EXTINCTION OF FLEMING-VIOT-TYPE PARTICLE SYSTEMS WITH STRONG DRIFT

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For any fixed domain $D \subset \mathbb{R}^d$ and an integer $N \geq 2$, the Fleming-Viot-type process $\mathbf{X}_t = (X_t^1, \ldots, X_t^N)$, $t \geq 0$, driven by a diffusion X, is defined as follows: let $\mathbf{X}_0 = (x_0^1, \ldots, x_0^N) \in D$ and then $X_t^1, \ldots, X_t^N$ move in D as independent copies of X until the time τ_1 when one of them hits the boundary ∂D of D. Then the particle on the boundary dies, and one of the remaining particles splits into two, or in other words the particle that hit the boundary jumps onto one of the remaining particles. Then again all N particles move as independent copies of X up to time τ_2, when one of them hits ∂D, and so on.

If $\{\tau_n,\, n \geq 1\}$ is the sequence of times of successive jumps, then the evolution of $\mathbf{X}_t$ is well defined only up to time

$$\tau_\infty = \lim_{n\to\infty} \tau_n,$$

as there is no natural way of extending this definition beyond τ_∞ and therefore it is of great interest to study the conditions when $\tau < \infty$ or $\tau_\infty = \infty$. Note that the condition $\tau_\infty < \infty$ can be thought of as extinction of the process in finite time. In [3] the authors claimed that for Fleming-Viot process driven by Brownian motion in arbitrary bounded domain $D \subset \mathbb{R}^d$ we have $\tau_\infty = \infty$. However, in [1] it is shown that the proof of the above claim is wrong, and moreover that the claim is true but for all Lipschitz domains $D \subset \mathbb{R}^d$ with Lipschitz constant less than or equal to a constant depending only on d.

The aim of this talk is to show examples when $\tau_\infty < \infty$. More precisely, we give outlines of the proofs of the following two theorems.

Theorem 1. *Let $\mathbf{X}$ be a Fleming-Viot process with N particles on $(0,\infty)$ driven by Bessel process of dimension $\nu \in \mathbb{R}$.*

(i) If $N = 2$ then $\tau_\infty < \infty$, a.s., if and only if $\nu < 0$.
(ii) If $N\nu \geq 2$ then $\tau_\infty = \infty$, a.s.

Consider the following SDE for a diffusion on $(0, 2]$,

$$X_t = x_0 + W_t - \int_0^t \frac{1}{\beta X_t^{\beta-1}}\, ds - L_t, \quad t \leq T_0,$$

where $x_0 \in (0, 2]$, $\beta > 2$, W is Brownian motion, T_0 is the first hitting time of 0 by X, and L_t is the local time of X at 2. We analyze a Fleming-Viot process on $(0, 2]$ driven by the diffusion defined above. The role of the boundary is played by the point 0, i.e. the particles jump only when they reach 0.

Theorem 2. *Fix any $\beta > 2$. For every $N \geq 2$, the N-particle Fleming-Viot process on $(0, 2]$ driven by X, has the property that $\tau_\infty < \infty$, a.s.*

References

[1] M. Bieniek, K. Burdzy, and S. Finch. Non-extinction of a Fleming-Viot particle model. *Probability Theory and Related Fields*, pages 1–40. DOI: 10.1007/s00440-011-0372-5.

[2] M. Bieniek, K. Burdzy, S. Pal. Extinction of Fleming-Viot-type particle systems with strong drift. *Electronic Journal of Probability* 17 (2012), no. 11, 1–15. DOI: 10.1214/EJP.v17-1770

[3] K. Burdzy, R. Hołyst, and P. March. A Fleming-Viot particle representation of the Dirichlet Laplacian. *Comm. Math. Phys.*, 214(3):679–703, 2000.