TWO-TERM ASYMPTOTICS FOR LÉVY OPERATORS IN INTERVALS

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The eigenvalues $(-\lambda_n)$ of the Dirichlet Laplace operator Δ in a domain $D \subseteq \mathbf{R}^d$ obey the Weyl law

$$\lambda_n = c_d |D|^{-2/d} n^{2/d} + o(n^{2/d}),$$

with $c_d = (2\pi)^2 / |B(0,1)|^{2/d}$. The second term is known for many smooth domains due to Ivrii [2]:

$$\lambda_n = c_d |D|^{-2/d} n^{2/d} + \tilde{c}_d |D|^{-1-1/d} |\partial D| n^{1/d} + o(n^{1/d}),$$

where $\tilde{c}_d$ is known explicitly.

Blumenthal and Getoor [1] proved that the eigenvalues $(-\lambda_n)$ of the fractional Laplace operator $-(-\Delta)^{\alpha/2}$ in a domain $D \subseteq \mathbf{R}^d$ with Dirichlet exterior condition satisfy the Weyl-type law

$$\lambda_n = c_{d,\alpha} |D| n^{\alpha/d} + o(n^{2/d}),$$

where $c_{d,\alpha} = (2\pi)^\alpha / |B(0,1)|^{\alpha/d}$. However, no Ivrii-type result is known in the fractional case. Due to lack of explicit expressions, even for the interval $D = (-a, a)$ the second term was not known until very recently. With Kamil Kaleta, Tadeusz Kulczycki, Jacek Małecki and Andrzej Stós [3, 4, 5] we proved that in this case

$$\lambda_n = \left(\tfrac{n\pi}{2a} - \tfrac{(2-\alpha)\pi}{8a}\right)^\alpha + O(\tfrac{1}{n}).$$

Generalizations to many other generators of one-dimensional symmetric Lévy processes are possible.

I will begin the talk with a brief review of known results in the area. Next I will discuss the above two-term asymptotic formula and its extensions. Finally, I will show the main ideas behind our results.

References

[1] R. M. Blumenthal, R. K. Getoor, *The asymptotic distribution of the eigenvalues for a class of Markov operators.* Pacific J. Math. 9(2) (1959): 399–408.

[2] V. Ya. Ivrii, *Second term of the spectral asymptotic expansion of the Laplace-Beltrami operator on manifolds with boundary.* Funct. Anal. Appl. 14 (1980): 98–106.

[3] K. Kaleta, M. Kwaśnicki, J. Małecki, *One-dimensional quasi-relativistic particle in the box.* Preprint, arXiv:1110.5887.

[4] M. Kwaśnicki, *Eigenvalues of the fractional Laplace operator in the interval.* J. Funct. Anal. 262(5) (2012): 2379–2402

[5] T. Kulczycki, M. Kwaśnicki, J. Małecki, A. Stós, *Spectral Properties of the Cauchy Process on Half-line and Interval.* Proc. London Math. Soc. 101(2) (2010): 589–622.