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Robustness of quadratic hedging strategies via BSDEs with jumps

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Outline

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Outline

Background

- model process consisting of continuous part + jumps, e.g., Lévy process
- BM + Poisson random measure
- small jumps (SJ) and large jumps
- jumps may have infinite activity
  in any finite time interval there are infinitely many jumps

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- algorithms to simulate general Lévy processes
- Asmussen and Rosinski (2001):

$$\frac{SJ - \mathbb{E}[SJ]}{\sqrt{Var[SJ]}} \sim BM$$

SJ $\approx$ scaled(ε) BM
convergence in distribution when $\varepsilon \to 0$

- approximation consisting of BM + compound Poisson
- practical purpose for simulations

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ROBUSTNESS of MODEL / SENSITIVITY

A. Khedher, PhD Thesis (2010)

- robustness of the option price
- robustness of the delta
- robustness of quadratic hedging strategies?

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Setting

- We consider a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ equipped with the $\mathbb{P}$-augmented filtration $\mathcal{F}_t$, $t \in [0, T]$.
- A risk free asset with price $S^0(t) > 0$.
  A risky asset with price $S^1(t)$.
- The discounted price process is given by

$$\widetilde{S}(t) = \frac{S^1(t)}{S^0(t)}.$$

- The value of a hedging portfolio is given by

$$V(t) = \underbrace{\chi(t)}_{\text{number of units of asset 1}} \widetilde{S}(t) + \underbrace{\eta(t)}_{\text{amount invested in asset 0}}.$$

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- We consider a portfolio strategy of the form $\varphi = (\chi, \eta)$.
- The gain process is given by

$$G(t,\chi) = \int_0^t \chi(s) d\widetilde{S}(s).$$

- The cost process is given by

$$C(t) = V(t) - G(t,\chi).$$

- A strategy is called self-financing if C is a constant or equivalently

$$V(t) = V(0) + G(t,\chi).$$

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Hedging problem

- We fix a contingent claim ξ. Example: European call option

$$\xi(\omega) = \max(\widetilde{S}(T,\omega) - K, 0).$$

Hedging in the Black-Scholes model:
$d\widetilde{S}(t) = \sigma \widetilde{S}(t) dW(t)$. (Under $\widetilde{\mathbb{P}}$).

$$\xi = \xi_0 + \int_0^T \chi(s) d\widetilde{S}(s). \qquad (\xi_0 \text{ is a constant!})$$

$\Longrightarrow$ The market is complete.
$\Longrightarrow$ Exact replication.

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Incomplete market

- Incomplete market: martingale representation does not hold, i.e., ξ_0 is not constant. It is in general not possible to find a self-financing strategy V with final value $V(T) = \xi$.
- Föllmer and Sondermann (1986).
  Quadratic hedging: introduce subjective criteria under which the strategies are chosen.
  Locally risk-minimizing strategies: $V(T) = \xi$. The strategy is not self-financing.
  Mean variance hedging strategies: we do not impose $V(T) = \xi$. We insist on the self-financing constraint.

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FS decomposition

- A semimartingale $\widetilde{S}$ is a process of the form

$$\widetilde{S} = \widetilde{S}(0) + \underbrace{M}_{\text{local martingale}} + \underbrace{A}_{\text{finite variation process}} .$$

- A special semimartingale $\widetilde{S}$ is a semimartingale with the decomposition

$$\widetilde{S} = \widetilde{S}(0) + M + A,$$

where A is a predictable process.

- Föllmer-Schweizer decomposition Föllmer and Schweizer (1991)

$$\xi = \xi_0 + \int_0^T \chi^{FS}(s) d\widetilde{S}(s) + \Phi_T^{FS},$$

where the process Φ^{FS} is a martingale orthogonal to M.

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Outline

Robustness of BSDEJs

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- W is a standard Wiener process
  $\widetilde{N} = \widetilde{N}(dt, dz)$ is a centered Poisson random measure.
- We introduce the $\mathbb{P}$-augmented filtration $\mathbb{F} = (\mathcal{F}_t)_{0 \le t \le T}$ by

$$\mathcal{F}_t = \sigma\Big\{ W(s), \int_0^s \int_A \widetilde{N}(du, dz), \;\; s \le t, \;\; A \in \mathcal{B}(\mathbb{R}_0) \Big\}$$

$$\begin{cases} -dX(t) = & f(t, X(t), Y(t), Z(t, \cdot))dt - Y(t)dW(t) \\ & - \int_{\mathbb{R}_0} Z(t,z)\widetilde{N}(dt, dz), \\ X(T) = & \xi. \end{cases} \tag{1}$$

A solution to (1) is (X, Y, Z) $\mathbb{F}$-adapted.
Existence and uniqueness result can be found in Tang and Li (1994).

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Two approximating BSDEJs

- $\lim_{\varepsilon\to 0}\xi_\varepsilon^0=\xi$
- *First candidate approximation*

$$\begin{cases} -dX_\varepsilon^0(t) = & f^0(t,X_\varepsilon^0(t),Y_\varepsilon^0(t),Z_\varepsilon^0(t,\cdot))dt - Y_\varepsilon^0(t)dW(t) \\ & -\int_{\mathbb{R}_0} Z_\varepsilon^0(t,z)\widetilde{N}(dt,dz), \\ X_\varepsilon^0(T) = & \xi_\varepsilon^0. \end{cases} \tag{2}$$

A solution to (2) is $(X_\varepsilon^0,Y_\varepsilon^0,Z_\varepsilon^0)$ $\mathbb{F}$-adapted.

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Two approximating BSDEJs

- B is a standard Wiener process independent of W
- We define the $\mathbb{P}$-augmented filtration $\mathbb{G} = (\mathcal{G}_t)_{0\leq t\leq T}$ by

$$\mathcal{G}_t = \sigma\Big\{W(s), B(s), \int_0^s \int_A \widetilde{N}(du, dz), \;\; s \leq t, \;\; A \in \mathcal{B}(\mathbb{R}_0)\Big\}.$$

- *Second candidate approximation*

$$\begin{cases} -dX^1_\varepsilon(t) = & f^1(t, X^1_\varepsilon(t), Y^1_\varepsilon(t), Z^1_\varepsilon(t,\cdot), \zeta_\varepsilon(t))dt - Y^1_\varepsilon(t)dW(t) \\ & - \int_{\mathbb{R}_0} Z^1_\varepsilon(t,z)\widetilde{N}(dt,dz) - \zeta_\varepsilon(t)dB(t), \\ X^1_\varepsilon(T) = & \xi^1_\varepsilon, \end{cases} \tag{3}$$

A solution to (3) is $(X^1_\varepsilon, Y^1_\varepsilon, Z^1_\varepsilon, \zeta_\varepsilon)$ $\mathbb{G}$-adapted.

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Convergence result

Theorem

Assume that f^0 and f^1 satisfy

$$|f(t,x_1,y_1,z_1) - f^0(t,x_2,y_2,z_2)| \leq C\Big(|x_1 - x_2| + |y_1 - y_2| + \|z_1 - z_2\|\Big), \quad \textit{for all } t,$$

$$|f(t,x_1,y_1,z_1) - f^1(t,x_2,y_2,z_2,\zeta)| \leq C\Big(|x_1 - x_2| + |y_1 - y_2| + \|z_1 - z_2\| + |\zeta|\Big) \quad \textit{for all } t.$$

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Convergence result

Theorem

Then we have for $t \in [0, T]$, and for $\rho = 0, 1$,

- $$\mathbb{E}\Big[\int_t^T |X(s) - X_\varepsilon^\rho(s)|^2 ds\Big] + \mathbb{E}\Big[\int_t^T |Y(s) - Y_\varepsilon^\rho(s)|^2 ds\Big] + \mathbb{E}\Big[\int_t^T \int_{\mathbb{R}_0} |Z(s,z) - Z_\varepsilon^\rho(s,z)|^2 \ell(dz) ds\Big] + \rho\mathbb{E}\Big[\int_t^T |\zeta_\varepsilon^1(s)|^2 ds\Big] \leq K\mathbb{E}\left[|\xi - \xi_\varepsilon^\rho|^2\right],$$

- $$\mathbb{E}\left[\sup_{0\leq t\leq T} |X(t) - X_\varepsilon^\rho(t)|^2\right] \leq K\mathbb{E}\left[|\xi - \xi_\varepsilon^\rho|^2\right]$$

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Outline

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Model

- A risk free asset: $dS^{(0)}(t) = S^{(0)}(t)r(t)dt$,
  A risky asset:
  $dS^{(1)}(t) = S^{(1)}(t)\Big[a(t)dt + b(t)dW(t) + \int_{\mathbb{R}_0} \gamma(t,z)\widetilde{N}(dt,dz)\Big]$.
- The dynamics of the discounted price process $\widetilde{S} = \frac{S^{(1)}}{S^{(0)}}$:

$$d\widetilde{S}(t) = \widetilde{S}(t)\Big[(a(t) - r(t))dt + b(t)dW(t) + \int_{\mathbb{R}_0} \gamma(t,z)\widetilde{N}(dt,dz)\Big].$$

- $\widetilde{S} = \widetilde{S}(0) + M + A$, where

$$M(t) = \int_0^t b(s)\widetilde{S}(s)dW(s) + \int_0^t \int_{\mathbb{R}_0} \gamma(s,z)\widetilde{S}(s)\widetilde{N}(ds,dz),$$

$$A(t) = \int_0^t (a(s) - r(s))\widetilde{S}(s)ds.$$

LRM strategy

We have

$$\widetilde{V}(t) = \mathbb{E}_{\mathbb{Q}}[\widetilde{\xi}|\mathcal{F}_t], \;\; \widetilde{\xi} = \xi/S^{(0)}(T)$$

where $\mathbb{Q}$ is a martingale measure.
In a risk-minimizing strategy, the martingale measure is the risk minimal martingale measure (Föllmer and Sondermann (1986)).
We assume for the MVT-process

$$K(t) = \int_0^t \underbrace{\frac{(a(s)-r(s))^2}{b^2(s)+\int_{\mathbb{R}_0}\gamma^2(s,z)\ell(dz)}}_{\alpha^2(s)} ds < C \;\; \mathbb{P}\text{-a.s.}$$

and

$$\widetilde{S}(t)\alpha(t)\gamma(t,z) > -1, \qquad \text{a.e. in } (t,z,\omega).$$

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FS-decomposition and LRM strategy

We have the following FS decomposition for $\widetilde{V}$ (Choulli et al. (1998))

$$\widetilde{V}(t) = \mathbb{E}_{\mathbb{Q}}[\widetilde{\xi}] + \int_0^t \chi^{FS}(s) d\widetilde{S}(s) + \phi^{FS}(t),$$

where ϕ^{FS} is a $\mathbb{P}$-martingale orthogonal to M.
Replacing $\widetilde{S}$ by its value we get

$$\begin{cases} d\widetilde{V}(t) = & \widetilde{\pi}(t)(a(t) - r(t))dt + \widetilde{\pi}(t)b(t)dW(t) \\ & + \int_{\mathbb{R}_0} \widetilde{\pi}(t)\gamma(t,z)\widetilde{N}(dt,dz) + d\phi^{FS}(t), \\ \widetilde{V}(T) = & \widetilde{\xi}, \end{cases}$$

where $\widetilde{\pi} = \chi^{FS}\widetilde{S}$.

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Martingale representation theorem

Kunita and Watanabe (1967).

Theorem

Every $\xi \in L^2_T$, $\mathcal{G}_T$-measurable has a representation of the form

$$\xi = \mathbb{E}[\xi] + \sum_{k=1}^{3} \int_0^T \int_{\mathbb{R}} \varphi_k(t,z)\mu_k(dt,dz), \tag{4}$$

where the stochastic integrators

$$\mu_1(dt,dz) = W(dt) \times \delta_0(dz), \quad \mu_2(dt,dz) = B(dt) \times \delta_0(dz),$$
$$\mu_3(dt,dz) = \widetilde{N}(dt,dz)\mathbf{1}_{[0,T]\times\mathbb{R}_0}(t,z),$$

are orthogonal martingale random fields on $[0,T] \times \mathbb{R}_0$.
For every $\xi \in L^2_T$, $\mathcal{F}_T$-measurable, (4) holds with $\mu_2(dt,dz) = 0$.

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Approximating LRM strategy

Applying the martingale representation theorem,

$$\phi^{FS}(t) = \underbrace{\mathbb{E}[\phi^{FS}(T)]}_{=0} + \int_0^t Y^{FS}(s)dW(s) + \int_0^t \int_{\mathbb{R}_0} Z^{FS}(s,z)\widetilde{N}(ds,dz).$$

In that case, we have

$$\begin{cases} d\widetilde{V}(t) = & \widetilde{\pi}(t)(a(t)-r(t))dt + \big(\widetilde{\pi}(t)b(t) + Y^{FS}(t)\big)dW(t) \\ & \qquad + \int_{\mathbb{R}_0} \big(\widetilde{\pi}(t)\gamma(t,z) + Z^{FS}(t,z)\big)\widetilde{N}(dt,dz), \\ \widetilde{V}(T) = & \widetilde{\xi}. \end{cases}$$

Approximating LRM strategy

Lemma

Let $\kappa(t) = b^2(t) + \int_{\mathbb{R}_0} \gamma^2(t,z)\ell(dz)$. *Assume that for all* $t \in [0,T]$

$$\frac{|a(t) - r(t)|}{\sqrt{\kappa(t)}} \leq C, \ \mathbb{P}\text{-a.s.}, \tag{5}$$

for a positive constant C. *Then BSDEJ for* $\widetilde{V}$ *is of type* (1).

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$$\begin{cases} -dX(t) = & f(t, X(t), Y(t), Z(t,\cdot))dt - Y(t)dW(t) \\ & - \int_{\mathbb{R}_0} Z(t,z)\widetilde{N}(dt,dz), \\ X(T) = & \xi. \end{cases}$$

$$\begin{cases} -d\widetilde{V}(t) = & -\widetilde{\pi}(t)(a(t) - r(t))dt - \big(\widetilde{\pi}(t)b(t) + Y^{FS}(t)\big)dW(t) \\ & - \int_{\mathbb{R}_0} \big(\widetilde{\pi}(t)\gamma(t,z) + Z^{FS}(t,z)\big)\widetilde{N}(dt,dz), \\ \widetilde{V}(T) = & \widetilde{\xi}. \end{cases}$$

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First candidate approximation to S

The discounted price process is given by

$$d\widetilde{S}_{1,\varepsilon}(t) = \widetilde{S}_{1,\varepsilon}(t)\Big\{(a(t)-r(t))dt + b(t)dW(t) + \int_{|z|>\varepsilon}\gamma(t,z)\widetilde{N}(dt,dz) + G(\varepsilon)\widetilde{\gamma}(t)dB(t)\Big\}.$$

Benth, Di Nunno, and Khedher (2011).

Let

$$\widetilde{V}_{1,\varepsilon}(t) = \mathbb{E}_{\mathbb{Q}_{1,\varepsilon}}[\widetilde{\xi}^{1}_{\varepsilon} \mid \mathcal{G}_t],$$

where $\mathbb{Q}_{1,\varepsilon}$ is the minimal martingale measure.

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First candidate approximation to LRM

The equation we obtain for the approximating problem is thus given by

$$\begin{cases} d\widetilde{V}_{1,\varepsilon}(t) = & \widetilde{\pi}_{1,\varepsilon}(t)(a(t)-r(t))dt + (\widetilde{\pi}_{1,\varepsilon}(t)b(t) + Y^{FS}_{1,\varepsilon}(t))dW(t) \\ & +(\widetilde{\pi}_{1,\varepsilon}(t)G(\varepsilon)\widetilde{\gamma}(t) + Y^{FS}_{2,\varepsilon}(t))dB(t) \\ & + \int_{|z|>\varepsilon} \left(\widetilde{\pi}_{1,\varepsilon}(t)\gamma(t,z) + Z^{FS}_{\varepsilon}(t,z)\right)\widetilde{N}(dt,dz), \\ \widetilde{V}_{1,\varepsilon}(T) = & \widetilde{\xi}^1_{\varepsilon}, \end{cases}$$

where $\widetilde{\pi}_{1,\varepsilon} = \chi^{FS}_{1,\varepsilon}\widetilde{S}_{1,\varepsilon}$.

$\Longrightarrow$ BSDEJ for $\widetilde{V}_{1,\varepsilon}$ is of type (3)

Daveloose, Khedher, Vanmaele (2013).

First candidate approximation to LRM

Theorem

Assume that for all $t \in [0, T]$ $\frac{|a(t)-r(t)|}{\sqrt{\kappa(t)}} \leq C$, $\mathbb{P}$*-a.s.. Then ,*

$$\mathbb{E}\Big[\sup_{0\leq t\leq T} |\widetilde{V}(t) - \widetilde{V}_{1,\varepsilon}(t)|^2\Big] \leq C\mathbb{E}[|\widetilde{\xi} - \widetilde{\xi}_\varepsilon^1|^2].$$

Assume that (5) *holds and that for all* $t \in [0, T]$

$$\inf_{t\leq s\leq T} \kappa(s) \geq K, \ \mathbb{P}\text{-a.s.} \tag{6}$$

Then for all $t \in [0, T]$*,*

$$\mathbb{E}\Big[\int_t^T |\widetilde{\pi}(s) - \widetilde{\pi}_{1,\varepsilon}(s)|^2 ds\Big] \leq C\mathbb{E}[|\widetilde{\xi} - \widetilde{\xi}_\varepsilon^1|^2].$$

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Robustness of remaining risk

Theorem

Assume that (5) and (6) hold and for all $t \in [0, T]$

$$\sup_{t \le s \le T} \widetilde{\gamma}^2(s) \le \widetilde{K}, \quad \sup_{t \le s \le T} \kappa(s) \le \widehat{K} < \infty \ \mathbb{P}\text{-a.s.}$$

Then for all $t \in [0, T]$

$$\mathbb{E}\Big[|\phi^{FS}(t) - \phi^{FS}_{1,\varepsilon}(t)|^2\Big] \le C\mathbb{E}[|\widetilde{\xi} - \widetilde{\xi}^1_\varepsilon|^2] + C' G(\varepsilon),$$

where C and C' are two positive constants.

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Robustness of cost process

Let

$$C(t) = \phi^{FS}(t) + \widetilde{V}(0)$$

and

$$C_{1,\varepsilon}(t) = \phi^{FS}_{1,\varepsilon}(t) + \widetilde{V}_{1,\varepsilon}(0).$$

Corollary

Under the assumption of the last lemma we have for all $t \in [0, T]$

$$\mathbb{E}[|C(t) - C_{1,\varepsilon}(t)|^2] \leq K\mathbb{E}[|\widetilde{\xi} - \widetilde{\xi}^1_\varepsilon|^2] + K'G(\varepsilon),$$

where K and K' are two positive constants.

Second candidate approximation to S

The discounted price process is given by

$$d\widetilde{S}_{0,\varepsilon}(t) = \widetilde{S}_{0,\varepsilon}(t)\Big[(a(t)-r(t))dt + (b(t)+G(\varepsilon)\widetilde{\gamma}(t))dW(t) + \int_{|z|>\varepsilon}\gamma(t,z)\widetilde{N}(dt,dz)\Big].$$

Let $\widetilde{V}_{0,\varepsilon}(t) = \mathbb{E}_{\mathbb{Q}_{0,\varepsilon}}[\widetilde{\xi}^0_\varepsilon \mid \mathcal{F}_t]$, where $\mathbb{Q}_{0,\varepsilon}$ is the minimal martingale measure. Thus

$$\begin{cases} d\widetilde{V}_{0,\varepsilon}(t) = & \widetilde{\pi}^1_{0,\varepsilon}(t)(a(t)-r(t))dt + \big(\widetilde{\pi}_{0,\varepsilon}(t)[b(t)+G(\varepsilon)\widetilde{\gamma}(t)] \\ & + Y^{FS}_\varepsilon(t)\big)dW(t) + \int_{|z|>\varepsilon}\big(\widetilde{\pi}_{0,\varepsilon}(t)\gamma(t,z) + Z^{FS}_\varepsilon(t,z)\big)\widetilde{N}(dt,dz), \\ \widetilde{V}_{0,\varepsilon}(T) = & \widetilde{\xi}^0_\varepsilon, \end{cases}$$

where $\widetilde{\pi}_{0,\varepsilon}(t) = \chi^{FS}_{0,\varepsilon}\widetilde{S}_{1,\varepsilon}$. $\Longrightarrow$ BSDEJ for $\widetilde{V}_{0,\varepsilon}$ is of type (2)

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- case $S_{1,\varepsilon}$
  - variance of continuous part $= b^2(t) + G^2(\varepsilon)\widetilde{\gamma}^2(t)$
    $=$ variance of continuous part $+$ variance of SJ in S
  - study approximation by embedding original model solution into larger filtration $\mathbb{G}$
- case $S_{0,\varepsilon}$
  - variance of continuous part $= (b(t) + G(\varepsilon)\widetilde{\gamma}(t))^2$
    $\neq$ variance of continuous part $+$ variance of SJ in S
  - approximation of solution in original filtration $\mathbb{F}$

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Mean-variance hedging

- self-financing strategy
- the shortfall or loss from hedging $\widetilde{\xi}$ is given by

$$\widetilde{\xi}-\widetilde{V}(T)=\widetilde{\xi}-\widetilde{V}(0)-\int_0^T \widetilde{\Gamma}(s)d\widetilde{S}(s), \quad \widetilde{V}(0)\in\mathbb{R}, \quad \widetilde{\Gamma}\in\Theta\,.$$

- minimize the latter quantity in the L^2-norm
- the finite variation process A in the decomposition of $\widetilde{S}$ is continuous
- assuming that the MVT process K with $K(t)=\int_0^t \alpha^2(s)ds$ is deterministic, the discounted number of risky assets is given by

$$\widetilde{\Gamma}(t)=\widetilde{\chi}^{FS}(t)+\alpha(t)\Big(\widetilde{V}(t)-\widetilde{V}(0)-\int_0^t \widetilde{\Gamma}(s)d\widetilde{S}(s)\Big), \quad (7)$$

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Mean-variance hedging

- $\widetilde{V}(t) = \mathbb{E}_{\widetilde{Q}}[\widetilde{\xi}|\mathcal{F}_t]$, $0 \le t \le T$, where $\widetilde{Q}$ is the minimal martingale measure
- the amount of wealth is given by

$$\widetilde{\Upsilon}(t) = \widetilde{\pi}(t) + h(t)\Big(\widetilde{V}(t) - \widetilde{V}(0) - \int_0^t \frac{\widetilde{\Upsilon}(s)}{\widetilde{S}(s)} d\widetilde{S}(s)\Big),$$

- the amount of wealth associated to $\widetilde{S}_{1,\varepsilon}$ is given by

$$\widetilde{\Upsilon}_{1,\varepsilon}(t) = \widetilde{\pi}_{1,\varepsilon}(t) + h(t)\Big(\widetilde{V}_{1,\varepsilon}(t) - \widetilde{V}_{1,\varepsilon}(0) - \int_0^t \frac{\widetilde{\Upsilon}_{1,\varepsilon}(s)}{\widetilde{S}_{1,\varepsilon}} d\widetilde{S}_{1,\varepsilon}(s)\Big).$$

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Robustness of mean-variance hedging

Theorem

Assume the mean-variance tradeoff process is deterministic and that (5) *and* (6) *hold. Then for all* $t \in [0, T]$,

$$\mathbb{E}\Big[|\widetilde{\Upsilon}(t) - \widetilde{\Upsilon}_{1,\varepsilon}(t)|^2\Big] \leq C\mathbb{E}[|\widetilde{\xi} - \widetilde{\xi}^1_\varepsilon|^2] + \widetilde{C}G^2(\varepsilon).$$

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Outline

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Conclusion

- we considered different models for the price process
- using BSDEJs we proved that the locally risk-minimizing and the mean-variance hedging strategies are robust towards the choice of the model
- results are given in terms of estimates containing $\mathbb{E}[|\widetilde{\xi} - \widetilde{\xi}_{\varepsilon}^{\rho}|^2]$, which is a quantity well studied by Benth, Di Nunno, and Khedher (2010) and Kohatsu-Higa and Tankov (2010).

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Conclusion

Ongoing research

- Discretizing the approximating BSDEJs.
- Computing the error coming from the combined effect discretization and approximation
- Apply this to study the robustness of discrete quadratic hedging strategies.

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References

- Benth, F. E., Di Nunno, G., and Khedher, A. (2011). Robustness of option prices and their deltas in markets modelled by jump-diffusions.*Comm. Stochastic Analysis.*, **5** (2), pp. 285–307.
- Benth, F. E., Di Nunno, G., and Khedher, A. (2013). A note on convergence of option prices and their Greeks for Lévy models. *Stochastics: An International Journal of Probability and Stochastic Processes.* **0** (0), pp 1–25
- Choulli, T., Krawczyk, L., Stricker, C. (1998). $\mathcal{E}$-martingales and their applications in mathematical finance. *Annals of Probability.*, **26**, 2, pp. 853–876.
- Daveloose, C., Khedher, A., Vanmaele, M. (2013). Robustness study of the hedging of European claims. Working paper

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References

- Föllmer, H., Sondermann, D. (1986). Hedging of non redundant contingent claims. In: W. Hildenbrand and A. Mas-Collel, editors, *Contributions to Mathematical Economics.* pp. 205–223. North-Holland, Elsevier, 1986.
- Kohatsu-Higa, A., and Tankov, P. (2010). jump-adapted discretization schemes for Lévy-driven SDEs. *Stochastic Processes and their Applications*, **120**, 11, pp. 2258–2285.
- Kunita, H., and Watanabe, S. (1967). On square integrable martingales. *Nagoya Math.*, **30**, pp. 209–245.
- S.Tang, X.Li. (1994). Necessary conditions for optimal control of stochastic systems with random jumps. *SIAM J. Control Optim*, **32**, (5), pp. 1447–1475.

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Thanks for your attention