STOCHASTIC VARIATIONAL INEQUALITIES DRIVEN BY POISSON RANDOM MEASURES

ADRIAN ZĂLINESCU

We consider jump-difussion variational inequalities of the following type:

$$dX_t + \partial\varphi\left(X_t\right)dt \ni b\left(t, X_t\right)dt + \sigma\left(t, X_t\right)dW_t + \int_{\mathbb{R}^d\setminus\{0\}} \gamma\left(t, X_{t-}, z\right)\tilde{N}_t\left(dz\right)dt.$$

Here, $\partial\varphi$ is the subdifferential of a proper, l.s.c., convex function $\varphi : \mathbb{R}^n \to \overline{\mathbb{R}}$, W is a Brownian motion in $\mathbb{R}^d$, and $\tilde{N}$ is the compensated measure of a Poisson random measure N, independent of W.

We show that, under Lipschitz assumptions on the coefficients b, σ and γ and a certain growth condition on φ, there exists a unique strong solution of the above equation. The existence of the solution is proven via a penalization method, by considering the Yosida regularization of φ; the main ingredient for the uniqueness is the monotonicity of the subdifferential operator.

Furthermore, by imposing only the continuity and linear growth of the coefficients, we are able to show that there exists a weak solution, by appealing the martingale problem method of Stroock and Varadhan.

References

[1] I. Asiminoaiei and A. Răşcanu, *Approximation and simulation of stochastic variational inequalities - splitting up method*, Numer. Funct. Anal. and Optimiz. **18** (1997), no. 3&4, 251–282.

[2] E. Cépa, *Problème de Skorohod multivoque*, Ann. Probab. 26, No.2 (1998), 500-532.

[3] J.-L. Menaldi and M. Robin, *Reflected Diffusion Processes with Jumps*, Ann. Probab. Volume 13, Number 2 (1985), 319-341.

[4] A. Zălinescu, *Weak solutions and optimal control for multivalued stochastic differential equations.*, Nonlinear Differ. Equ. Appl. **15** (2008), 511–533.