Energy Derivatives with Volume Control

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Introduction

- Electricity producers sell their production in open power markets.
- They need hedging instruments to cover both price and volume risk.
- We study financial option contracts, formulated as a stochastic control problem, with a payoff structure of a call option.
- Case I: Maximal and minimal total volume constraint.
- Case II: Maximal total volume constraint and penalty.

Aim

Study the optimal exercise policy under Case I and Case II.
Maximal constraint has been studied in *Benth, F.E., Lempa, J., Nilssen, T. (2010), On Optimal Exercise of Swing Options in Electricity Market.*

Case I

Maximal constraint

- There is a maximum amount of power that can be produced.
- Agreements in the contract.

Minimal constraint

In the contract agreement a certain amount of power is guaranteed to be delivered until maturity.

Consequence:

- The producer have to adapt his production to meet a minimal constraint at maturity.
- He might have to produce power even if prices are low.

Case II

Introduce a penalty if the total produced volume is below the agreement.
Consequence:

- The producer tries to adapt his production to meet the agreements in the contract.
- Have the option to take the penalty if that is more profitable.

Outline

- Define the controlspace.
  This will specify the total volume constraints.
- Define the value of the contract.
  Formulated as a stochastic control problem. It will be an option paying money according to price levels and volume decisions.
- The associated HJB-equation.
  Focus on the optimal exercise policy.
- Properties of the marginal value.
  From the HJB-equation, the marginal value plays a key role in the optimal exercise policy.

Controlspace

Define the cumulative control $Z(t)$ for $t \in [0, T]$ as

$$Z(t) = \int_0^t u(s)ds, \tag{1}$$

where u is bounded and progressively measurable w.r.t the filtration generated by the underlyning price process X. Z represents the total volume and we can think of u as the production rate.

Admissible controls

Define the set $\mathcal{U}_m(t, T)$ of admissible controls $u(s)$ for $s \in [t, T]$ as:

- (1) holds and $u(s) \in [0, \bar{u}]$.
- $Z(T) \leq M$ (Maximal constraint when $M < \bar{u}T$)
- $Z(s) \geq \ell(s) := m - \bar{u}(T - s)$ (Minimal constraint when $m > 0$)

Note that the last condition is always fullfilled for $t \in [0, \tilde{t}]$, $\tilde{t} := T - \frac{m}{\bar{u}}$, since $Z(0) = 0$ and a non-decreasing process.

The stochastic control problem

Define the price process X on $(\Omega, \mathcal{F}, \mathbb{F}, \mathbf{P})$ as a strong solution to

$$dX(s) = \mu(s, X(s))ds + \sigma(s, X(s))dW(s), \quad X(t) = x. \tag{2}$$

We define the value function on $\mathcal{S} := [0, T] \times [\ell(s)\mathbf{1}(s > \tilde{t}), M] \times \mathbf{R}$.

Value function: Case I

$$V(t, z, x) = \sup_{u \in \mathcal{U}_m(t,T)} \mathbf{E}_{txz} \left[\int_t^T e^{-r(s-t)}(X_s - K)u_s ds \right] \tag{3}$$

Value function: Case II

$$V(t, z, x) = \sup_{u \in \mathcal{U}_0(t,T)} \mathbf{E}_{txz} \left[\int_t^T e^{-r(s-t)}(X_s - K)u_s ds + g(Z(T)) \right] \tag{4}$$

Interpretation

In both cases we have an option paying out the accumulated value of the difference between the price and the strike K. This payment is scaled by the production rate. Thus, an option paying money according to price levels and volume decisions.

Case II

- Increase the set of admissible controls to $\mathcal{U}_0(t, T)$.
- There is no restriction on the set of admissible controls such that we are guaranteed to have produced a minimal volume of m at maturity.
- Interpret the terminal cost function g as a penalty function if $Z(T) < m$. Define

$$g(Z(T)) := \alpha(m - Z(T))^+, \quad \alpha < 0. \tag{5}$$

- We can think think of m as an "indirect" minimal constraint, appearing as a penalty boarder.

The HJB equation

Via dynamic programing and a verification theorem, the solution to the controlproblems is given as a solution to the HJB equation

$$V_t(t,z,x)+\frac{1}{2}\sigma^2(t,x)V_{xx}(t,z,x)+\mu(t,x)V_x(t,z,x)-rV(t,z,x) \\ +\sup_u\{u(t)(x-K+V_z(t,z,x))\}=0, \quad (6)$$

with appropriate boundary conditions, which are different for the two cases.

Optimal exercise rule

The boundary $x-K+V_z(t,z,x)$ plays a key role. Define

$$\hat{u}(t)=\begin{cases}\bar{u}, & X(t)-K>-V_z(t,Z(t),X(t)),\\ 0, & X(t)-K\leq -V_z(t,Z(t),X(t)),\end{cases} \quad (7)$$

for all $t\in[0,T]$.

We now investigate the marginal value, $\frac{\partial V}{\partial z}$, for case I and II separately.

Case I: Marginal Value.

Define the time for which the total minimal volume constraint is reached as

$$t_m := \inf\{s \in [t, T] : Z_s \geq m\}. \quad (8)$$

Proposition

Let $0 < m < M < \bar{u}T$, then

$$\frac{\partial V}{\partial z} \geq 0 \quad \text{for} \quad t \in [0, t_m)$$
$$\frac{\partial V}{\partial z} \leq 0 \quad \text{for} \quad t \in [t_m, T]$$

Case I: Interpretation

Recall the optimal exercise rule $X(t) - K > -V_z(t, Z(t), X(t))$.

If $z < m$:

- The usage of the option increase its value.
- Since the marginal value is non-negative, it is optimal to exercise even for a non-positvie payoff.

If $z \geq m$:

- The usage of the option will lower its value.
- Since the marginal value is non-positive, it is optimal to exercise only for non-negative payoff.

If $m = 0$, i.e we have no minimal constraint, then the marginal value is non-positive.

The minimal constraint advance the optimal exercise of the option.

For the case $m \equiv 0$ it has been shown that the introduction of a maximal constraint postpone the optimal exercise.

Case II: Marginal Value.

Recall:

$$V(t,z,x) = \sup_{u \in \mathcal{U}_0(t,T)} \mathbf{E}_{txz} \left[\int_t^T e^{-r(s-t)}(X_s - K)u_s ds + \alpha(m - Z(T))^+ \right]. \tag{9}$$

Proposition

For $M < \bar{u}T$

$$\frac{\partial V}{\partial z}(t,z,x) \leq -\alpha \mathbf{1}(m - z > 0) \tag{10}$$

Case II: Interpretation

Suppose we start below the penalty boarder, i.e $\frac{\partial V}{\partial z}(t, z, x) \leq -\alpha$.

- Due to the penalty, the optimal exercise may be advanced.
- The producer may exercise for a negative payoff to avoid penalty, or choose to take a penalty (If that is more profitable).

Suppose we know that $Z(T) < m$: Then it can be shown that $\frac{\partial V}{\partial z}(t, z, x) = -\alpha$. I.e, it is optimal to exercise as long as the payoff is greater than $\alpha < 0$.

Summary

- Given a mathematical formulation for contracts, to hedge for price and volume risk. This is particulary usefull in the powermarket.
- Deduced that the marginal value plays a key role in finding the optimal exercise strategy.
- The introduction of a total maximal volume constraint postpone the optimal exercise policy.
- The introduction of a total minimal volume constraint or a penalty advance the optimal exercise policy.

Some References

- Pham, H. (2009) Continuous-time Stochastic Control and Optimization with Finacial Applications, Springer.
- Fleming, W.H., Soner, H.M. (2006) Controlled Markov Processes and Viscosity Solutions, Springer.
- Benth, F.E., Lempa, J., Nilssen, T. (2010), On Optimal Exercise of Swing Options in Electricity Market.
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