Scenario Aggregation

for Solvency Regulation

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Outline

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Insurance Solvency in 1990s

- Deregulation of insurance markets
- Growth and competition based on unrealistic guarantees
- Solvency regulation with focus on liabilities
- No asset risk assessment
- Dot-com bubble: insurers buy equities ...
- Insolvencies world-wide (Nissan Mutual Life, Equitable Life, HIH Insurance Group,...)

Insurance Solvency in 2000s

2001: Solvency II project initiated

2003: Swiss Solvency Test (SST) initiated

- Quantitative impacts studies (Solvency II and SST)

2008: SST mandatory for large insurers

2011: SST in force

- ...

2016: Enforcement of Solvency II

SST: Some Key Principles

- Risk based: market, insurance, and credit risks are quantified
- Market consistent valuation of assets and liabilities
- Total balance sheet approach
- **Stress scenarios**: to be aggregated for capital requirement
- **Internal models** encouraged: to be approved by regulator

SST: Available and Required Capital

- Available capital $C = \text{Assets} - \text{Liabilites}$
- Annual loss $L = C(0) - C(1)$
- Required capital $K = \mathrm{ES}[L]$ (expected shortfall)
- Capital requirement: $C > K$

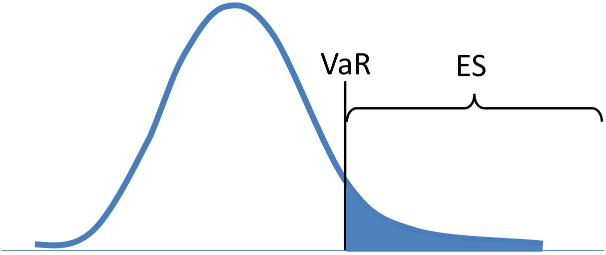


Figure: VaR = 99%-Value at Risk, ES = 99%-Expected Shortfall

Outline

SST Results 2011–2012

- Significant difference between business models

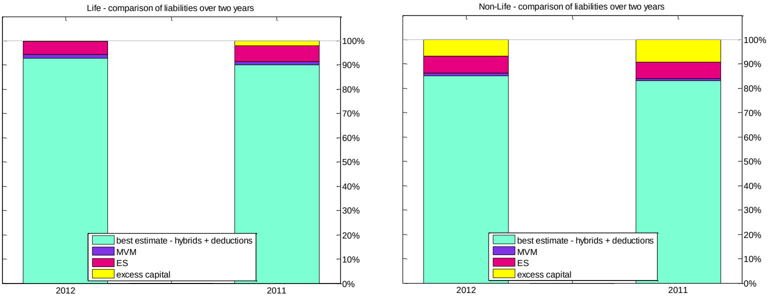
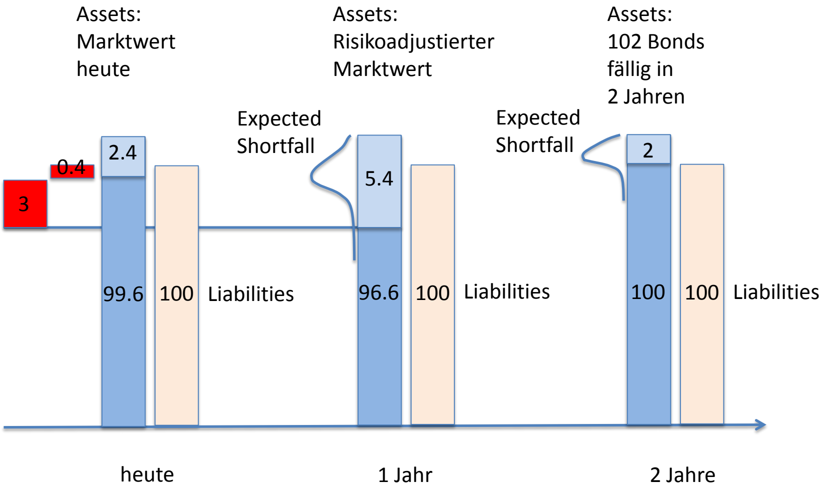


Figure: Life and non-life results. Source: FINMA SST Survey 2012

Example

- Liabilities: 100 due in 2 years
- Risk-free discount rate is zero
- Assets: 102 corporate bonds (BBB) well diversified
- Maturity of bonds in 2 years
- Default probability: 0.5%
- Recovery rate: 40%
- Spread today: 2%
- Absolute spread volatility: 1.4%

Example: Figure



Example: Result

- Expected one-in-one hundred year loss (expected shortfall) at maturity in 2 years is fully absorbed
- Even so there results an additional capital requirement of 3.4
- Explanation: market consistency means
  1. Positive spot market value at any time (liquidation vs. going-concern view)
  2. Spread risk is fully charged ("fictitious risk"?)
- Are these costs justified, or is SST economically inefficient?

Example: Relevance

- Spread risk accounts up to 30–50% of total risk

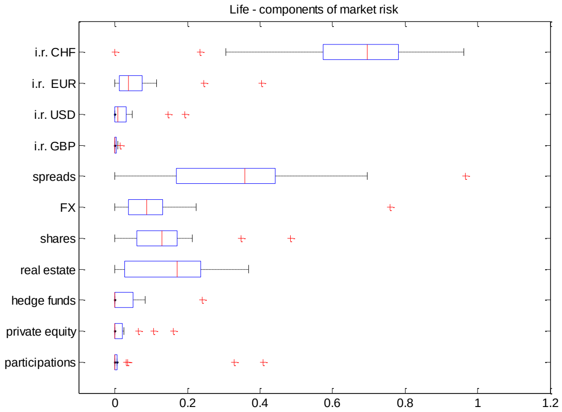


Figure: Components of market risk. Source: FINMA SST Survey 2012

Outline

Internal Models

- Probability space $(\Omega, \mathcal{F}, \mathbb{P})$
- Annual loss L: random variable assigning a loss $L(\omega)$ to any possible state of the world $\omega \in \Omega$

$\hookrightarrow$ $(\Omega, \mathcal{F})$ is universal

$\hookrightarrow$ $\mathbb{P}$ and L is insurer specific (internal model)

Assumption:

Regulator trusts the mapping $L : \Omega \to \mathbb{R}$ (no ambiguity)

Fact:

Regulator wants to challenge $\mathbb{P}$, or distribution $F_L(x) = \mathbb{P}[L \leq x]$

Scenarios

A scenario is an event $S \in \mathcal{F}$: *a narrative description of a possible state of the world*

For factor models, this notion includes

- **point scenarios** $S = \{\mathbf{x} \in \mathbb{R}^n \mid x_j = c_j \text{ for some } j\}$, e.g.
  - $\hookrightarrow$ 1-year loss of EUR 100 mio in Eurowind,
  - $\hookrightarrow$ 1-year drop of -20% for SMI,
- **quadrants** $S = \{\mathbf{x} \in \mathbb{R}^n \mid x_j \geq c_j \text{ for some } j\}$, e.g.
  - $\hookrightarrow$ 1-year change in EURCHF $\leq -20\%$,
  - $\hookrightarrow$ 1-year change European Credit Spreads AAA $\geq 50\%$.

Scenarios from a Regulatory point of view

"Mit den SST-Szenarien sollen die Mängel aus verteilungsbasierten Modellen korrigiert werden. So können analytische Modelle extreme Ereignisse oft nur unzureichend abbilden, sowohl in Bezug auf die Heavy-Tailedness der Randverteilungen als auch in Bezug auf die so genannte Tail-Dependency."

— FINMA, *Wegleitung für die Erarbeitung des SST-Berichtes 2013*, 2012

SST Scenario Aggregation

Given by FINMA

- Scenarios $S_1, \ldots, S_d$ along with auxiliary probabilities $\pi_i > 0$
  - $\hookrightarrow$ In general, $\pi_i \neq \mathbb{P}[S_i]$
  - $\hookrightarrow$ It is natural to set $S_0 = \Omega$ and $\pi_0 = 1 - \sum_{i=1}^{d} \pi_i$

Scenario	**Probability of occurrence**
Industrial	0.5%
Pandemic	1%
Accident on a works outing	0.5%
Accident: Panic in a football stadium	Type 2: not relevant for target capital.
Hail scenario	Type 2: not relevant for target capital.
Disability	0.5%
Daily allowance for sickness	0.5%

Source: FOPI, *SST Technical Documents*, 2006

SST Scenario Aggregation

Ansatz

- Scenario S_i causes an extra-ordinary loss

$$\ell_i = \mathbb{E}[L \mid S_i] - \mathbb{E}[L]$$

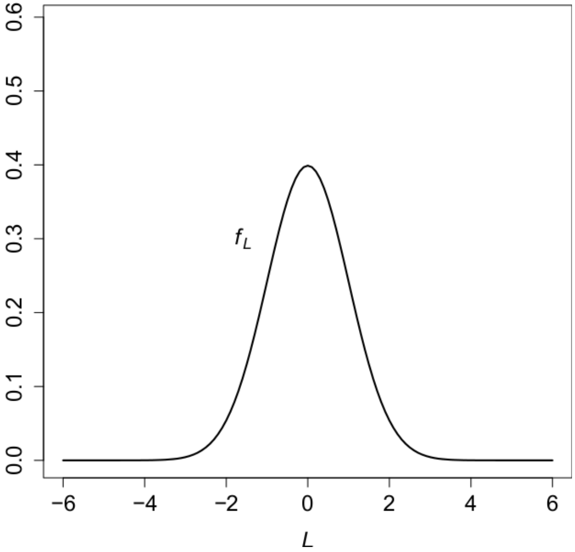
  to be determined by actuary, with $\ell_0 = 0$
- Loss distribution conditional on scenario S_i is $F_L(x - \ell_i)$

Aggregation

- Replace $F_L(x)$ by aggregated loss distribution

$$F_L^{\mathrm{aggr}}(x) = \sum_{i=0}^{d} \pi_i F_L(x - \ell_i)$$

SST Scenario Aggregation



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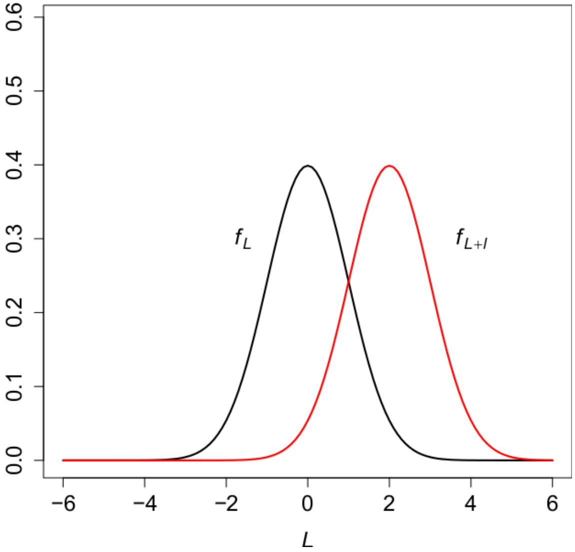

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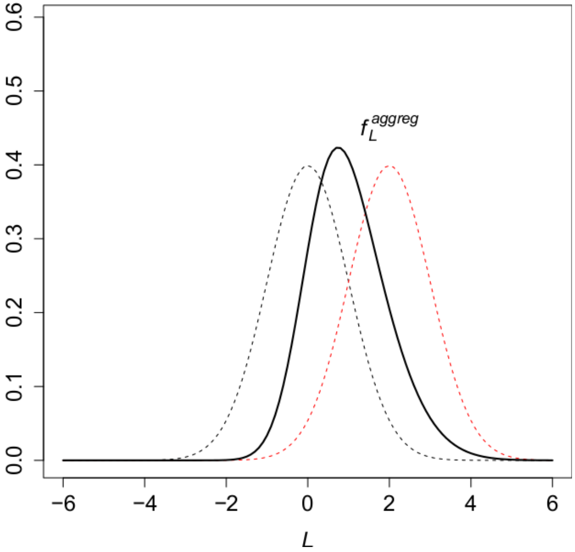

SST Method: Discussion

- Fact: $F_L^{\mathrm{aggr}}(x)$ is the cdf of $L + Z$ for an independent extra-ordinary loss random variable Z with $\mathbb{P}[Z = \ell_i] = \pi_i$
- Lemma:

$$\mathrm{ES}^{\mathrm{aggr}}[L] = \mathrm{ES}[L + Z] \geq \mathrm{ES}[L] + \mathbb{E}[Z]$$

- Consequence: if $\mathbb{E}[Z] > 0$ then

$$\mathrm{ES}^{\mathrm{aggr}}[L] > \mathrm{ES}[L]$$

  no matter how conservative the internal model for L is

$\Rightarrow$ scenario aggregation penalizes conservative internal models

SST Method: Discussion

- No control on how far $F_L^{\mathrm{aggr}}(x)$ from $F_L(x)$ is
- No control on how far $\mathrm{ES}^{\mathrm{aggr}}(L)$ from $\mathrm{ES}(L)$ is
- Confusion among stakeholders about “double-counting”
- High degree of subjectivity about auxiliary weights π_i
- Capital is increased even if scenario is not in tail loss event
- Aggregation is on the level of $F_L(x)$, not $\mathbb{P}$

Outline

A Moment's Reflection on Stress Tests

- Stress test: selected states of the world $\omega_i \in \Omega$
- Leads to a maximal insurer specific loss $\ell = \max_i L(\omega_i)$
- Internal model (null hypothesis) $\mathbb{P}$ passes the stress test if not rejected on significance level $1 - \alpha = 1\%$. That is, if

$$\ell \leq \mathrm{VaR}_\alpha(L)$$

- Equivalently,

$$\mathbb{P}[S] \geq 1 - \alpha$$

  for the scenario $S = \{L \geq \ell\}$

Outline

Definition of Views

- Given a collection of scenarios $S_1, \ldots, S_d \in \mathcal{F}$
- Given a vector of target probabilities $\mathbf{c} = (c_1, \ldots, c_d)^\top$
- Define $S_0 = \Omega \setminus \cup_{i=1}^d S_i$
- Denote $\mathcal{M} =$ set of probability measures on $(\Omega, \mathcal{F})$
- **Views** on $\mathbb{Q} \in \mathcal{M}$:

$$\mathbb{Q}[S_i] \geq c_i, \quad i = 1, \ldots, d \qquad \text{(views)}$$

Views in Terms of Atoms

- Let $U_0, \ldots, U_n$ be the atoms of $\mathcal{S} = \sigma(S_1, \ldots, S_d)$:

$$S_0 = U_0, \quad S_i = \cup_{j \in J(i)} U_j, \quad i = 1, \ldots, d$$

- **Views** on $\mathbb{Q} \in \mathcal{M}$:

$$\sum_{j \in J(i)} q_j \geq c_i, \quad i = 1, \ldots, d \qquad \text{(views)}$$

  for vector $q_j = \mathbb{Q}[U_j]$

- **Views** in matrix form:

$$A\,\mathbf{q} \geq \mathbf{c} \qquad \text{(views)}$$

  for matrix $A_{ij} = 1_{J(i)}(j)$

Scenario Aggregation

- Modification of internal model: find minimizer for

$$\begin{aligned} \text{minimize} \quad & d(\mathbb{Q}, \mathbb{P}) \\ \text{subject to} \quad & \text{(views)} \end{aligned}$$

  with domain $\mathcal{M}$

- $d(\cdot, \mathbb{P})$ measures the difference from $\mathbb{P}$ on $\mathcal{M}$

ϕ-Divergence

- ϕ-divergence

$$d(\mathbb{Q}, \mathbb{P}) = \begin{cases} \mathbb{E}[\phi(d\mathbb{Q}/d\mathbb{P})], & \text{if } \mathbb{Q} \ll \mathbb{P} \\ +\infty, & \text{otherwise} \end{cases}$$

  where ϕ is convex, and strictly convex at 1 with $\phi(1) = 0$

- Standard measure for difference of $\mathbb{Q}$ from $\mathbb{P}$ in statistics, Csiszar (1963)

- Fact: $d(\mathbb{Q}, \mathbb{P})$ is not a metric, but convex in $\mathbb{Q}$

Examples and Facts

- Examples:

$$\phi(t) = \begin{cases} t \log t, & \text{relative entropy} \\ (\sqrt{t} - 1)^2, & \text{Hellinger distance} \\ |t - 1|^p, & L^p\text{-distance, } p \geq 1 \end{cases}$$

- Facts:

$$\begin{aligned} \|d\mathbb{Q}/d\mathbb{P} - 1\|_1 &\leq \sqrt{2d_E(\mathbb{Q}, \mathbb{P})} \\ d_H(\mathbb{Q}, \mathbb{P}) \leq \|d\mathbb{Q}/d\mathbb{P} - 1\|_1 &\leq \sqrt{2d_H(\mathbb{Q}, \mathbb{P})} \\ \|d\mathbb{Q}/d\mathbb{P} - 1\|_1 &= 2\, d_{TV}(\mathbb{Q}, \mathbb{P}) \quad \text{total variation} \end{aligned}$$

- Fact: all but the L^1-distance are strictly convex in $\mathbb{Q}$

Outline

Robustness Check

- Question: is the capital requirement robust under minimum ϕ-divergence scenario aggregation?
- In other words: is VaR and ES continuous with respect to $d(\mathbb{Q}, \mathbb{P})$?

Recall Definitions of VaR and ES

- Value at risk

$$\mathrm{VaR}(X) = q_\alpha^-(X), \quad \text{left } \alpha\text{-quantile}$$

- Expected shortfall

$$\begin{aligned}\mathrm{ES}(X) &= \frac{1}{1-\alpha}\mathbb{E}\left[(X-q)^+\right] + q \\ &= \frac{1}{1-\alpha}\left(\mathbb{E}[X\,1_{\{X>q\}}] + q\,(\mathbb{P}[X \le q] - \alpha)\right)\end{aligned}$$

for any α-quantile $q \in [q_\alpha^-(X), q_\alpha^+(X)]$

- Folk theorem: VaR is more robust than ES ...

Lemma for Value at Risk

Lemma 4.1.
If $d\mathbb{P}_n/d\mathbb{P} \to 1$ *in* L^1 *then*

$$\sup_x |\mathbb{P}_n[X \leq x] - \mathbb{P}[X \leq x]| \to 0$$

and

$$q_\alpha^-(X) \leq \liminf_n q_n \leq \limsup_n q_n \leq q_\alpha^+(X)$$

for any sequence (q_n) *of* α*-quantiles of* X*, for any* $X \in L^0$*.*

Non-Robustness of Value at Risk: Example

- Define $X = 0$ or 1 with $\mathbb{P}[X = 0] = \alpha$
- Define

$$d\mathbb{P}_n/d\mathbb{P} = \begin{cases} 1 + (1-\alpha)(-1)^n/(\alpha n), & \text{on } \{X = 0\} \\ 1 + (-1)^{n+1}/n, & \text{on } \{X = 1\} \end{cases}$$

- VaR does not converge:

$$\mathrm{VaR}_n(X) = \begin{cases} 0 = q_\alpha^-(X), & \text{for } n \text{ even} \\ 1 = q_\alpha^+(X), & \text{for } n \text{ odd} \end{cases}$$

Robustness of Expected Shortfall

Theorem 4.2.
Let $p \in [1, \infty]$. If $d\mathbb{P}_n/d\mathbb{P} \to 1$ in L^p then

$$\mathrm{ES}_n(X) \to \mathrm{ES}(X)$$

for all $X \in L^r$, where $p^{-1} + r^{-1} = 1$.

Proof.
Using previous lemma and

$$\begin{aligned}(1-\alpha)\left|\mathrm{ES}_n(X) - \mathrm{ES}(X)\right| &\leq \mathbb{E}\left[|Z_n - 1|(X - q_n)^+\right] \\ &\quad + \mathbb{E}\left[\left|(X - q_n)^+ - (X - q)^+\right|\right] + (1-\alpha)\left|q_n - q\right|\end{aligned}$$

for any converging (sub-)sequence of α-quantiles $q_n \to q$. $\square$

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Back to the Optimization Problem

- Modification of internal model: find minimizer for

$$\begin{aligned} \text{minimize} \quad & d(\mathbb{Q}, \mathbb{P}) \\ \text{subject to} \quad & \text{(views)} \end{aligned} \tag{P}$$

  with domain $\mathcal{M}$

Lemma 4.3.
For every $\mathbb{R} \ll \mathbb{P}$ satisfying the views there exists a

$$\mathbb{R}' \in \mathcal{Q} := \{\mathbb{Q} \ll \mathbb{P} \mid d\mathbb{Q}/d\mathbb{P} \textit{ is } \mathcal{S}\textit{-measurable}\}$$

satisfying the views and $d(\mathbb{R}', \mathbb{P}) \le d(\mathbb{R}, \mathbb{P})$.

Proof.
Set $d\mathbb{R}'/d\mathbb{P} = \mathbb{E}\left[d\mathbb{R}/d\mathbb{P} \mid \mathcal{S}\right]$ and use Jensen's inequality. □

Existence and Uniqueness

- Note $\dim \mathcal{Q} = n + 1$: identify $\mathbb{Q} \in \mathcal{Q}$ with $\mathbf{q}$ by

$$q_j = \mathbb{Q}[U_j], \quad j = 0, \ldots, n$$

Theorem 4.4.
There exists a solution of (P) *in* $\mathcal{Q}$. *Moreover, if* ϕ *is strictly convex then the solution is unique.*

Solution of Optimization Problem

- Define $\mathbf{p} \in (0,1)^{n+1}$ by $p_j = \mathbb{P}[U_j]$
- The optimization problem (P) reduces to

$$\begin{aligned} \text{minimize} \quad & \textstyle\sum_{j=0}^{n} p_j \, \phi(q_j/p_j) \\ \text{subject to} \quad & A\,\mathbf{q} \geq \mathbf{c} \\ & \mathbf{1}^\top \mathbf{q} = \mathbf{1} \end{aligned} \tag{P}$$

  with domain $(0,1)^{n+1}$
- Solution via dual problem or Kuhn–Tucker (FOC) conditions
- Reference e.g. Boyd and Vandenberghe (2004)

Special Case: $d = 1$ Scenario

Corollary 4.5.

For $d = 1$ scenario S_1 a (the) solution to (P) *is given by*

$$\frac{d\mathbb{Q}^*}{d\mathbb{P}} = \frac{1 - \max\{c_1, p_1\}}{p_0} 1_{S_0} + \frac{\max\{c_1, p_1\}}{p_1} 1_{S_1}$$

independently of the choice of the (strictly) convex divergence function ϕ.

Proof.

Convexity of ϕ implies that $q_1 \mapsto d(\mathbb{Q}(q_1), \mathbb{P})$ is non-decreasing in q_1 for $q_1 > p_1$. $\square$

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Proof.

Convexity of ϕ implies that $q_1 \mapsto d(\mathbb{Q}(q_1), \mathbb{P})$ is non-decreasing in q_1 for $q_1 > p_1$. $\square$

Special Case: Stress Testing ($d = 1$ Scenario)

- Recall stress testing is equivalent to $d = 1$ views

$$\mathbb{Q}[S_1] \geq 1 - \alpha$$

on the scenario $S_1 = \{L \geq \ell\}$ with $\ell = \max_i L(\omega_i)$

- In this case we obtain a closed form expression for ES:

Corollary 4.6.

The expected shortfall under $\mathbb{Q}^*$ given in Corollary 4.5 satisfies

$$\mathrm{ES}_{\mathbb{Q}^*,\,\alpha}(L) = \mathrm{ES}_{\mathbb{P},\,\max\{\mathbb{P}[L<\ell],\alpha\}}(L).$$

Outline

Relative Entropy: Dual Problem

- The Lagrangian function is

$$L(\mathbf{q}, \lambda, \nu) = \sum_{j=0}^{n} q_j \log \frac{q_j}{p_j} + \lambda^\top (\mathbf{c} - A\mathbf{q}) + \nu \left(\mathbf{1}^\top \mathbf{q} - 1 \right)$$

- The dual problem is

$$\begin{array}{ll} \text{minimize} & \sum_{j=0}^{n} p_j \mathrm{e}^{(A^\top \lambda)_j - \nu - 1} - \mathbf{c}^\top \lambda + \nu \\ \text{subject to} & \lambda \geq 0 \end{array} \qquad \text{(DP)}$$

with domain $\lambda \in \mathbb{R}^d$ and $\nu \in \mathbb{R}$

Relative Entropy: Slater's Condition

- Slater's condition: $\exists \mathbf{q} > 0$ such that $A\,\mathbf{q} \geq \mathbf{c}$ and $\mathbf{1}^\top \mathbf{q} = 1$
- If Slater's condition holds then there exists a unique minimizer (λ^*, ν^*) of the dual problem (DP), and strong duality holds:

$$q_j^* = p_j e^{(A^\top \lambda^*)_j - \nu^* - 1} \quad j = 0, \ldots, d$$

Relative Entropy: Kuhn–Tucker Conditions

If Slater's condition holds then the Kuhn–Tucker conditions are necessary and sufficient for optimality:

$$\lambda \geq \mathbf{0}, \quad A\,\mathbf{q} \geq \mathbf{c}, \quad \lambda^\top (A\,\mathbf{q} - \mathbf{c}) = \mathbf{0}$$
$$\mathbf{1}^\top q = 1$$
$$\log \mathbf{q} - \log \mathbf{p} + \mathbf{1} - A^\top \lambda + \nu \mathbf{1} = \mathbf{0}$$

Relative Entropy: Explicit Solution for Disjoint Scenarios

Lemma 4.7.
If the d scenarios $S_1, \ldots, S_d$ are mutually disjoint then the unique solution to (P) *is given by*

$$q_j^* = \max\left\{ c_j,\ p_j \frac{1 - \sum_{i=k^*+1}^{d} c_i}{\sum_{i=0}^{k^*} p_i} \right\}$$

where k^* is the integer determined by

$$\sum_{i=0}^{k^*+1} p_i \left(\frac{c_{k^*+1}}{p_{k^*+1}} - \frac{c_i}{p_i} \right) > 1 - \sum_{i=0}^{d} c_i \geq \sum_{i=0}^{k^*} p_i \left(\frac{c_{k^*}}{p_{k^*}} - \frac{c_i}{p_i} \right)$$

and we assume w.l.o.g. that $\frac{c_0}{p_0} < \cdots < \frac{c_d}{p_d}$ with $c_0 := 0$.

Relative Entropy: Example

- $d = 2$ disjoint scenarios S_1, S_2
- Target probabilities $\mathbf{c} = (0.2, 0.2)^\top$

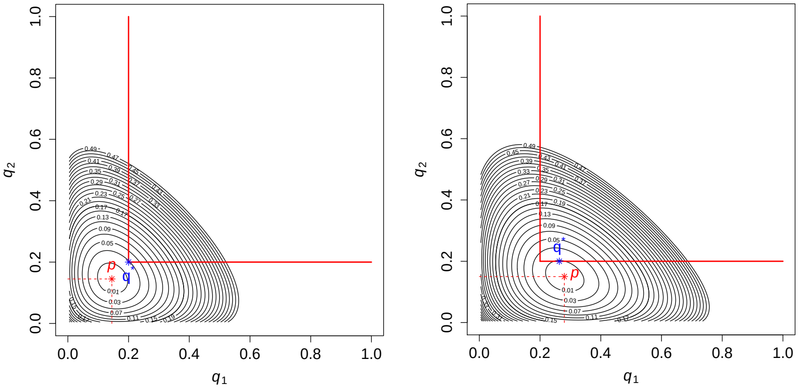

Figure: Contour plots for $(p_1, p_2)^\top \leq \mathbf{c}$ (left), $p_1 > c_1$, $p_2 < c_2$ (right)

Relative Entropy: More on Solutions

- In the paper we also provide explicit solutions for $d = 2$ overlapping scenarios S_1, S_2
- In general: numerical solution of dual problem

Outline

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Case Study 1: Setup

- Loss $L \sim \mathcal{N}(0, 10.2)$
- Compare scenario aggregation using SST and minimum entropy method

Case Study 1: SST Method

- $d = 1$ scenario with probability c and extra-ordinary loss ℓ
- Recall: $F_L^{\mathrm{aggr}}(x) = (1-c)\,F_L(x) + c\,F_L(x-\ell)$

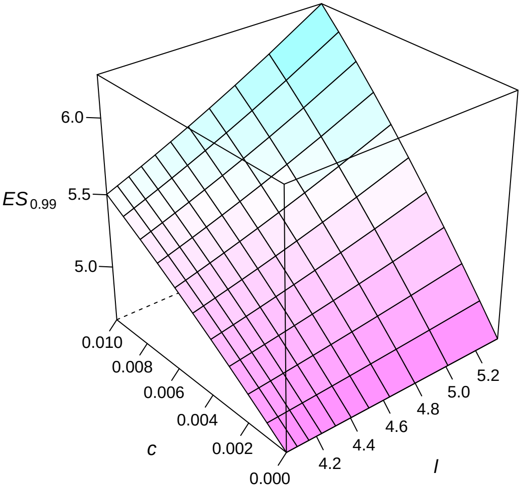

Figure: Sensitivity of $\mathrm{ES}^{\mathrm{aggr}}[L]$ with respect to c and ℓ

Case Study 1: Minimum Entropy Method

- $d = 1$ scenario $S_1 = \{L \geq \ell\}$
- View: $\mathbb{Q}[L \geq \ell] \geq c$ for some auxiliary level c

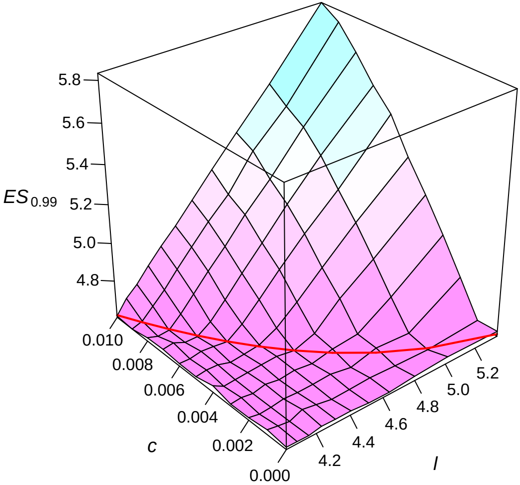

Figure: Sensitivity of $\mathrm{ES}_{\mathbb{Q}^*}[L]$ with respect to c and ℓ

Case Study 1: Difference SST - Minimum Entropy Method

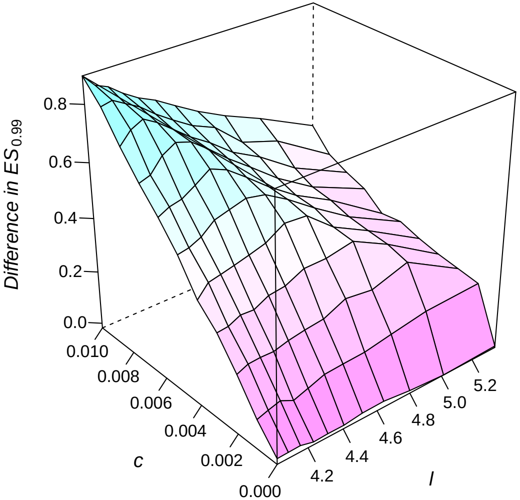

Figure: $\mathrm{ES}^{\mathrm{aggr}}[L] - \mathrm{ES}_{\mathbb{Q}^*}[L]$ as function of c and ℓ

Case Study 1 (Stress Test): Minimum Entropy Method

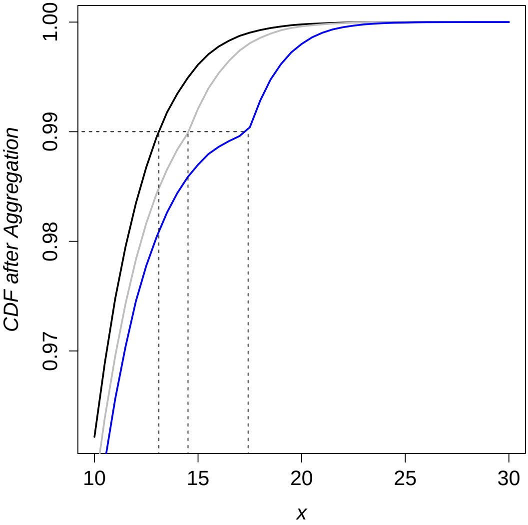

Figure: Impact on cumulative distribution function for $\ell = \mathrm{VaR}_\alpha(L)$ with $\alpha = 0.99$ (black), $\alpha = 0.995$ (grey), $\alpha = 0.999$ (blue)

Outline

Case Study 2: Setup

- Two risk factors (X_1, X_2) normal with mean $\mathbf{0}$, and $\mathrm{var}(X_1) = 1$, $\mathrm{var}(X_2) = 4$, and $\mathrm{corr}(X_1, X_2) = -0.5$
- X_1: change in interest rates
- X_2: risk factor related to CAT events with reinsurance
- Loss

$$L = \max\{X_1, -1\} + \max\{\min\{X_2, 5\}, -1\}$$

  is capped in X_2 (reinsurance), and gains are capped at 1
- $d = 2$ scenarios

$$S_1 = \{X_1 \geq 1,\ X_2 \geq 1\} \quad \text{and} \quad S_2 = \{X_1 < -2\}$$

Case Study 2: Shortfall Region

- Shortfall region $W = \{L > \mathrm{VaR}_{0.99}(L)\}$ overlaps with S_1,

$$W \cap S_1 \neq \emptyset$$

  but not with S_2,

$$W \cap S_2 = \emptyset$$

- Extra-ordinary losses caused by S_1 and S_2 are positive

$$\ell_1 = \mathbb{E}[L \mid S_1] - \mathbb{E}[L] = 2.7, \quad \ell_2 = \mathbb{E}[L \mid S_2] - \mathbb{E}[L] = 0.9$$

- SST aggregation of S_2 leads to a capital increase even though S_2 does not intersect with the shortfall region W

Case Study 2: Scenarios and Shortfall Region

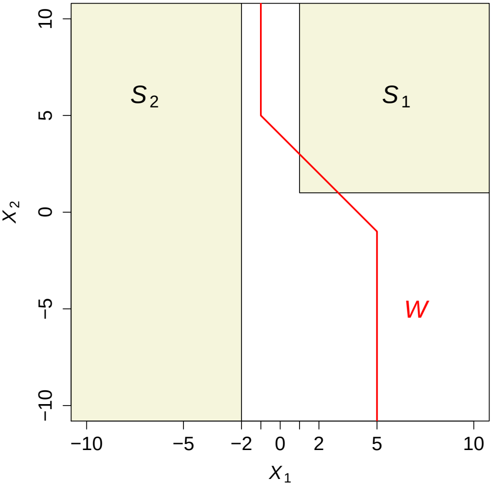

Figure: Scenarios S_1, S_2, shortfall region W

Case Study 2: SST Method

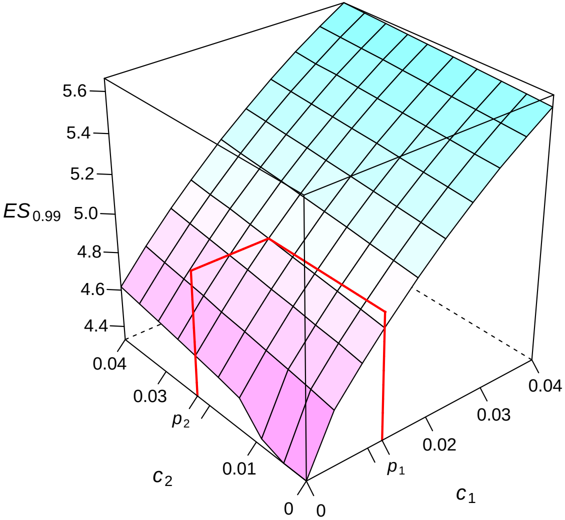

Figure: Sensitivity of $\mathrm{ES}^{\mathrm{aggr}}[L]$ with respect to c_1 and c_2

Case Study 2: Minimum Entropy Method

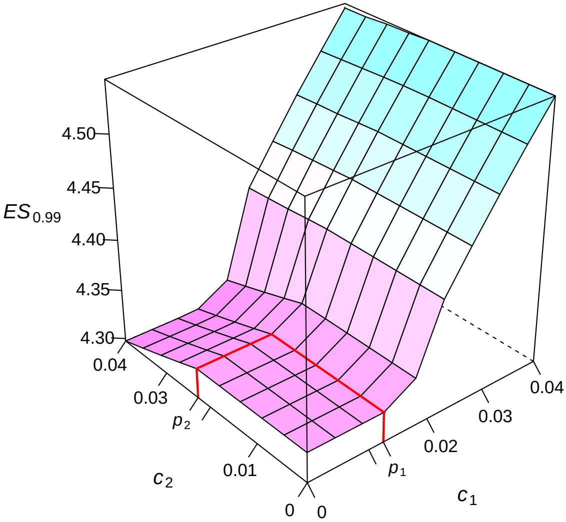

Figure: Sensitivity of $\mathrm{ES}_{\mathbb{Q}^*}[L]$ with respect to c_1 and c_2

Case Study 2: Difference SST - Minimum Entropy Method

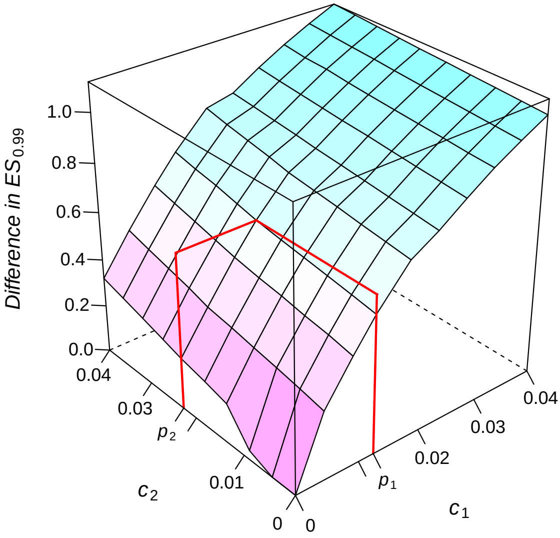

Figure: $\mathrm{ES}^{\mathrm{aggr}}[L] - \mathrm{ES}_{\mathbb{Q}^*}[L]$ as function of c_1 and c_2

Conclusion

- Risk-based market-consistent solvency regime comes along with intended and unintended consequences
- Scenario aggregation is vital part of risk-based solvency regulation
- Current SST method subject to critical review
- Minimum ϕ-divergence approach is a coherent scenario aggregation method:
  - No penalty for conservative internal models
  - Focus on tail loss events
  - Control over distance from internal model
  - Robustness of capital requirement
  - Highly tractable (closed form solutions sometimes)