Numerical approximation for a portfolio optimization problem under liquidity risk and costs.

presented by:

M’hamed GAIGI

Joint work with:

Vathana LY VATH, Mohamed MNIF and Salwa TOUMI
Motivation

Problem formulation

Discretized problem

Convergence of the numerical scheme

Numerical results
Plan

1. Motivation
2. Problem formulation
3. Discretized problem
4. Convergence of the numerical scheme
5. Numerical results
1. Motivation
2. Problem formulation
3. Discretized problem
4. Convergence of the numerical scheme
5. Numerical results
1. Motivation
2. Problem formulation
3. Discretized problem
4. Convergence of the numerical scheme
5. Numerical results
Plan

1. Motivation
2. Problem formulation
3. Discretized problem
4. Convergence of the numerical scheme
5. Numerical results
Ly Vath V., Mnif M. and H. Pham.
A model of optimal portfolio selection under liquidity risk and price impact.

Ly Vath V., Mnif M. and H. Pham.
A model of optimal portfolio selection under liquidity risk and price impact.

Control problem of portfolio optimization under liquidity risk and price impact.
Ly Vath V., Mnif M. and H. Pham.
A model of optimal portfolio selection under liquidity risk and price impact.


- Control problem of portfolio optimization under liquidity risk and price impact.
- The value function is the unique continuous viscosity solution of some HJB equation.
Ly Vath V., Mnif M. and H. Pham.
A model of optimal portfolio selection under liquidity risk and price impact.

- Control problem of portfolio optimization under liquidity risk and price impact.
- The value function is the unique continuous viscosity solution of some HJB equation.
- Numerical resolution of the impulse control problem under state constraints based on a probabilistic method.
Plan

1. Motivation
2. Problem formulation
3. Discretized problem
4. Convergence of the numerical scheme
5. Numerical results
Let \((\Omega, \mathcal{F}, P)\) be a probability space equipped with a filtration \((\mathcal{F}_t)_{0 \leq t \leq T}\) supporting an one-dimensional Brownian motion \(W\) on a finite horizon \([0, T]\), \(T < \infty\).

We consider a continuous time financial market model. We denote by \(X_t\) the amount of money and by \(Y_t\) the number of shares in the stock held by the investor at time \(t\). The price process of the risky asset is denoted by \(P_t\).

We model the investor's trades through an impulse control strategy \(\alpha = (\tau_n, \zeta_n)_{n \geq 1}\), where the non-decreasing \(\tau_1 \leq ... \tau_n < T\) represent the intervention times and \((\zeta_n)_{n \geq 1}\) are \(\mathcal{F}_{\tau_n}\)-measurable real valued r.v. and represent the number of stock trade at these times.
Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space equipped with a filtration $(\mathcal{F}_t)_{0 \leq t \leq T}$ supporting an one-dimensional Brownian motion $W$ on a finite horizon $[0, T]$, $T < \infty$. 

M’hamed GAIGI

AMaMeF and Banach Center Conference, Juin 2013
Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space equipped with a filtration $(\mathcal{F}_t)_{0 \leq t \leq T}$ supporting an one-dimensional Brownian motion $W$ on a finite horizon $[0, T]$, $T < \infty$.

We consider a continuous time financial market model. We denote by $X_t$ the amount of money and by $Y_t$ the number of shares in the stock held by the investor at time $t$. The price process of the risky asset is denoted by $P_t$. 
Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space equipped with a filtration $(\mathcal{F}_t)_{0 \leq t \leq T}$ supporting an one-dimensional Brownian motion $W$ on a finite horizon $[0, T]$, $T < \infty$.

We consider a continuous time financial market model. We denote by $X_t$ the amount of money and by $Y_t$ the number of shares in the stock held by the investor at time $t$. The price process of the risky asset is denoted by $P_t$.

We model the investor’s trades through an impulse control strategy $\alpha = (\tau_n, \zeta_n)_{n \geq 1}$, where the non-decreasing s.t. $\tau_1 \leq \ldots \tau_n \leq \ldots < T$ represent the intervention times and $(\zeta_n)_{n \geq 1}$ are $\mathcal{F}_{\tau_n}$-measurable real valued r.v. and represent the number of stock trade at these times.
Dynamics of $Y$

\[ Y_s = Y_{\tau_n}, \quad \tau_n \leq s < \tau_{n+1} \]
\[ Y_{\tau_{n+1}} = Y_{\tau_{n+1}^-} + \zeta_{n+1} \]
Dynamics of $Y$

\[ Y_s = Y_{\tau_n}, \quad \tau_n \leq s < \tau_{n+1} \]

\[ Y_{\tau_{n+1}} = Y_{\tau_{n+1}-} + \zeta_{n+1} \]

---

Dynamics of $P$

\[ dP_s = P_s (b ds + \sigma dW_s), \quad \tau_n \leq s < \tau_{n+1} \]

\[ P_{\tau_{n+1}} = e^{\lambda \zeta_{n+1}} P_{\tau_{n+1}-} \]
Dynamics of $Y$

\[ Y_s = Y_{\tau_n}, \quad \tau_n \leq s < \tau_{n+1} \]
\[ Y_{\tau_{n+1}} = Y_{\tau_{n+1}}^- + \zeta_{n+1} \]

Dynamics of $P$

\[ dP_s = P_s (bsd + \sigma dW_s), \quad \tau_n \leq s < \tau_{n+1} \]
\[ P_{\tau_{n+1}} = e^{\lambda \zeta_{n+1}} P_{\tau_{n+1}}^- \]

Dynamics of $X$

\[ dX_s = rX_s ds \quad \tau_n \leq s < \tau_{n+1} \]
\[ X_{\tau_{n+1}} = X_{\tau_{n+1}}^- - \zeta_{n+1} e^{\lambda \zeta_{n+1}} P_{\tau_{n+1}}^- - k \]
State process

\[ Z_s^{\alpha,t,z} = (X_s^{\alpha,t,x}, Y_s^{\alpha,t,y}, P_s^{\alpha,t,p}) \quad \forall s \in [t, T] \]
State process

\[ Z_s^\alpha,t,z = (X_s^\alpha,t,x, Y_s^\alpha,t,y, P_s^\alpha,t,p) \quad \forall s \in [t, T] \]

The investor’s net wealth

\[ L(z) = \max[L_0(z), L_1(z)] \chi_{y \geq 0} + L_0(z) \chi_{y < 0} \]

where

\[ L_0(z) = x + ype^{-\lambda y} - k, \quad \text{and} \quad L_1(z) = x. \]
### State process

\[ Z_s^{\alpha,t,z} = (X_s^{\alpha,t,x}, Y_s^{\alpha,t,y}, P_s^{\alpha,t,p}) \quad \forall s \in [t, T] \]

### The investor’s net wealth

\[ L(z) = \max[L_0(z), L_1(z)]1_{y \geq 0} + L_0(z)1_{y < 0} \]

where

\[ L_0(z) = x + ype^{-\lambda y} - k, \quad \text{and} \quad L_1(z) = x. \]

### Solvency region

\[ S = \{z = (x, y, p) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}^*_+ : L(z) > 0\}, \]

with

\[ \partial S = \{z = (x, y, p) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}^*_+ : L(z) = 0\} \quad \text{and} \quad \bar{S} = S \cup \partial S. \]
Value Function

\[ v(t, z) = \sup_{\alpha \in A(t, z)} \mathbb{E} \left[ e^{-r(T-t)} U_L(Z_T^{\alpha, t, z}) \right], \quad (t, z) \in [0, T] \times \bar{S}. \]

with

\[ U_L(z) = U(L(z)) = K(L(z))^\gamma, \quad \gamma \in ]0, 1[. \]
**Value Function**

$$v(t, z) = \sup_{\alpha \in A(t, z)} \mathbb{E} \left[ e^{-r(T-t)} U_L(Z_T^{\alpha, t, z}) \right], \quad (t, z) \in [0, T] \times \bar{S}. $$

with

$$U_L(z) = U(L(z)) = K(L(z))^{\gamma}, \quad \gamma \in ]0, 1[. $$_

**HJB-QVI**

$$\min \left[ -\frac{\partial v}{\partial t} - \mathcal{L}v, \ v - \mathcal{H}v \right] = 0 \quad \text{sur} \quad [0, T) \times S$$

$$\mathcal{L} \varphi = rx \frac{\partial \varphi}{\partial x} + bp \frac{\partial \varphi}{\partial p} + \frac{1}{2} \sigma^2 p^2 \frac{\partial^2 \varphi}{\partial p^2} - r \varphi,$$

$$\mathcal{H} \varphi(t, z) = \sup_{\zeta \in C(z)} \varphi(t, \Gamma(z, \zeta)), \quad (t, z) \in [0, T] \times \bar{S}$$

$$\Gamma(z, \zeta) = (x - \zeta pe^{\lambda \zeta} - k, y + \zeta, pe^{\lambda \zeta}).$$
Theorem [Ly Vath, Mnif and Pham]

The value function $v$ is continuous on $[0, T) \times S$ and is the unique (in $[0, T) \times S$) constrained viscosity solution to HJB-QVI satisfying the boundary and terminal conditions:

$$
\lim_{(t', z') \to (t, z), z' \in S} v(t', z') = 0, \quad \forall (t, z) \in [0, T) \times D_0
$$

$$
\lim_{(t', z') \to (T, z), z' \in S} v(t', z') = \max[U_L(z), HU_L(z)], \quad \forall z \in \bar{S},
$$

and

$$
|v(t, z)| \leq K \left(1 + \left(x + \frac{p}{\lambda}\right)\right)^\gamma, \quad \forall (t, z) \in [0, T) \times S
$$

where $D_0 = \{(0, 0)\} \times \mathbb{R}^*_+$ and $K < \infty$.

$$
G_\gamma([0, T] \times \bar{S}) = \left\{v : [0, T] \times \bar{S} \to \mathbb{R}; \sup_{[0,T] \times \bar{S}} \frac{|v(t, z)|}{(1 + (x + \frac{p}{\lambda}))^\gamma} < \infty \right\}
$$
Plan

1. Motivation
2. Problem formulation
3. Discretized problem
4. Convergence of the numerical scheme
5. Numerical results
A classical way for a numerical approximation

Finite difference scheme

\[
\frac{\partial \varphi}{\partial x}(x) \sim \frac{\varphi(x + \delta) - \varphi(x - \delta)}{2\delta}
\]

\[
\frac{\partial^2 \varphi}{\partial x^2}(x) \sim \frac{\varphi(x + \delta) - 2\varphi(x) + \varphi(x - \delta)}{\delta^2}
\]
A classical way for a numerical approximation

**Finite difference scheme**

\[
\frac{\partial \varphi(x)}{\partial x} \approx \frac{\varphi(x + \delta) - \varphi(x - \delta)}{2\delta}
\]

\[
\frac{\partial^2 \varphi(x)}{\partial x^2} \approx \frac{\varphi(x + \delta) - 2\varphi(x) + \varphi(x - \delta)}{\delta^2}
\]

\[
\varphi(t, z) = \max(\mathcal{L}_\delta \varphi(t, z); \sup_{\zeta \in \mathcal{C}_\delta} \varphi(t, \Gamma(z, \zeta))
\]

M'hamed GAIGI
AMaMeF and Banach Center Conference, Juin 2013
Discretization scheme

\[ S^h(t, z, \psi, \varphi) := \begin{cases} 
\min \left[ \psi - \mathbb{E}[\varphi(t + h, Z_{t+h}^{0, t, z})], \psi - \mathcal{H}\varphi(t, z) \right]; & t \in [0, T - h] \\
\min \left[ \psi - \mathbb{E}[\varphi(T, Z_T^{0, t, z})], \psi - \mathcal{H}\varphi(t, z) \right]; & t \in (T - h, T) \\
\min \left[ \psi - U_L(z), \psi - \mathcal{H}U_L(z) \right]; & t = T 
\end{cases} \]
**Discretization scheme**

\[
S^h(t, z, \psi, \varphi) := \begin{cases} 
min \left[ \psi - \mathbb{E}[\varphi(t + h, Z_{t+h}^0, z)], \psi - \mathcal{H}\varphi(t, z) \right]; & t \in [0, T - h] \\
min \left[ \psi - \mathbb{E}[\varphi(T, Z_{T}^0, z)], \psi - \mathcal{H}\varphi(t, z) \right]; & t \in (T - h, T) \\
min \left[ \psi - U_L(z), \psi - \mathcal{H}U_L(z) \right]; & t = T 
\end{cases}
\]

\[
v^h(T, z) = \max \left[ U_L(z), \mathcal{H}U_L(z) \right]
\]

\[
v^h(t_i, z) = \max \left[ \mathbb{E}[v^h(t_i + h, Z_{t_i+h}^0, z)], \mathcal{H}v^h(t_i, z) \right],
\]

where \( h = T/m \) and \( i \in \{0, \ldots, m - 1\} \).
Localized domain

\[ \tilde{S}_{loc} = \tilde{S} \cap ([x_{\text{min}}, x_{\text{max}}] \times [y_{\text{min}}, y_{\text{max}}] \times [0, p_{\text{max}}]) \]

\[ R := \min \left( |x_{\text{min}}|, |x_{\text{max}}|, |y_{\text{min}}|, |y_{\text{max}}|, |p_{\text{max}}| \right). \]
Localized domain

\[ \bar{S}_{loc} = \bar{S} \cap ([x_{min}, x_{max}] \times [y_{min}, y_{max}] \times [0, p_{max}]) \]

\[ R := \min \left( |x_{min}|, |x_{max}|, |y_{min}|, |y_{max}|, |p_{max}| \right). \]

Space grid

\[ Z_l = \{ z = (x, y, p) \in X_l \times Y_l \times P_l; z \in \bar{S}_{loc} \} \]

where \( X_l \) is the uniform grid on \([x_{min}, x_{max}]\) of step \( \frac{x_{max} - x_{min}}{l} \) and similarly for \( Y_l \) and \( P_l \).
Localized domain

\[ \tilde{S}_{loc} = \bar{S} \cap ([x_{min}, x_{max}] \times [y_{min}, y_{max}] \times [0, p_{max}]) \]

\[ R := \min \left( |x_{min}|, |x_{max}|, |y_{min}|, |y_{max}|, |p_{max}| \right). \]

Space grid

\[ \mathbb{Z}_l = \{ z = (x, y, p) \in X_l \times Y_l \times P_l; z \in \tilde{S}_{loc} \} \]

where \( X_l \) is the uniform grid on \([x_{min}, x_{max}]\) of step \( \frac{x_{max} - x_{min}}{l} \) and similarly for \( Y_l \) and \( P_l \).

Grid of the admissible controls

\[ \mathcal{C}_{M,R}(z) = \{ \zeta_i = \zeta_{min} + \frac{i}{M} (\zeta_{max} - \zeta_{min}); 0 \leq i \leq M/\Gamma(z, \zeta_i) \in \tilde{S}_{loc} \} \]

where \( \zeta_{min} < \zeta_{max} \in \mathbb{R} \) and \( M \in \mathbb{N}^* \) are fixed constants.
**Functional Quantization**

\[
\mathcal{E}^{N,R}[v^h(t, Z^0_s, z)] := \sum_{i_1=1}^{N_1} \cdots \sum_{i_{d(N)}=1}^{N_{d(N)}} \mathbb{P}_{i_1 \cdots i_{d(N)}} v^h(t, Z^0_{N,R}(t)) \quad \forall \ s \leq t
\]

\[
Z^0_{N,R}(t) := (x, y, \text{proj}_{[0,p_{max}]}(p \exp \{(b - \frac{\sigma^2}{2})(t - s) + \sigma W^N_{i_1 \cdots i_{d(N)}}(t - s)\}))
\]

\[
W^N_{i_1 \cdots i_{d(N)}}(t) = \sum_{n=1}^{d(N)} \sqrt{\lambda_n} x_{i_n} e_n(t) = \sum_{n=1}^{d(N)} \frac{\sqrt{2T}}{\pi(n - \frac{1}{2})} x_{i_n} \sin \left( \frac{\pi t}{T} \left(n - \frac{1}{2}\right) \right)
\]
Functional Quantization

\[ \mathcal{E}_{N,R}^{N} \left[ v^h(t, Z^{0}_{t}, z) \right] := \sum_{i_1=1}^{N_1} \ldots \sum_{i_d(N)=1}^{N_d(N)} P_{i_1 \ldots i_d(N)} v^h(t, Z_{N,R}^{0,s,z}(t)) \quad \forall \, s \leq t \]

\[ Z_{N,R}^{0,s,z}(t) := \left( x, y, \text{proj}_{[0,p_{\text{max}}]}(p \exp \left\{ (b - \frac{\sigma^2}{2})(t - s) + \sigma W_{i_1 \ldots i_d(N)}^{N}(t - s) \right\}) \right) \]

\[ W_{i_1 \ldots i_d(N)}^{N}(t) = \sum_{n=1}^{d(N)} \sqrt{\lambda_n} x_{i_n} e_n(t) = \sum_{n=1}^{d(N)} \frac{\sqrt{2T}}{\pi(n - \frac{1}{2})} x_{i_n} \sin \left( \frac{\pi t}{T} (n - \frac{1}{2}) \right) \]

The optimal grid \((x_{i_n})\) and the associated weights \(P_{i_1 \ldots i_d(N)}\) are downloaded from the website: http://www.quantize.maths-fi.com/downloads.
H. Luschgy and G. Pages.
Functional quantization of Gaussian processes.
H. Luschgy and G. Pages.
Functional quantization of Gaussian processes.

**Discretization scheme**

\[ v^h(T, z) = \max \left[ U_L(z), \sup_{\zeta \in \mathcal{C}_{M,R}(z)} U_L(\Gamma(z, \zeta)) \right] \]

\[ v^h(t_i, z) = \max \left[ \mathcal{E}^{N,R}[v^h(t_{i+1}, Z_{t_i+1}^0, z)], \mathcal{H}^{M,R}v^h(t_i, z) \right] \]
\[ v_n(t, z) := \sup_{\alpha \in A_n(t, z)} \mathbb{E}[U_L(Z_T)] \quad (t, z) \in [0, T] \times \bar{S}. \]
Motivation
Problem formulation
Discretized problem
Convergence of the numerical scheme
Numerical results

\[ v_n(t, z) := \sup_{\alpha \in \mathcal{A}_n(t, z)} \mathbb{E}[U_L(Z_T)] \quad (t, z) \in [0, T] \times \bar{S}. \]

Iterative scheme

We define the sequence \( \varphi_n(t, z) \), solution of stopping time problems, as follows:

\[
\varphi_{n+1}(t, z) = \sup_{\tau \in \mathcal{S}_{t, T}} \mathbb{E}[\mathcal{H}\varphi_n(\tau, Z^0_{\tau, t, z})],
\]

\[
\varphi_0(t, z) = v_0(t, z),
\]

and we show that

\[
\varphi_n(t, z) = v_n(t, z).
\]
\[ v_n(t, z) := \sup_{\alpha \in \mathcal{A}_n(t, z)} \mathbb{E}[U_L(Z_T)] \quad (t, z) \in [0, T] \times \bar{S}. \]

**Iterative scheme**

We define the sequence \( \varphi_n(t, z) \), solution of stopping time problems, as follows:

\[
\varphi_{n+1}(t, z) = \sup_{\tau \in S_{t,T}} \mathbb{E}[\mathcal{H}\varphi_n(\tau, Z^0_{\tau}, t, z)],
\]

\[
\varphi_0(t, z) = v_0(t, z),
\]

and we show that

\[
\varphi_n(t, z) = v_n(t, z).
\]

**Theorem**

\( v_n \) (hence \( \varphi_n \)) converges towards \( v \) when \( n \) goes to \( +\infty \).
Approximation scheme

\[ v^h_{n+1}(T, z) = \max \left[ U_L(z), \sup_{\zeta \in \mathcal{C}_{M,R}(z)} U_L(\Gamma(z, \zeta)) \right] \]

\[ v^h_{n+1}(t_i, z) = \max \left[ \mathcal{E}^{N,R}[v^h_{n+1}(t_{i+1}, Z_{t_{i+1}}^{0,t_i,z})], \mathcal{H}^{M,R}v^h_{n}(t_i, z) \right] \]

for \( i = 0, \ldots, m - 1 \); \( z = (x, y, p) \in \mathbb{Z}_l \) and starting from

\[ v^h_0(t, z) = \mathcal{E}^{N,R}[U_L(Z_T^{0,t,z})] \]
Plan

1. Motivation
2. Problem formulation
3. Discretized problem
4. Convergence of the numerical scheme
5. Numerical results
Barles G. and P.E. Souganidis.  
Convergence of approximation schemes for fully nonlinear second order equations.  

Monotonicity + Stability + Consistency \(\rightarrow\) Convergence
Barles G. and P.E. Souganidis.
Convergence of approximation schemes for fully nonlinear second order equations.

Monotonicity + Stability + Consistency $\rightarrow$ Convergence

Convergence

For all $(t, z) \in [0, T) \times S$ we have that

$$\lim_{(t', z') \to (t, z)} v_{h,M,N,R}^{h,M,N,R}(t', z') = v(t, z),$$

where $v_{h,R,N,M}^{h,R,N,M}$ is the solution of the discretized scheme and $v$ is the solution of the HJB-QVI.
Monotonicity

∀ h > 0, \((t, z) \in [0, T] \times \bar{S}, \ g \in \mathbb{R} \) and \(\varphi, \psi \in G_{\gamma}\) s.t. \(\varphi \leq \psi\) we have that

\[ S^{h,R,N,M}(t, z, g, \varphi) \geq S^{h,R,N,M}(t, z, g, \psi) \]
### Monotonicity

\[ \forall h > 0, \ (t, z) \in [0, T] \times \bar{S}, \ g \in \mathbb{R} \text{ and } \varphi, \psi \in \mathcal{G}_\gamma \text{ s.t. } \varphi \leq \psi \text{ we have that} \]

\[ S_{h,R,N,M}^h(t, z, g, \varphi) \geq S_{h,R,N,M}^h(t, z, g, \psi) \]

### Stability

For all \( h > 0 \), there exists a unique solution \( \nu_{n}^{h,R,N,M} \in \mathcal{G}_\gamma([0, T] \times \bar{S}) \) to the discretized scheme and the sequence \( (\nu_{n}^{h,R,N,M})_h \) is uniformly bounded in \( \mathcal{G}_\gamma([0, T] \times \bar{S}) \) i.e. there exists \( w \in \mathcal{G}_\gamma([0, T] \times \bar{S}) \) s.t. \( |\nu_{n}^{h,R,N,M}| \leq |w| \) for all \( h > 0 \).
(i) \( \forall (t, z) \in [0, T) \times \bar{S} \) and Lipschitz function \( \phi \in C^{1,2}([0, T) \times \bar{S}) \) we have

\[
\limsup_{(h,t',z') \to (0,t,z)} \min_{(M,N,R) \to +\infty} \left\{ \frac{\phi(t', z') - E^{N,R}[\phi(t' + h, Z^{0,t',z'}_{t'+h})]}{h}, \phi(t', z') - H^{M,R}\phi(t', z') \right\}
\]

\[
\leq \min \left\{ \left( -\frac{\partial \phi}{\partial t} - L\phi \right)(t, z), (\phi(t, z) - H\phi(t, z)) \right\}
\]

and

\[
\liminf_{(h,t',z') \to (0,t,z)} \min_{(M,N,R) \to +\infty} \left\{ \frac{\phi(t', z') - E^{N,R}[\phi(t' + h, Z^{0,t',z'}_{t'+h})]}{h}, \phi(t', z') - H^{M,R}\phi(t', z') \right\}
\]

\[
\geq \min \left\{ \left( -\frac{\partial \phi}{\partial t} - L\phi \right)(t, z), (\phi(t, z) - H\phi(t, z)) \right\}
\]
(ii) \( \forall z \in \bar{S} \) and Lipschitz function \( \phi \in C^{1,2}([0, T] \times \bar{S}) \) we have

\[
\limsup_{(h, t', z') \to (0, T, z)} \min \left\{ \phi(t', z') - U_L(z'), \left( \phi(t', z') - \mathcal{H}^{M, R} U_L(z') \right) \right\} \\
\leq \min \left\{ \phi(T, z) - U_L(z), \left( \phi(T, z) - \mathcal{H} U_L(z) \right) \right\}
\]

and

\[
\liminf_{(h, t', z') \to (0, T, z)} \min \left\{ \phi(t', z') - U_L(z'), \left( \phi(t', z') - \mathcal{H}^{M, R} U_L(z') \right) \right\} \\
\geq \min \left\{ \phi(T, z) - U_L(z), \left( \phi(T, z) - \mathcal{H} U_L(z) \right) \right\}
\]
Plan

1. Motivation
2. Problem formulation
3. Discretized problem
4. Convergence of the numerical scheme
5. Numerical results
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maturity</td>
<td>1 year</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>5.00E(-07)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.25</td>
</tr>
<tr>
<td>$b$</td>
<td>0.1</td>
</tr>
<tr>
<td>$k$</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{min}$</td>
<td>-100</td>
</tr>
<tr>
<td>$x_{max}$</td>
<td>200</td>
</tr>
<tr>
<td>$y_{min}$</td>
<td>-4</td>
</tr>
<tr>
<td>$y_{max}$</td>
<td>20</td>
</tr>
<tr>
<td>$p_{min}$</td>
<td>0</td>
</tr>
<tr>
<td>$p_{max}$</td>
<td>50</td>
</tr>
<tr>
<td>$l$</td>
<td>20</td>
</tr>
<tr>
<td>$m$</td>
<td>40</td>
</tr>
<tr>
<td>$M$</td>
<td>100</td>
</tr>
<tr>
<td>$N$</td>
<td>96</td>
</tr>
<tr>
<td>$\bar{\varepsilon}$</td>
<td>$10^{-3}$</td>
</tr>
</tbody>
</table>

**Table 1: Parameters**
**Figure:** *Value Function for fixed P*
**Figure:** The Optimal Policy sliced in XY
Figure: The Optimal Policy sliced in XY for $\lambda = 5.00E(-03)$
**Convergence in N**

**Figure**: Relative error of the value function computed when $N = 96$ Vs. $N = 200$. 

M'hamed GAIGI  
AMaMeF and Banach Center Conference, Juin 2013
**Convergence in M**

**Figure**: Relative error of the value function when $M = 200$ Vs. $M = 250$. 
Convergence in $R$

Some values of the value function for two different values $R_1$ and $R_2$ of $R$ where $R_1$ is chosen as in Table 1 and we choose $R_2$ as follows:

$$R_2 = \min \left( |x_{\min} = -257.90|, |x_{\max} = 342.10|, |y_{\min} = -16.63|, |y_{\max} = 31.36|, |p_{\max} = 100| \right)$$

<table>
<thead>
<tr>
<th>$R$</th>
<th>$R_1$</th>
<th>$R_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v(t, z_1)$</td>
<td>20.0038</td>
<td>19.9966</td>
</tr>
<tr>
<td>$v(t, z_2)$</td>
<td>24.5429</td>
<td>24.5340</td>
</tr>
</tbody>
</table>

*Table 2: Values of the value function for different values of $R$ and $z$.***
Kyle A.
Continuous auctions and insider trading.

Korn R.
Portfolio optimization with strictly positive transaction costs and impulse control.

Oksendal B. and A. Sulem.
Optimal consumption and portfolio with both fixed and proportional transaction costs.
Ly Vath V., Mnif M. and H. Pham.
A model of optimal portfolio selection under liquidity risk and price impact.

H. Luschgya and G. Pages.
Functional quantization of Gaussian processes.

Barles G. and P.E. Souganidis.
Convergence of approximation schemes for fully nonlinear second order equations.
Thank you for your attention.