Normalized solutions to a class of $(2,q)$-Laplacian equations

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Abstract

In this talk, we will disclose the main results contained in a recent paper written jointly with Tao Yang (Zhejiang Normal University, P.R. China).

Motivated by the fact that physicists are often interested in normalized solutions, we will discuss some recent results concerning existence and nonexistence of solutions $(\lambda, u) \in \mathbb{R} \times X$, with $X := H^1(\mathbb{R}^N) \cap D^{1,q}(\mathbb{R}^N)$ to the following $(2,q)$-Laplacian equation in the entire Euclidean space

$$-\Delta u - \Delta_q u = \lambda u + |u|^{p-2}u \quad \text{in } \mathbb{R}^N$$

under the constraint

$$\int_{\mathbb{R}^N} |u|^2 dx = c^2,$$

where $\Delta_q u = div(|\nabla u|^{q-2}\nabla u)$ is the q-Laplacian of u, $c > 0$, $1 < q < N$, $q \neq 2$, $2 < p < \min\{2^*, q^*\}$ and $s^* := sN/(N-s)$ is the critical Sobolev's exponent, for every $1 < s < N$.

Our main theorems cover the mass subcritical, critical, and supercritical cases, in the sense of the critical exponents $2(1+2/N)$, $q(1+2/N)$. In the mass subcritical case, we study a global minimization problem obtaining a ground state solution. While, in the mass critical cases, we prove several nonexistence results by using asymptotic decays of particular externals.

It is known that there are many difficulties in treating the supercritical case because of the unboundedness of the energy functional on the constraint. In order to overcome this obstacle, a suitable Pohožaev constraint approach and a natural stretched functional are taken into account to derive, respectively, a ground state solution and infinitely many radial solutions. Indeed, the radial setting allows us to overtake some restrictions in order to obtain a wider result. Moreover, the different behaviour of the quasilinear q-Laplacian depending on the cases $q < 2$ and $q > 2$ requires a more cumbersome and accurate analysis with respect to the classical Schrödinger equation where $q = 2$. Our proofs rely on variational tools.