Optimal Liquidation under stochastic liquidity and regime shifting

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Motivations : Selling an illiquid asset

An investor selling an illiquid asset,

1. "Impatient" trader

   - Finds a buyer for the asset

   - Immediately sells even at a discounted price w.r.t. the theoritical/fair price

2. "Patient" trader

   - Continuously estimates the theoritical/fair price

   - Waits for a buyer at that price

3. "Mixed" trading strategy

   - Continuously estimates the theoritical/fair price

   - Waits a buyer at that price

   - If a buyer proposes an "acceptable" discounted price, immediately sell

Motivations : Selling an illiquid asset

- "Impatient" trading : Optimal portfolio selection with transaction costs, Optimal portfolio liquidation,..

  $\longrightarrow$ the trader pays liquidity costs.

- "Patient" trading : Optimal portfolio liquidation with limit orders (Bayraktar, Ludkowski 2012)

  $\longrightarrow$ the asset is sold at a random time τ (execution and inventory risks).

- "Mixed" trading : Market making (Guilbaud, Pham 2011)

  $\longrightarrow$ tradeoff between liquidation costs and execution risks

Motivations : Optimal stopping problem

To address liquidation problem of an asset :

- Determine the optimal stopping time to sell one asset at a discounted price.

  $\longrightarrow$ Optimal stopping problem with random maturity :
  Carr (98) and Bouchard, El Karoui, Touzi (05)

- Incorporate a diffusion process for liquidity discount.
  $\longrightarrow$ Multi-dimensional optimal stopping problem
  Broadie, Detemple (97),...

- Incorporate regime switching for the intensity of buy orders arrival.
  $\longrightarrow$ Optimal stopping problem with regime switching
  Guo, Zhang (04)

Related papers on Optimal liquidation include : Schied (09), Bouchard and Dang (11), and Guéant and Lehalle (12).

The financial market

- We consider a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with a filtration $\mathcal{F} = (\mathcal{F}_t)_{t\geq 0}$ satisfying the usual conditions.
- Let W and B be two correlated $(\mathcal{F}_t)$-Brownian motions, with correlation ρ.
- Theoretical/fair value of an illiquid asset evolving according a positive process S, which may be written as $S_t := \exp(X_t)$, where the process X is governed by the following s.d.e.

$$\begin{aligned} dX_t &= \mu(X_t)dt + \sigma(X_t)dB_t \\ X_0 &= x, \end{aligned}$$

where μ and σ are two Lipschitz functions with linear growth conditions.

Liquidity costs

- **(Il)liquidity process** Y, is a mean-reverting non-negative process and governed by the following s.d.e.

$$\begin{aligned} dY_t &= \alpha(Y_t)dt + \gamma(Y_t)dW_t, \\ Y_0 &= y, \end{aligned}$$

  where α and γ are locally Lipschitz function on $\mathbb{R}^+$, more precisely, they may be Lipschitz on $[\epsilon, \infty)$, for any $\epsilon > 0$.

- **Liquidity discount factor** : $(f(Y_t))_{t\geq 0}$, where f is a positive, continuous and decreasing function defined on $\mathbb{R}^+ \longrightarrow [0, 1]$, and satisfies the following conditions :

$$\begin{aligned} f(0) &= 1 \\ \lim_{y\to\infty} f(y) &= 0 \end{aligned}$$

- **Discounted price** : Should the investor decide to sell immediately the assets at the highest available bid price, i.e. at a discounted price, he would obtain a cash-flow of $S_t f(Y_t)$.

Liquidity regimes

- **Liquidity regimes** :
  Let L be a continuous time, time homogenous, irreductible Markov chain, independent of W and B, with $m+1$ states.
  The generator of the chain L under $\mathbb{P}$ is denoted by $A = (\vartheta_{i,j})_{i,j=0,\ldots n}$. Here $\vartheta_{i,j}$ is the constant intensity of transition of the chain L from state i to state j.

- **Market orders arrival** :
  The market order arrival time, denoted by τ, is defined as the first jump time of a Cox process with an intensity $\left(\lambda_{L_t}\right)_{t\geq 0}$.
  τ is independent of W and B and, without lost of generality we assume

$$\lambda_0 \geq \lambda_1 \geq \ldots \geq \lambda_m > 0$$

Utility function

- **Classical assumptions** :
  - $U : \mathbb{R}^+ \longrightarrow \mathbb{R}$, is non-decreasing, concave and belongs to $\mathcal{C}^2(\mathbb{R}^+)$.
  - U has the following behavior

$$\lim_{s\to 0} \; s\, U'(s) \quad < \quad +\infty$$

- **Supermeanvalued utility** : (Dinkyn, Oksendal) The investor is coherent and rationnal : we suppose that U is supermeanvalued w.r.t. S, i.e.

$$U(s) \geq \mathbb{E}^s[U(S_\theta)]$$

  for $s \geq 0$ and any stopping time $\theta \in \mathcal{T}$ where $\mathcal{T}$ is the collection of all $\mathcal{F}-$stopping times.

An optimal stopping problem with regime switching

- Maximizing the expected utility of the wealth received from the sales of the illiquid assets.

Objective function : For $x \in \mathbb{R}$, $y \in \mathbb{R}^+$, $i \in \{0, \ldots, m\}$, we set

$$v(i, x, y) \quad := \quad \sup_{\theta \in \mathcal{T}} \mathbb{E}^{i,x,y} \left[h(X_\theta, Y_\theta)\, \mathbb{I}_{\theta \leq \tau} + U(e^{X_\tau})\, \mathbb{I}_{\theta > \tau} \right] ,$$

where $\mathbb{E}^{i,x,y}$ denotes the flow with initial condition $X_0 = x$, $Y_0 = y$ and $L_0 = i$ and $h(x, y) = U(\exp(x) f(y))$.

Continuity of the objective functions

Continuity of the objective functions

The value functions v_i are continuous on $\mathbb{R} \times \mathbb{R}^+$ and satisfy :

$$\lim_{(u,y)\to(x,0^+)} v_i(u,y) = v_i(x,0) = U(e^x).$$

Proof

- Continuity of stochastic flows (up to $\xi_y := \inf\{t > 0,\ Y_t^y = 0\}$).

Lemma

There exists an optimal stopping time $\theta^*_{i,x,y}$ such that

$$v(i,x,y) = \mathbb{E}^{i,x,y}\left[h(X_{\theta^*_{i,x,y}}, Y_{\theta^*_{i,x,y}})\, 1_{\theta^*_{i,x,y} \le \tau\wedge\xi_y} + U(e^{X_\tau})\, 1_{\theta^*_{i,x,y} > \tau\wedge\xi_y} \right]. \qquad (1)$$

Moreover, on $\{\xi_y \le \tau\}$, we have $\theta^*_{i,x,y} \le \xi_y$.

- Supermeanvalue assumption
- $\lim_{s\to 0}\ s\, U'(s) < +\infty$

Viscosity Characterization of objective function

Theorem

The value functions v_i, $i \in \{0, \ldots, m\}$, are the unique continuous viscosity solutions on $\mathbb{R} \times \mathbb{R}^+$ with growth condition $|v_i(x, y)| \leq |U(e^x)| + |U(e^x f(y)|$, and boundary data $\lim_{y \downarrow 0} v_i(x, y) = U(e^x)$, to the system of variational inequalities :

$$\min \Big[-\mathcal{L}v(i, x, y) - \mathcal{G}_i v(., x, y) - \mathcal{J}_i v(., x, y)\,,\ v(i, x, y) - h(x, y) \Big] = 0,$$

where, for functions φ : $\{0, \ldots, m\} \times \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}$ with $\varphi(i, \cdot, \cdot) \in \mathcal{C}^2(\mathbb{R} \times \mathbb{R}^+)$ for all $i \in \{0, \ldots, m\}$, we have set

$$\mathcal{L}\phi(x, y) = \mu(x)\frac{\partial \phi}{\partial x} + \alpha(y)\frac{\partial \phi}{\partial y} + \frac{1}{2}\sigma^2(x)\frac{\partial^2 \phi}{\partial x^2} + \rho\gamma(y)\sigma(x)\frac{\partial^2 \phi}{\partial x \partial y} + \frac{1}{2}\gamma^2(y)\frac{\partial^2 \phi}{\partial y^2}\,.$$

and $\mathcal{G}_i$ and $\mathcal{J}_i$ act on functions φ :

$$\begin{aligned}
\mathcal{G}_i\varphi(., x, y) &= \sum_{j \neq i} \vartheta_{i,j} \left(\varphi(j, x,\, y) - \varphi(i, x,\, y)\right) \\
\mathcal{J}_i\varphi(., x, y) &= \lambda_i \left(U(e^x) - \varphi(i, x,\, y)\right).
\end{aligned}$$

Dynamic Programming Principle

We deduce from the following Dynamic Programming Principle (**DPP**) that the objective functions are solution of the previous variationnal inequalities system :

Dynamic Programming Principle

For any $(i, x, y) \in \{0, \ldots, n\} \times \mathbb{R} \times (0, \infty)$, for all $\nu \in \mathcal{T}$, we have

$$v(i, x, y) = \sup_{\theta \in \mathcal{T}} \mathbb{E}^{i,x,y} \Big[h(X_\theta, Y_\theta) \mathbb{1}_{\theta \leq \tau \wedge \nu} + U(e^{X_\tau}) \mathbb{1}_{\tau < \theta \wedge \nu} + v(L_\nu, X_\nu, Y_\nu) \mathbb{1}_{\nu < \theta} \mathbb{1}_{\nu \leq \tau} \Big].$$

Proof of uniqueness

Comparaison principle

Let $(\phi_i)_{0\leq i\leq m}$ (resp. $(\psi_i)_{0\leq i\leq m}$) a family of continuous subsolution (resp. super solution) of the VI system on $\mathbb{R}\times\mathbb{R}^+$ satisfying the following growth conditions on $\{0,..,m\}\times\mathbb{R}\times\mathbb{R}^+$

$$|\phi(i,x,y)| + |\psi(i,x,y)| \leq |U(e^x)| + |U(e^x)f(y)| \text{ on } \mathbb{R}\times\mathbb{R}^+$$

and $\lim_{y\to 0^+}\phi_i(x,y) \leq \lim_{y\to 0^+}\psi_i(x,y)$. We have $\phi\leq\psi$ on $\{0,..,m\}\times\mathbb{R}\times\mathbb{R}^+_*$.

Proof.

- **Step 1.** Construction of strict super-solutions to the system with suitable perturbations of ψ_i :

  -

  $$\psi_i^\gamma = (1-\gamma)\psi_i + \gamma\eta_i$$

  - $(\psi_i^\gamma)_{(i=1,\dots,n)}$ is a strict super-solution to the VI system.

- **Step 2.** It suffices to show (by contradiction) that for all $\gamma\in(0,1)$:

$$\max_{i\in\{0,..,m\}}\ \sup_{\mathbb{R}\times\mathbb{R}^+_*}(\phi_i - \psi_i^\gamma) \leq 0,$$

Proof of uniqueness

- Technical point : Construct a strict super solution which dominates ϕ_i and ψ_i.

$\rightarrow$ The following function is a strict super solution which dominates $U(e^x)$

$$g(x,y) = \begin{cases} ax^4 + by^n + k + U(1)\theta(0) + A_1 x + \frac{1}{2}A_2 x^2 & x \leq 0 \\ ax^4 + by^n + k + U(e^x)\theta(x) & x > 0. \end{cases}$$

$\rightarrow$ Existence of such strict super solution follows from characterization of $\mathbb{E}^{i,x}[U(e^{X_\tau})]$ as solution of a system of PDE.

Execution and continuation regions

- **Execution and continuation regions**

$$\begin{aligned} \mathcal{E} &= \{(i,x,y) \in \{0,...,m\} \times \mathbb{R} \times \mathbb{R}^+ \mid v(i,x,y) = h(x,y)\} \\ \mathcal{C} &= \{0,...,m\} \times \mathbb{R} \times \mathbb{R}^+ \setminus \mathcal{E}. \end{aligned}$$

We also define the $(i,x)-$sections for every $(i,x) \in \{0,...,m\} \times \mathbb{R}$ by

$$\mathcal{E}_{(i,x)} = \{y \in (0,+\infty) \mid v(i,x,y) = h(x,y)\} \text{ and } \mathcal{C}_{(i,x)} = \mathbb{R}^+ \setminus \mathcal{E}_{(i,x)}.$$

- **Optimal execution time** : $\theta^*_{ixy} = \inf\left\{u \geq 0 \mid \left(L^i_u, X^x_u, Y^y_u\right) \in \mathcal{E}\right\}$.

Properties of execution region

Let $(i,x) \in \{0,...,m\} \times \mathbb{R}$.

- If $\mathbb{E}^{i,x}[U(e^{X_\tau})] = U(e^x)$, then, for all $y \in \mathbb{R}^+$, $v(i,x,y) = U(e^x)$ and $\mathcal{E}_{(i,x)} = \{0\}$.
- If $\mathbb{E}^{i,x}[U(e^{X_\tau})] < U(e^x)$, then $\exists x_0$ such that $\mathcal{E}_{(i,x_0)} \setminus \{0\} \neq \emptyset$ and $\bar{y}^*(i,x) := \sup \mathcal{E}_{(i,x)} < +\infty$.

Logarithmic utility

Throughout this section we assume that

- $U(s) = \ln(s)$ on $\mathbb{R}_*^+$.
- X and Y solutions of the following SDEs :

$$\begin{array}{rcl} dX_t & = & \mu dt + \sigma(X_t)dB_t;\ X_0 = x \\ dY_t & = & \kappa\,(\beta - Y_t)\,dt + \gamma\sqrt{Y_t}dW_t;\ Y_0 = y \end{array}$$

Remarks

- The supermean value assumption implies that $\mu \leq 0$.
- If $\mu = 0$, we have seen that $v(i,x,y) = U(e^x)$ and $\mathcal{E}_{(i,x)} = \{0\}$

Dimension reduction

Proposition

For $(i, y) \in \{1, ..., m\} \times \mathbb{R}^+$ we define the function :

$$w(i, y) = \sup_{\theta \in \mathcal{T}_{L,W}} \mathbb{E}^{i,y}[\mu(\theta \wedge \tau) + \ln(f(Y_\theta)) \mathbb{I}_{\{\theta \leq \tau\}}],$$

where $\mathcal{T}_{L,W}$ is the set of stopping times with respect to the filtration generated by (L, W). We have

$$v(i, x, y) = x + w(i, y) \text{ on } \{1, ..., m\} \times \mathbb{R} \times \mathbb{R}^+.$$

Proof.

- On $\{0, ..., m\} \times \mathbb{R} \times \mathbb{R}^+$, we have

$$v(i, x, y) = \sup_{\theta \in \mathcal{T}} \mathbb{E}^{i,x,y}[X_{\theta \wedge \tau} + \ln(f(Y_\theta)) \mathbb{I}_{\{\theta \leq \tau\}}].$$

- We prove that $\frac{\partial v}{\partial x}(i, x, y) = 1$ then $v(i, x, y) = x + \phi(i, y)$
- An optimal stopping time is $\theta^*_{ixy} = \inf\{t \geq 0 : \phi(L^i_t, Y^y_t) = \ln(f(Y^y_t))\}$, which belongs to $\mathcal{T}_{L,W}$. We obtain $\phi = w$.

Viscosity characterization

Corollary

$(w(i,\cdot))_{0\leq i\leq m}$ are the unique continuous viscosity solutions of the following system of equation which satisfy $\ln(f(y)) \leq w(i,y) \leq 0$:

$$\min\left[-\overline{\mathcal{L}}w(i,y) + \lambda_i w(i,y) - \sum_{j\neq i} \vartheta_{i,j}\left(w(j,\ y) - w(i,\ y)\right)\ ,\ w(i,y) - \ln(f(y))\right] = 0\,.$$

where , for $\phi \in \mathcal{C}^1(\mathbb{R}^+)$,

$$\overline{\mathcal{L}}\phi(y) = \frac{1}{2}\gamma^2 y \frac{\partial^2\phi}{\partial y^2} + \alpha(\beta - y)\frac{\partial\phi}{\partial y} + \mu.$$

Smooth fit

For all $i \in \{0,..,m\}$, $w(i,\cdot)$ is continuously differentiable.

Execution region

Proposition

Let $i \in \{0, ..., m\}$ and $g(y) = \ln(f(y))$ on $\mathbb{R}_*^+$. We set

$$\hat{y}_i = \inf\{y \geq 0 : \mathcal{H}_i g(y) \geq 0\} \text{ with } \mathcal{H}_i g(y) = \overline{\mathcal{L}} g(y) - \lambda_i g(y) + \sum_{j \neq i} \vartheta_{i,j} \left(w(j, y) - g(y)\right).$$

There exists $y_i^* \geq 0$ such that for all $x \in \mathbb{R}$, $[0, y_i^*] = \mathcal{E}_{(i,x)} \cap [0, \hat{y}_i]$.
Moreover, if $\mathcal{H}_i g(y) > 0$ on $(\hat{y}_i, +\infty)$ then $[0, y_i^*] = \mathcal{E}_{(i,x)}$.

Proposition

If the function $y \to \overline{\mathcal{L}} g(y)$ is non-decreasing then $w(i, \cdot) - g(\cdot)$ is also non decreasing.
Consequently, there exists $y_i^* \geq 0$ such that for all $x \in \mathbb{R}$, $[0, y_i^*] = \mathcal{E}_{(i,x)}$.
Moreover, if $\mu < 0$, $y_i^* > 0$.

Exemple

If we set $f(y) = e^{-y}$ on $\mathbb{R}^+$, we have $\overline{\mathcal{L}} g(y) = \alpha(y - \beta)$ and then $[0, y_i^*] = \mathcal{E}_{(i,x)}$.

No regime switch

Let $i \in \{1, ..., m\}$. Throughout this section, we shall assume that $\vartheta_{i,j} = 0 \; \forall i \neq j$ and that there exists $0 < y_i^*$ such that $\mathcal{E}_{(i,x)} = [0, y_i^*]$.

Proposition

y_i^* is the solution of the following equation

$$\frac{g(y_i^*) - \frac{\mu}{\lambda_i}}{g'(y_i^*)} = -\frac{\gamma^2}{2\lambda_i} \frac{\Psi\left(\frac{\lambda_i}{\alpha}, \frac{2\alpha\beta}{\gamma^2}, \frac{2\alpha}{\gamma^2} y_i^*\right)}{\Psi\left(\frac{\lambda_i}{\alpha} + 1, \frac{2\alpha\beta}{\gamma^2} + 1, \frac{2\alpha}{\gamma^2} y_i^*\right)}.$$

the function $w(i, \cdot)$ is given by

$$w(i, y) = \begin{cases} g(y) & y \leq y_i^* \\ \dfrac{g(y_i^*) - \frac{\mu}{\lambda_i}}{\Psi\left(\frac{\lambda_i}{\alpha}, \frac{2\alpha\beta}{\gamma^2}, \frac{2\alpha}{\gamma^2} y_i^*\right)} \Psi\left(\frac{\lambda_i}{\alpha}, \frac{2\alpha\beta}{\gamma^2}, \frac{2\alpha}{\gamma^2} y\right) + \dfrac{\mu}{\lambda_i} & y > y_i^* \end{cases}$$

where Ψ denotes the confluent hypergeometric function of second kind

Two regimes

We assume that $m = 1$, $\vartheta_{0,1}\vartheta_{1,0} \neq 0$ and that, for $i \in \{0, 1\}$, there exists $y_i^* > 0$ such that $\mathcal{E}_{(i,s)} = [0, y_i^*]$.

Proposition

We can show that $y_0^* \leq y_1^*$

Let Λ be the matrix

$$\Lambda = \begin{pmatrix} \lambda_0 + \vartheta_{0,1} & -\vartheta_{0,1} \\ -\vartheta_{1,0} & \lambda_1 + \vartheta_{1,0} \end{pmatrix}$$

As $\vartheta_{0,1}\vartheta_{1,0} > 0$ it is easy to check that Λ has two eigenvalues $\widetilde{\lambda}_0$ and $\widetilde{\lambda}_1 < \widetilde{\lambda}_0$. Let $\widetilde{\Lambda} = P^{-1}\Lambda P$ be the diagonal matrix with diagonal $(\widetilde{\lambda}_0, \widetilde{\lambda}_1)$. The transition matrix P is denoted by

$$P = \begin{pmatrix} p_0^0 & p_1^0 \\ p_0^1 & p_1^1 \end{pmatrix}.$$

Two regimes

Proposition

The function $w(1,\cdot)$ is given by

$$w(1,y) = \begin{cases} g(y) & y \in [0, y_1^*] \\ p_0^1 \left[\widehat{e}\Psi\left(\frac{\widetilde{\lambda}_0}{\alpha}, \frac{2\alpha\beta}{\gamma^2}, \frac{2\alpha}{\gamma^2}x \right) + \frac{\mu}{\widetilde{\lambda}_0} \right] & y \in (y_1^*, \infty) \\ \qquad +p_1^1 \left[\widehat{f}\Psi\left(\frac{\widetilde{\lambda}_1}{\alpha}, \frac{2\alpha\beta}{\gamma^2}, \frac{2\alpha}{\gamma^2}y \right) + \frac{\mu}{\widetilde{\lambda}_1} \right] & \end{cases}$$

where Ψ denotes the confluent hypergeometric function of second kind, $\mathcal{I}$ is a particular solution to the non-homogeneous confluent differential equation.

Two regimes

Proposition

The function $w(0,\cdot)$ is given by

$$w(0,y)=\begin{cases} g(y) & y\in[0,y_0^*] \\ \widehat{c}\Phi\left(\frac{\lambda_0+\vartheta_{0,1}}{\alpha},\frac{2\alpha\beta}{\gamma^2},\frac{2\alpha}{\gamma^2}y\right)+\widehat{d}\Psi\left(\frac{\lambda_0+\vartheta_{0,1}}{\alpha},\frac{2\alpha\beta}{\gamma^2},\frac{2\alpha}{\gamma^2}y\right) & y\in(y_0^*,y_1^*] \\ \quad +\mathcal{I}\left(\frac{2\alpha}{\gamma^2},\beta,-2\frac{\lambda_0+\vartheta_{0,1}}{\gamma^2},2\frac{\vartheta_{0,1}g(\cdot)+\mu}{\gamma^2}\right)(y) & \\ p_0^0\left[\widehat{e}\Psi\left(\frac{\widetilde{\lambda}_0}{\alpha},\frac{2\alpha\beta}{\gamma^2},\frac{2\alpha}{\gamma^2}y\right)+\frac{\mu}{\widetilde{\lambda}_0}\right] & y\in(y_1^*,\infty) \\ \quad +p_1^0\left[\widehat{f}\Psi\left(\frac{\widetilde{\lambda}_1}{\alpha},\frac{2\alpha\beta}{\gamma^2},\frac{2\alpha}{\gamma^2}y\right)+\frac{\mu}{\widetilde{\lambda}_1}\right] & \end{cases}$$

where Φ and Ψ denote respectively the confluent hypergeometric function of first and second kind, $\mathcal{I}$ is a particular solution to the non-homogeneous confluent differential equation.

Power utility

Throughout this section we assume that

- $U(s) = s^a$ on $\mathbb{R}_*^+$ with $0 < a \leq 1$.
- There exists μ and σ in $\mathbb{R}$ s.t. X is solution of the following sde :

$$dX_t = \mu dt + \sigma dB_t$$

Remarks

- The supermean value assumption implies that $\mu a + \frac{\sigma^2}{2} a^2 \leq 0$.
- If $\mu a + \frac{\sigma^2}{2} a^2 = 0$, we have seen that $v(i, x, y) = U(e^x)$ and $\mathcal{E}_{(i,x)} = \{0\}$

Dimension reduction

Proposition

For $(i, y) \in \{1, ..., m\} \times \mathbb{R}^+$ we define the function :

$$u(i,y) = \sup_{\theta \in \mathcal{T}_{L,W}} \mathbb{E}^{i,y}[e^{(\mu+(1-\rho^2)\frac{\sigma^2}{2})(\theta\wedge\tau)+\rho\sigma W_{\theta\wedge\tau}} \left(\mathbb{I}_{\{\theta>\tau\}} + g(Y_\theta)\mathbb{I}_{\{\theta\le\tau\}}\right)].$$

We have

$$v(i,x,y) = e^{ax} u(i,y) \text{ on } \{1, ..., m\} \times \mathbb{R} \times \mathbb{R}^+.$$

$(u(i,\cdot))_{0\le i\le m}$ are the unique continuously differentiable viscosity solutions of the system of equation :

$$\min\Big[-\tilde{\mathcal{L}}u(i,y) - \lambda_i(1-u(i,y)) - \sum_{j\neq i} \vartheta_{i,j}\left(u(j,\,y) - u(i,\,y)\right)\,,\, u(i,y) - g(y)\Big] = 0\,.$$

where we have set $g(y) = (f(y))^a$ and, for $\phi \in \mathcal{C}^1(\mathbb{R}^+)$,

$$\tilde{\mathcal{L}}\phi(y) = \frac{1}{2}\gamma^2 y \frac{\partial^2\phi}{\partial y^2} + \Big[\alpha(\beta - y) + \rho\sigma\gamma a\sqrt{y}\Big]\frac{\partial\phi}{\partial y} + \Big[\frac{\sigma^2 a^2}{2} + \mu a\Big]\phi(y).$$

Conclusions

- Study of a liquidation problem of an illiquid asset
- Mathematical characterisation of the objective function
- Explicit solutions for specific utility functions (power and exponential)

Thank you for your attention !