THE METHOD OF FEYNMAN FORMULAE FOR APPROXIMATION OF MARKOV EVOLUTION

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We present a new method to investigate and to describe Markov evolution. This method is based on representations of corresponding evolution semigroups (or, what is the same, representations of solutions of the corresponding evolution equations) by Feynman formulae, i.e. by limits if iterated n-fold integrals when n tends to infinity. Sometimes one succeeds to get Feynman formulae containing only integrals of elementary functions. Such Feynman formulae allow to calculate solutions of evolution equations directly, to approximate transition probabilities of underlying stochastic processes, to model stochastic and quantum dynamics numerically.

The limits in Feynman formulae often coincide with some functional integrals with respect to probability measures[1] or Feynman type pseudomeasures. Hence, Feynman formulae provide a tool to establish some new Feynman–Kac formulae, to calculate functional integrals, to find new connections between quantum mechanics and stochastic analysis.

In the talk we present different types of Feynman formulae for Feller semigroups and semigroups generated by stochastic processes in a domain with absorption on the boundary. We show how the method works for additive and multiplicative perturbations of semigroups. We also discuss some Feynman–Kac formulae and phase space Feynman path integrals related to the obtained Feynman formulae.

Some of the results are obtained in collaboration with O.G. Smolyanov, R.L. Schilling and M.G. Grothaus.

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