

BROWNIAN COUPLINGS AND APPLICATIONS

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The technique of coupling is a useful probabilistic tool for obtaining various estimates or comparisons of certain quantities associated with the processes involved.

In this talk I will present some of the main couplings of reflecting Brownian motions in smooth domains: synchronous, mirror and scaling coupling of reflecting Brownian motions.

As applications, I will present a resolution of the Hot spots conjecture (Jeffrey Rauch, 1974) in the case of smooth convex domains with symmetries, a unifying proof of Isaac Chavel's conjecture (1986) on the domain monotonicity of the Neumann heat kernel in the case of convex domains satisfying the intermediate ball condition, and a resolution of the Laugesen-Morpurgo conjecture (1998) on the radial monotonicity of the diagonal of the Neumann heat kernel of the unit ball in \mathbb{R}^n .

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