

On a sufficient condition for large deviations of additive functionals (This is joint work with Zhen-Qing Chen.)

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Preliminaries		results and examples
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Dirichlet form and PCAF		

Define

$$egin{array}{rll} {\cal E}(u,u)&=& \displaystyle{\iint_{{\mathbb R}^d imes {\mathbb R}^dackslash d}}(u(x)-u(y))^2J(x,y)dxdy,\ {\cal F}&=& \overline{C_0^\infty({\mathbb R}^d)}^{\sqrt{{\cal E}_1}}. \end{array}$$

 $(\mathcal{E}, \mathcal{F})$: a regular Dirichlet form. $M = (P_x, X_t)$: the symmetric pure jump Markov (Hunt) process on \mathbb{R}^d generated by $(\mathcal{E}, \mathcal{F})$.

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 μ : a positive smooth measure associated with M. A_t^{μ} : the positive continuous additive functional corresponding with μ :

For any positive Borel function f on \mathbb{R}^d and $\gamma\text{-excessive}$ function h $(\gamma\geq 0)$,

$$\lim_{t
ightarrow 0}rac{1}{t}E_{h\cdot m}\left(\int_{0}^{t}f(X_{s})dA_{s}^{\mu}
ight)=\int_{\mathbb{R}^{d}}f(x)h(x)\mu(dx)$$

(Revuz correspondence)

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spectral function		

We define a spectral function by

$$egin{array}{rl} C(heta)&=&-\infiggl\{ \mathcal{E}(u,u)- heta\int_{\mathbb{R}^d}u^2d\mu\ :u\in\mathcal{F},\ \int_{\mathbb{R}^d}u^2dm=1iggr\}. \end{array}$$

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 $-C(\theta)$ is the bottom of the spectrum of $H_0 + \theta \mu$, where H_0 is the generator of M.

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$$\begin{array}{c|c} \label{eq:product} \end{picture} \begin{tabular}{|c|c|c|} \hline \end{picture} \en$$

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Preliminaries
our aim
our aim
Theorem (Large deviation for
$$A_t^{\mu}$$
)
(1) (lower bound) For any open set $G \subset \mathbb{R}$,
 $\lim_{t\to\infty} \frac{1}{t} \log P_x \left(\frac{A_t^{\mu}}{t} \in G\right) \ge -\inf_{\lambda \in G} I(\lambda)$.
(11) (upper bound) For any closed set $F \subset \mathbb{R}$,
 $\lim_{t\to\infty} \frac{1}{t} \log P_x \left(\frac{A_t^{\mu}}{t} \in F\right) \le -\inf_{\lambda \in F} I(\lambda)$,
where $I(\lambda) = \sup_{\theta \in \mathbb{R}} (\theta \lambda - C(\theta))$.

What are conditions to satisfy the above theorem?

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The assumptions of \boldsymbol{M}

- (I) M is irreducible conservative and transient.
- (II) M has doubly Feller property.
- (III) P_t is bounded from $L^2(\mathbb{R}^d)$ to $L^{\infty}(\mathbb{R}^d)$.
- (IV) The embedding $(\mathcal{F}_e, \mathcal{E}) \to L^2(\mu)$ is compact.
- (\vee) The Harnack inequality for M holds.
- (VI) G(x,0) is not in $L^2(\mathbb{R}^d)$, where G(x,y) is the Green function of M.

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The assumption of μ

For any $\epsilon > 0$, there is a Borel subset $K = K(\epsilon)$ of finite μ -measure and a constant $\delta = \delta(\epsilon) > 0$ such that $\sup_{(x,z)\in (\mathbb{R}^d\times \mathbb{R}^d)\backslash d} \int_{K^c} \frac{G(x,y)G(y,z)}{G(x,z)} \mu(dy) \leq \epsilon$ and for all measurable sets $B \subset K$ with $\mu(B) < \delta$, $\sup_{(x,z)\in (\mathbb{R}^d imes \mathbb{R}^d)ackslash d}\int_B rac{G(x,y)G(y,z)}{G(x,z)}\mu(dy)\leq \epsilon.$

We denote the class of above measures by \mathcal{S}_{∞} .

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<u>Theorem 1</u>

Let J(x,y) be the Lévy kernel of M: there are constants $\alpha_0\in(0,2),\ r_0>0$ and $c_0>0$ so that

$$J(x,y) \geq c_0 |x-y|^{-d-lpha_0} \quad ext{for } |x-y| \leq r_0$$

and that

$$\sup_{x\in \mathbb{R}^d}\int_{\mathbb{R}^d\setminus\{x\}}(1\wedge |x-y|^2)J(x,y)dy<\infty.$$

Assumption (IV) (i.e., the compact embedding from \mathcal{F}_e to $L^2(\mu)$) holds for any $\mu \in \mathcal{S}_{\infty}$.

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Suppose that above assumptions hold.

<u>Theorem 2</u> The spectral function $C(\theta)$ is differentiable.

Theorem 3

By the Gärtner-Ellis theorem, the large deviation of A^{μ}_t holds.

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Examples

- 1. M : symmetric stable processes, $lpha < d \leq 2lpha$, $(H_0 = -(-\Delta)^{lpha/2})$
- 2. M : relativistic stable processes, $2 < d \le 4$, $(H_0 = m - (m^{2/lpha} - \Delta)^{lpha/2} \ m > 0)$

3. M : truncated (finite range) stable processes, $2 < d \leq 4$, $(J(x,y) = |x-y|^{-d-lpha} 1_{|x-y| \leq 1})$

$$egin{aligned} \mu(dx) &= V(x) dx & \Longrightarrow & A^{\mu}_t = \int_0^t V(X_s) ds \ \mu(dx) &= \sigma_r(dx) & \Longrightarrow & A^{\mu}_t ext{ is the local time at } \partial B_r. \end{aligned}$$

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Thank you for your attention!

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