Applications of the Likelihood Theory in Finance: Modelling and Pricing

Arnold Janssen (joint work with Martin Tietje)

Heinrich-Heine-Universität Düsseldorf

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Heinrich-Heine-Universität Düsseldorf, Universitätsstr. 1, 40225 Düsseldorf, Germany
E-mails: janssena@math.uni-duesseldorf.de, tietje@math.uni-duesseldorf.de

Summary

This paper discusses the connection between mathematical finance and statistical modelling which turns out to be more than a formal mathematical correspondence. We like to figure out how common results and notions in statistics and their meaning can be translated to the world of mathematical finance and vice versa. A lot of similarities can be expressed in terms of LeCam's theory for statistical experiments which is the theory of the behaviour of likelihood processes. For positive prices the

Outline

- 1. Motivation
- 2. Representation of financial models as statistical experiments
- 3. Option prices as functions of power functions of tests
- 4. Convergence of option prices

1. Motivation

Goal:

1. Application of statistical results in finance.
2. Review parallel working in finance and statistical modelling (Le Cam theory).

Similarities:

- Filtered likelihood processes
- Regression models
- Contiguity
- Completeness
- ...

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Examples of statistical concepts in finance

- geometric Brownian motion
- Contiguity: asymptotic arbitrage
  Kabanov / Kramkov, Hubalek / Schachermeyer
- Neyman Pearson tests
  Föllmer / Leukert, Schied, Rudloff / Karatzas
- binary experiments
  Gushin / Mordecki
- Le Cam's third Lemma in finance
  Shiryaev

2. Representation of financial models as statistical experiments

Filtered statistical experiment:

- $E = (\Omega, \mathcal{F}, \{P_\vartheta : \vartheta \in \Theta\})$
- Filtration $(\mathcal{F}_t)_{t \geq 0}$.

Financial model:

- Time intervall: $I \subset [0, T]$ with $T < \infty$, $\{0, T\} \subset I$,
- Filtered probability space $(\Omega, \mathcal{F}, P)$ with filtration $(\mathcal{F}_t)_{t \in I}$, $\mathcal{F} = \mathcal{F}_T$, $\mathcal{F}_0 = \{N : P(N) = 0 \text{ or } P(N) = 1\}$
- **Adapted, positive, discounted price processes** $(X_t^i)_{t \in I}$, $1 \leq i \leq d$.

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Martingale measure:
Q Martingale measure, if $(X_t^i)_{t\in I}$ Q-martingale $\forall 1 \le i \le d$.

Theorem 1

Let Q be a probability measure equivalent to P. The following assertions are equivalent:

(1) *There are probability measures* $Q_1, ..., Q_d$ *on* $(\Omega, \mathcal{F})$ *satisfying*

$$\frac{\mathbf{dQ_{i|\mathcal{F}_t}}}{\mathbf{dQ_{|\mathcal{F}_t}}} = \frac{\mathbf{X_t^i}}{\mathbf{X_0^i}}, \quad \mathbf{t} \in \mathbf{I}. \tag{1}$$

and $Q_i \ll Q$ *for all* $1 \le i \le d$.

(2) *Q is a martingale measure.*

From now on: financial models which allow a martingale measure

Notation:

- $(\Omega, \mathcal{F}, \{Q_1, ..., Q_d, Q, P\})$ together with $(\mathcal{F}_t)_{t \in I}$ is called a financial experiment.
- The processes $\left(\frac{dQ_{i|\mathcal{F}_t}}{dQ_{|\mathcal{F}_t}} \right)_{t \in I}$ are called filtered likelihood processes.

Dictionary

Finance	*Statistics*
price process	*filtered likelihood process*

Example (Cox-Ross-Rubinstein model)

- *Q, Q_1 products of Bernoulli distributions (parameters τ, κ)*
- *$Q := ((1-\tau)\varepsilon_0 + \tau\varepsilon_1)^N$ and $Q_1 := ((1-\kappa)\varepsilon_0 + \kappa\varepsilon_1)^N$*
- *Result:*

$$\frac{dQ_{1|\mathcal{F}_n}}{dQ_{|\mathcal{F}_n}}(k_n) = \left(\frac{\kappa}{\tau}\right)^{k_n}\left(\frac{1-\kappa}{1-\tau}\right)^{n-k_n} = \tilde{u}^{k_n}\tilde{d}^{n-k_n} = \frac{X_n(k_n)}{X_0(k_n)}.$$

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Example (Itô type price processes)

Discounted price processes:

$$\mathbf{X_t^i} = \mathbf{X_0^i} \exp\left(\int_0^t \sigma_i'(s) dW(s) + \int_0^t \left(\mu_i(s) - \rho(s) - \frac{\|\sigma_i(s)\|^2}{2} \right) ds \right)$$

where W is a d-dimensional Brownian motion

Usual assumptions:

- *Parameter space Θ: space of volatility matrices $\sigma = (\sigma_{ij})_{i,j=1,\ldots,d}$ which are progressively measurable, uniformly positive definite processes such that an integrability condition holds.*
  $\sigma_i := (\sigma_{i1}, \ldots, \sigma_{id})'$
- *Interest rate and drift: ρ and $\mu = (\mu_1, \ldots, \mu_d)'$ progressively measurable processes*
- *Bond price:*
  $V_t^0 = \exp\left(\int_0^t \rho(s) ds \right), \quad 0 \le t \le T$

$$\frac{\mathbf{X^i_T}}{\mathbf{X^i_0}} = \frac{\mathbf{dQ_i}}{\mathbf{dQ}} := \exp\left(\int_0^{\mathbf{T}} \sigma'_{\mathbf{i}}(\mathbf{s})\mathbf{d\overline{W}(s)} - \frac{\mathbf{1}}{\mathbf{2}}\int_0^{\mathbf{T}} \|\sigma_{\mathbf{i}}(\mathbf{s})\|^{\mathbf{2}}\mathbf{ds}\right)$$

Changing martingale measure:
$1\!\!1 = (1, ..., 1)' \in \mathbb{R}^d$ and $\theta(s) := \sigma^{-1}(s)[\rho(s)1\!\!1 - \mu(s)]$

- Set $\frac{dQ}{dP} := \exp\left(\int_0^T \theta'(s)dW(s) - \frac{1}{2}\int_0^T \|\theta(s)\|^2 ds\right)$
  By Girsanov's Theorem $\bar{W}(t) = W(t) - \int_0^t \theta(s)ds$ is a d-dimensional Brownian motion with respect to Q

Statistical meaning: regression model in survival analysis

- $\xi(\mathbf{t}) = \mathbf{W(t)} + \int_0^{\mathbf{t}}(\tau(\mathbf{s}) - \theta(\mathbf{s}))\mathbf{ds} = \mathbf{\overline{W}(t)} + \int_0^{\mathbf{t}} \tau(\mathbf{s})\mathbf{ds}$
  $\mathbf{0} \le \mathbf{t} \le \mathbf{T}$
- $\mathbf{Q} = \mathcal{L}\left((\xi(\mathbf{t}))_{\mathbf{t}\le\mathbf{T}} | \tau = \mathbf{0}\right)$
- $\mathbf{Q_i} = \mathcal{L}\left((\xi(\mathbf{t}))_{\mathbf{t}\le\mathbf{T}} | \tau = \sigma_{\mathbf{i}}\right),\ \sigma_{\mathbf{i}} := (\sigma_{\mathbf{i1}}, ..., \sigma_{\mathbf{id}})'$

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Finance	Statistics
Itô process models of Black-Scholes type	regression models
volatility	hazard parameters

Definition

$\{P_\vartheta : \vartheta \in \Theta\}$ is $\mathcal{G}$-complete w.r.t. some class of function $\mathcal{G}$, if for $g \in \mathcal{G}$ we have $\int g\, dP_\vartheta =$ constant for all P_ϑ implies $g =$ constant a.e.

Finance	Statistics
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3. Option prices as functions of power functions of tests

Example (European Call option)

- *Payoff European Call:*

  $$H_C = (X_T^1 - K)^+ = (X_T^1 - K)\mathbf{1}_{\{X_T^1 > K\}} = (X_T^1 - K)\phi_C\left(\frac{dQ_1}{dQ}\right)$$

  where $\phi_C\left(\frac{dQ_1}{dQ}\right) = \mathbf{1}_{\left\{\frac{dQ_1}{dQ} > \frac{K}{X_0^1}\right\}}$ *is a* **Neyman Pearson test** *for the null hypothesis* $\{Q\}$ *versus* $\{Q_1\}$

- *Fair price:*

  $$p(H_C) = X_0^1 E_{Q_1}\left(\phi_C\left(\frac{dQ_1}{dQ}\right)\right) - KE_Q\left(\phi_C\left(\frac{dQ_1}{dQ}\right)\right)$$

- $p(H_C) + K$ **Bayes risk** *of the test* ϕ_C *with respect to the prior* $\Lambda_0 = X_0^1$ *and* $\Lambda_1 = K$

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Option:
Random payoff H at time T.
Assumption *(A)*: H is of the form

$$H = \sum_{j=1}^{m} \sum_{i=1}^{d} [a_{ij} X_T^i - K_{ij}] \phi_{ij} \left(\left(\frac{X_t^i}{X_0^i} \right)_{t \in I} \right),$$

where $\phi_{ij} : \mathbb{R}^I \to [0,1]$ tests and a_{ij}, K_{ij} real coefficients $1 \leq i \leq d$, $1 \leq j \leq m$.

Examples:

- European Call, European Put
- Strangle Option, Straddle Option, Bull-Spread Option
- Digital Option

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Theorem 2

Under assumption (A) and for a fixed martingale measure Q the option price p(H) of H is given by

$$\begin{aligned} p(H) &= \sum_{j=1}^{m}\sum_{i=1}^{d}\Bigg[a_{ij}X_0^i E_{Q_i}\left(\phi_{ij}\left(\left(\frac{dQ_{i|\mathcal{F}_t}}{dQ_{|\mathcal{F}_t}}\right)_{t\le T}\right)\right) \\ &- K_{ij}E_Q\left(\phi_{ij}\left(\left(\frac{dQ_{i|\mathcal{F}_t}}{dQ_{|\mathcal{F}_t}}\right)_{t\le T}\right)\right)\Bigg]. \end{aligned}$$

- $E_Q\left(\phi_{ij}\left(\left(\frac{dQ_{i|\mathcal{F}_t}}{dQ_{|\mathcal{F}_t}}\right)_{t\le T}\right)\right)$ level of the test ϕ_{ij}
- $E_{Q_i}\left(\phi_{ij}\left(\left(\frac{dQ_{i|\mathcal{F}_t}}{dQ_{|\mathcal{F}_t}}\right)_{t\le T}\right)\right)$ power of the test ϕ_{ij}

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&- \left.K_{ij}E_Q\left(\phi_{ij}\left(\left(\frac{dQ_{i|\mathcal{F}_t}}{dQ_{|\mathcal{F}_t}}\right)_{t\le T}\right)\right)\right].
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Dictionary

Finance	*Statistics*
options	*tests*
option price	*by power functions of test*
European call option	*Neyman Pearson test*
Black-Scholes price	*Bayes risk*

4. Convergence of option prices

Le Cam: Convergence of experiments (likelihood processes)
Consequences: Convergence of Neyman Pearson Tests
Convergence of Bayes risks

- $E_n = \{P_{n,\vartheta} : \vartheta \in \Theta\}$ sequence of experiments
- weak convergence of experiments $E_n \to E = \{P_\vartheta : \vartheta \in \Theta\}$ weak convergence of all finite dimensional likelihood ratio processes

$$\mathcal{L}\left(\left(\frac{dP_{n,\vartheta}}{dP_{n,\vartheta_0}}\right)_\vartheta \middle| P_{n,\tau}\right) \longrightarrow \mathcal{L}\left(\left(\frac{dP_\vartheta}{dP_{\vartheta_0}}\right)_\vartheta \middle| P_\tau\right) \quad \text{for all } \tau$$

- compact topology (on the set of classes of experiments)

Example

Central limit Theorem for statistical experiments (LAN) "local asymptotic normality" under some condition:

- $E_n \to E = \{Q_{\vartheta g} : \vartheta \in \mathbb{R}\}$ *weakly*
- $\frac{dQ_{\vartheta g}}{dQ_0} = \exp(\vartheta L(g) - \vartheta^2 \| g \|^2 / 2)$ *Gaussian shift,* $g \in L_2$
- $g \mapsto L(g)$ *Gaussian process, mean zero and* $Cov(L(g_1), L(g_2)) = \langle g_1, g_2 \rangle$ *under* Q_0
- *Covers the geometric Brownian motion*

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- *Covers the geometric Brownian motion*

Example (Brownian motion regression model)

- $g : [0,1] \to \mathbb{R}$, $P_0 = \lambda\!\!\lambda_{|[0,1]}$
- $X_t = B(t) + \int_0^t g(u)du, \ 0 \le t \le 1$ *noise + signal*
- $Q_g = \mathcal{L}((X_t)_{t \le 1} \mid g)$
- $L(g) = \int_0^1 g\, B(dt)$
- $\frac{dQ_g}{dQ_0} = \exp\left(\int_0^1 g\, B(dt) - \frac{\|g\|^2}{2} \right)$

For simplicity, one asset $d = 1$

$$X_{n,t} = \frac{dQ_{1,n|\mathcal{F}_{t,n}}}{dQ_{n|\mathcal{F}_{t,n}}}, \quad n \in \mathbb{N}_0$$

Theorem 3

Suppose that $X_{n,t}$ "converges weakly to" $X_{0,t}$ (in terms of financial experiments). $X_{0,t}$ is a price process iff $Q_{1,n} \lhd Q_n$ and $Q_n \lhd Q_{1,n}$ (Contiguity).

Contiguity $P_n \lhd Q_n$:
When $Q_n(A_n) \to 0$ then $P_n(A_n) \to 0$ holds.

"Asymptotic arbitrage freeness"

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Example

$t_k = \frac{k}{n} T$ *discrete times*

$$X_{n,t_k} = \prod_{j=1}^{k} Z_{n,j}, \quad Z_{n,j} - 1 = \frac{X_{n,t_k} - X_{n,t_{k-1}}}{X_{n,t_{k-1}}} \text{ returns}$$

Under regularity assumptions:
Convergence to the Gaussian shift (= geometric Brownian motion)

Concrete Example: Convergence of Cox-Ross-Rubinstein models

Theorem (Le Cam) (Main Theorem of Testing)

Suppose that $\{P_{n,\vartheta} : \vartheta \in \Theta\} \to \{P_\vartheta : \vartheta \in \Theta\}$weakly. $P_\vartheta \ll P_{\vartheta_0}$
Let $\varphi_n : \Omega_n \to [0, 1]$ be a sequence of tests with

$$\lim_{n\to\infty} E_{P_{n,\vartheta}}(\varphi_n) = a_\vartheta \text{ exists.}$$

Then there exists a test φ for $(P_\vartheta)_\vartheta$ with

$$E_{P_\vartheta}(\varphi) = a_\vartheta \text{ for all } \vartheta.$$

φ is related to the option for the “limit model”.

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All kind of convergence results are known in statistics.

Application in finance:

- Convergence of financial experiments
  (price processes)
- Convergence of power functions
  Neyman Pearson tests, Bayes risks
- Convergence of option prices
- Discrete approximation of complicated option prices

Dictionary

Finance	*Statistics*
price process	*filtered likelihood process*
Itô process models of Black-Scholes type	*regression models*
volatility	*hazard parameters*
complete markets	*completeness of experiments*
options	*tests*
option price	*by power functions of test*
European call option	*Neyman Pearson test*
Black-Scholes price	*Bayes risk*
martingale measure	*null hypothesis*
approximation of continuous time price models	*convergence of experiments*
asymptotic arbitrage free models	*contiguity*

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Thank you for your attention!

Appendix

$$X_{n,t_k} = \prod_{j=1}^{k} Z_{n,j}, \quad Z_{n,j} - 1 = \frac{X_{n,t_k} - X_{n,t_{k-1}}}{X_{n,t_{k-1}}} \text{ returns}$$

$$\frac{dQ_{1,n|\mathcal{F}_{t_k,n}}}{dQ_{n|\mathcal{F}_{t_k,n}}} = X_{n,t_k} = \prod_{j=1}^{k} Z_{n,j} = \frac{d\bigotimes_{j=1}^{k} Q_{1,n}(j)}{d\bigotimes_{j=1}^{k} Q_n(j)}$$

$\frac{dQ_{1,n}(j)}{dQ_n(j)} - 1 = Z_{n,j} - 1$ “returns at stage $\frac{t_j}{n}$ for the n-th price process”