On a sufficient condition for large deviations of additive functionals (This is joint work with Zhen-Qing Chen.)

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Define

$$\mathcal{E}(u,u) = \iint_{\mathbb{R}^d\times\mathbb{R}^d\setminus d} (u(x)-u(y))^2 J(x,y)\,dx\,dy,$$

$$\mathcal{F} = \overline{C_0^\infty(\mathbb{R}^d)}^{\sqrt{\mathcal{E}_1}}.$$

$(\mathcal{E},\mathcal{F})$: a regular Dirichlet form.

$M=(P_x, X_t)$: the symmetric pure jump Markov (Hunt) process on $\mathbb{R}^d$ generated by $(\mathcal{E},\mathcal{F})$.

μ : a positive smooth measure associated with M.
A_t^μ : the positive continuous additive functional corresponding with μ:

For any positive Borel function f on $\mathbb{R}^d$ and γ-excessive function h $(\gamma \geq 0)$,

$$\lim_{t\to 0} \frac{1}{t} E_{h\cdot m}\left(\int_0^t f(X_s)dA_s^\mu\right) = \int_{\mathbb{R}^d} f(x)h(x)\mu(dx)$$

(Revuz correspondence)

We define a spectral function by

$$C(\theta) = -\inf\left\{\mathcal{E}(u,u) - \theta\int_{\mathbb{R}^d} u^2 d\mu : u \in \mathcal{F},\ \int_{\mathbb{R}^d} u^2 dm = 1\right\}.$$

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$$\left. : u \in \mathcal{F}, \int_{\mathbb{R}^d} u^2 dm = 1\right\}.$$

$-C(\theta)$ is the bottom of the spectrum of $H_0 + \theta\mu$, where H_0 is the generator of M.

Theorem (Large deviation for A_t^μ)

(I) (lower bound) For any open set $G \subset \mathbb{R}$,

$$\liminf_{t\to\infty} \frac{1}{t} \log P_x\left(\frac{A_t^\mu}{t} \in G\right) \geq -\inf_{\lambda\in G} I(\lambda).$$

(II) (upper bound) For any closed set $F \subset \mathbb{R}$,

$$\limsup_{t\to\infty} \frac{1}{t} \log P_x\left(\frac{A_t^\mu}{t} \in F\right) \leq -\inf_{\lambda\in F} I(\lambda),$$

where $I(\lambda) = \sup_{\theta\in\mathbb{R}}(\theta\lambda - C(\theta))$.

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Problem

What are conditions to satisfy the above theorem?

The assumptions of M

(I) M **is irreducible conservative and transient.**

(II) M **has doubly Feller property.**

(III) P_t **is bounded from** $L^2(\mathbb{R}^d)$ **to** $L^\infty(\mathbb{R}^d)$**.**

(IV) **The embedding** $(\mathcal{F}_e, \mathcal{E}) \to L^2(\mu)$ **is compact.**

(V) **The Harnack inequality for** M **holds.**

(VI) $G(x, 0)$ **is not in** $L^2(\mathbb{R}^d)$**, where** $G(x, y)$ **is the Green function of** M**.**

The assumption of μ

For any $\epsilon > 0$, there is a Borel subset $K = K(\epsilon)$ of finite μ-measure and a constant $\delta = \delta(\epsilon) > 0$ such that

$$\sup_{(x,z)\in(\mathbb{R}^d\times\mathbb{R}^d)\setminus d}\int_{K^c}\frac{G(x,y)G(y,z)}{G(x,z)}\mu(dy)\le\epsilon$$

and for all measurable sets $B\subset K$ with $\mu(B)<\delta$,

$$\sup_{(x,z)\in(\mathbb{R}^d\times\mathbb{R}^d)\setminus d}\int_{B}\frac{G(x,y)G(y,z)}{G(x,z)}\mu(dy)\le\epsilon.$$

We denote the class of above measures by $\mathcal{S}_\infty$.

Theorem 1

Let $J(x,y)$ be the Lévy kernel of M: there are constants $\alpha_0 \in (0,2)$, $r_0 > 0$ and $c_0 > 0$ so that

$$J(x,y) \geq c_0 |x-y|^{-d-\alpha_0} \quad \text{for } |x-y| \leq r_0$$

and that

$$\sup_{x \in \mathbb{R}^d} \int_{\mathbb{R}^d \setminus \{x\}} (1 \wedge |x-y|^2) J(x,y) dy < \infty.$$

Assumption (IV) (i.e., the compact embedding from $\mathcal{F}_e$ to $L^2(\mu)$) holds for any $\mu \in \mathcal{S}_\infty$.

Suppose that above assumptions hold.

Theorem 2

The spectral function $C(\theta)$ is differentiable.

Theorem 3

By the Gärtner-Ellis theorem, the large deviation of A_t^{μ} holds.

Examples

1. M : symmetric stable processes, $\alpha < d \leq 2\alpha$, $(H_0 = -(-\Delta)^{\alpha/2})$
2. M : relativistic stable processes, $2 < d \leq 4$, $(H_0 = m - (m^{2/\alpha} - \Delta)^{\alpha/2} \ m > 0)$
3. M : truncated (finite range) stable processes, $2 < d \leq 4$, $(J(x, y) = |x - y|^{-d-\alpha} 1_{|x-y| \leq 1})$

$$\mu(dx) = V(x)dx \quad \Longrightarrow \quad A_t^\mu = \int_0^t V(X_s)ds$$

$\mu(dx) = \sigma_r(dx) \quad \Longrightarrow \quad A_t^\mu$ is the local time at ∂B_r.

Thank you for your attention!