REVISITING CLARK'S ROBUSTNESS PROBLEM OF NONLINEAR FILTERING

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In the late seventies, Clark pointed out that it would be natural for π_t, the probability measure that is the solution of the stochastic filtering problem, to depend continuously on the observed data $\{Y_s : s \in [0,t]\}$. Indeed, if the signal and the observation noise are independent one can show that, for any suitably chosen test function f, there exists a continuous map θ_t^f defined on the space of continuous paths endowed with the uniform convergence topology such that $\pi_t = \theta_t^f(Y)$ a.s. Unfortunately, for general correlated noise and multidimensional observations such a representation does not hold. By using the theory of rough paths we provide a solution: if the observation is "lifted" to a process $\boldsymbol{Y}$ that includes the Lévy-area process one can show that $\pi_t = \tilde{\theta}_t^f(\boldsymbol{Y})$ a.s. where $\tilde{\theta}_t^f$ is a *continuous* map defined on a suitably chosen space of rough paths. Further, we give a similar approach on the level of the Zakai SPDE by using the theory of viscosity solutions. (Joint work with D.Crisan,J.Diehl,P.Friz).

References

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