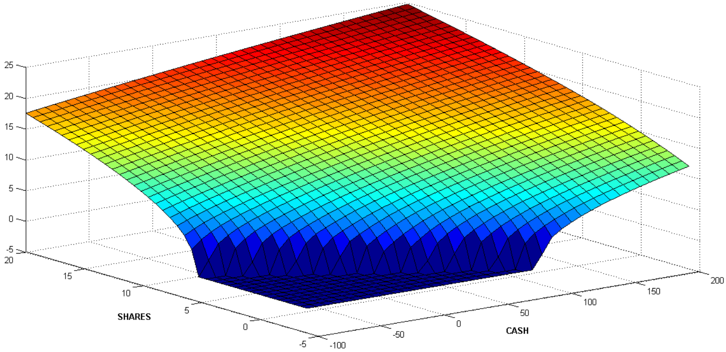

FIGURE : *Value Function for fixed P*

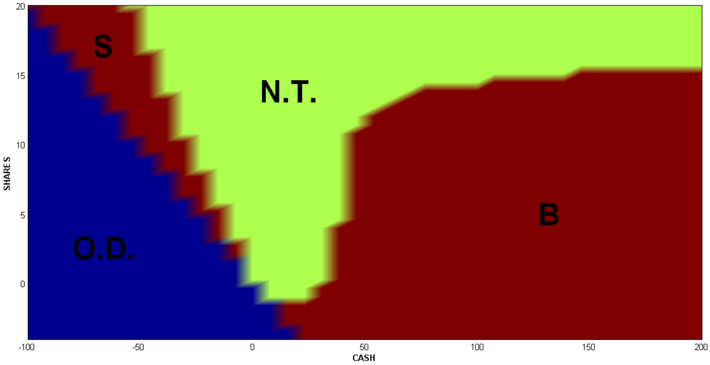

FIGURE : *The Optimal Policy sliced in XY*

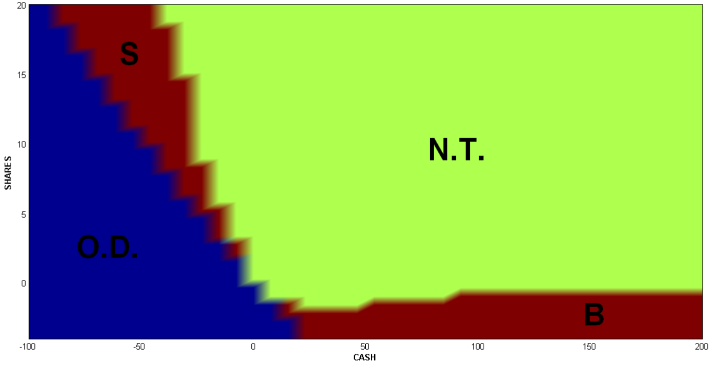

FIGURE : *The Optimal Policy sliced in XY for* $\lambda = 5.00E(-03)$

Convergence in N

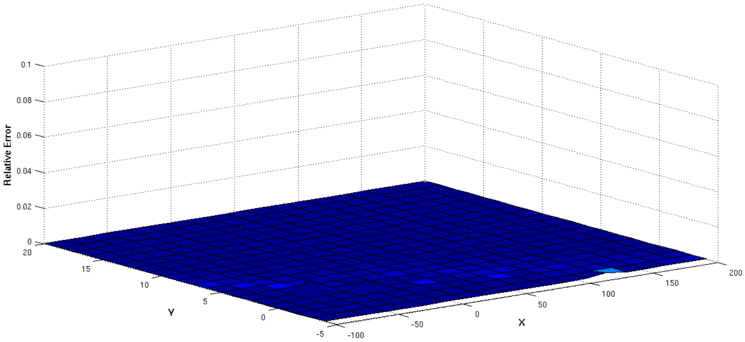

FIGURE : *Relative error of the value function computed when* $N = 96$ *Vs.* $N = 200$.

Convergence in M

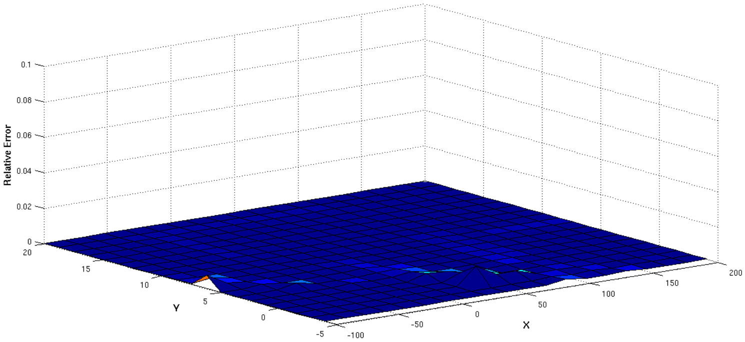

FIGURE : *Relative error of the value function when* $M = 200$ *Vs.* $M = 250$.

Convergence in R

Some values of the value function for two different values R_1 and R_2 of R where R_1 is chosen as in Table 1 and we choose R_2 as follows :

$$R_2 = \min \Big(\mid x_{min} = -257.90 \mid, \mid x_{max} = 342.10 \mid, \mid y_{min} = -16.63 \mid, \mid y_{max} = 31.36 \mid, \mid p_{max} = 100 \mid \Big)$$

R	R_1	R_2
$v(t, z_1)$	20.0038	19.9966
$v(t, z_2)$	24.5429	24.5340

Table 2 : *Values of the value function for different values of R and z.*

Kyle A.
Continuous auctions and insider trading.
Econometrica , 53, 1315-1335, 1985.

Korn R.
Portfolio optimization with strictly positive transaction costs and impulse control.
Finance and Stochastics, 2, 85-114, 1998.

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Thank you for your attention.